Uncertainty Quantification, Decision Theory, and the Economics of Climate Change

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IMSI Conference on the Economic Impacts of Climate Change

December 7, 2022

Based on:

- “Risk, Ambiguity, and Misspecification: Decision Theory, Robust Control, and Statistics,” joint with Sargent
- “Making Decisions under Model Misspecification,” joint with Cerreia-Viglio, Maccheroni, and Marinacci
- “How Should Climate Change Uncertainty Impact Social Valuation and Policy,” (in progress) joint with Barnett and Brock

Statistician’s wisdom

Now it would be very remarkable if any system existing in the real world could be exactly represented by any simple model. However, cunningly chosen parsimonious models often do provide remarkably useful approximations.

George Box (1979)

In what circumstances is a minimax solution reasonable? I suggest that it is reasonable if and only if the least favorable initial distribution is reasonable according to your body of beliefs.

Irving J. Good (1952)

Pretense of knowledge

"Even if true scientists should recognize the limits of studying human behaviour,

as long as the public has expectations, there will be people who

pretend or believe that they can do more to meet popular demand

than what is really in their power."

From Hayek's Nobel Address (1974)

Emphasis is mine.

Decision theories

- Statistical decision theory - Wald, Savage, Ferguson
  - axiomatic - Savage
  - partial ordering - admissibility
  - complete class theory
- Extensions of axiomatic decision theory (within economics) that formalize ambiguity aversion as distinct from risk aversion
  - max-min utility - Gilboa and Schmeidler
  - smooth ambiguity aversion - Klobanoff, Marinacci, and Mukerji
  - variational preferences - Maccheroni, Marinacci and Rustichini
  - objective and subjective rationality - Gilboa, Maccheroni, Marinacci and Schmeidler

Our starting point

Likelihoods and priors are central objects in the statistics literature but are obscure in the economics literature that is motivated by revealed preferences.

Findings

- modifications of Savage-style axiomatic formulations in the economics literature open doors to extending notions of uncertainty beyond risk in ways that make contact with applied econometric challenges
- we distinguish concerns about potential misspecifications of likelihoods from concerns about robustness of alternative priors
- we integrate both likelihood and prior uncertainty into a formal treatment of uncertainty quantification

Anscombe-Aumann

- preferences defined over acts
- act: states $\rightarrow$ probabilities over outcomes

In a static framework, we presume:

- a state is a parameter value that indexes a particular model
- each model induces a probability distribution over outcomes
- a probability distribution over "states" is a prior distribution

Remark: for us a statistical model conditions on the unknown parameter value

Static decision theory

- consider a parameterized model of a random vector with realization w: $\ell(w \mid \theta) d\tau_o(w)$ where

$$\int_W \ell(w \mid \theta) d\tau_o(w) = 1$$

  for $\theta \in \Theta$ and Θ is a parameter space
- Θ can be infinite dimensional
- a prize rule $\gamma(w)$ depicts an uncertain outcome (prize) of a decision
- $\gamma(w)$'s which are restricted to a set of functions indexed by a decision, d,

$$\gamma(w) = \Gamma(d, w) \qquad d \in D$$

Subjective expected utility

Represent preferences over γ using

$$\int_{\Theta} \left[\int_{W} u[\gamma(w)] \ell(w \mid \theta) d\tau_o(w) \right] d\pi_o(\theta)$$

Sets of likelihoods and priors

- likelihoods: Let $m(w \mid \theta) \geq 0$ in $\mathcal{M}$ satisfy

$$\int_W m(w \mid \theta)\ell(w \mid \theta)d\tau_o(w) = 1$$

- priors: Let $n(\theta) \geq 0$ in $\mathcal{N}$ satisfy

$$\int_\Theta n(\theta)d\pi_0(\theta) = 1$$

Use these constructs to explore two forms of misspecification.

Statistical Divergences

Use convex functions ϕ_m and ϕ_p for constructing divergence between probability measures. Each ϕ is a convex function with $\phi(1) = 0$ and $\phi''(1) = 1$ (normalization).

- ▷ For each θ, form statistical divergence

$$\int \phi_m[m(w \mid \theta)]\ell(w \mid \theta)d\tau_o(w) \geq 0.$$

- ▷ For priors over Θ, form

$$\int \phi_p[n(\theta)]d\pi_o(\theta) \geq 0.$$

We often use relative entropy, e.g. $\phi_m(m) = m \log m$.

Prior divergence I

- Let π_o be a baseline prior.
- Consider alternative priors of the form $d\pi(\theta) = n(\theta) d\pi_o(\theta)$ for $n \in \mathcal{N}_o$, a closed and convex subset of $\mathcal{N}$

Represent preferences over γ with:

$$\min_{n \in \mathcal{N}_o} \int_\Theta \left(\int_W u[\gamma(w)] \ell(w \mid \theta) d\tau_o(w) \right) n(\theta) d\pi_o(\theta)$$

A special case of max – min utility.

Prior divergence II

Represent preferences over γ with:

$$\min_{n \in \mathcal{N}} \int_\Theta \left(\int_W u[\gamma(w)] \ell(w \mid \theta) d\tau_o(w) \right) n(\theta) d\pi_o(\theta) \\ + \xi_p \int_\Theta \phi_p[n(\theta)] d\pi_o(\theta)$$

for $\xi_p > 0$. A special case of variational preferences

Remark: with relative entropy divergence, the implied preference ordering agrees with smooth ambiguity preferences but is rationalized in a fundamentally different way

Potential misspecification

- condition on a specific θ
- replace $\ell(w \mid \theta) d\tau_o(w)$ with $m(w \mid \theta)\ell(w \mid \theta) d\tau_o(w)$ and explore consequences.
- rank alternative γ's conditioned on θ by solving:

$$\min_{m \in \mathcal{M}} \int_W \left(u[\gamma(w)] m(w \mid \theta) + \xi_m \phi_m[m(w \mid \theta)] \right) \ell(w \mid \theta) d\tau_o(w)$$

for $\xi_m > 0$.

Remarks:

- a special case of variational preferences
- links to parts fo robust control theory

Robust Bayes with model misspecification, I

Let $\mathcal{N}_o$ be a closed and convex subset of $\mathcal{N}$.

Represent preferences over γ using:

$$\min_{n \in \mathcal{N}_o} \min_{m \in \mathcal{M}} \int_\Theta \left(\int_W u[\gamma(w)] m(w \mid \theta) \ell(w \mid \theta) d\tau_o(w) \right) n(\theta) d\pi_o(\theta) + \xi_m \int_\Theta \left(\int_W \phi_m[m(w \mid \theta)] \ell(w \mid \theta) d\tau_o(w) \right) d\pi(\theta)$$

Remark: The contribution of the divergence is zero whenever $m = 1$ for some $n \in \mathcal{N}_o$.

Robust Bayes with model misspecification, II

Represent preferences over γ with:

$$\min_{n\in\mathcal{N}} \min_{m\in\mathcal{M}} \int_{\Theta} \left(\int_{W} u[\gamma(w)] m(w \mid \theta) \ell(w \mid \theta) d\tau_o(w) \right) n(\theta) d\pi_o(\theta)$$
$$+ \xi_m \int_{\Theta} \left(\int_{W} \phi_m[m(w \mid \theta)] \ell(w \mid \theta) d\tau_o(w) \right) n(\theta) d\pi_o(\theta)$$
$$+ \xi_p \int_{\Theta} \phi_p[n(\theta)] d\pi_o(\theta)$$

Joint divergence over (m, n).

Extending uncertainty quantification

- Entertain prior and likelihood uncertainty subject to the penalization within the setting of a decision problem
- Employ min-max theorem and produce the worst-case likelihoods and priors for alternative penalty parameters
- Vary the penalty parameters to produce a two-dimensional family of worst-case likelihood-prior pairs and corresponding optimal choice of the prize rules
- Assess the plausibility of these likelihood-prior pairs using statistical reasoning or scientific expertise

Observation: the penalty parameters pin down the aversion to uncertainty

Dynamic counterparts

- Anderson, Hansen, and Sargent, *American Economic Review* , 2001
- Epstein and Schneider *Journal of Economic Theory*, 2003
- Maccheroni, Marinacci, and Rustichini *Journal of Economic Theory*, 2006
- Hansen and Miao, *Proceedings of the National Academy of Sciences*, 2018 and *Economic Theory*, 2022
- Hansen and Sargent, *Journal of Economic Theory*, 2022

Dynamics

Use conditional counterparts to the previous analysis

- explore the consequences of misspecifying the Markov transition dynamics - introduce positive martingales as potential changes in probabilities
- explore the consequences of misspecifying the priors/posteriors over alternative parameters
- address dynamic consistency
  - recursive construction of potential conditional probabilities over parameterized models
  - recursive construction of statistical divergences and their set counterpart

In Hansen-Sargent (JET, 2022), we expose and confront the tension between dynamic consistency and admissibility

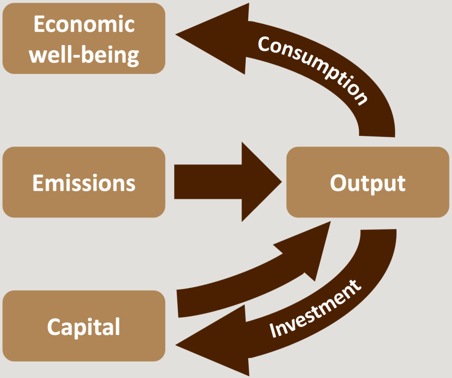
Formal approach

- Entertain uncertainty about probabilities from multiple sources
- Pose formally a stochastic control problem that includes two penalty parameters
  - ambiguity over how to weight alternative models
  - potential model misspecification
- Solve a max-min dynamic decision problem with two outputs:
  - an altered probability specification that isolates the uncertainty components that are most consequential
  - a decision rule that performs well for a range of alternative probability specifications

Climate policy under uncertainty

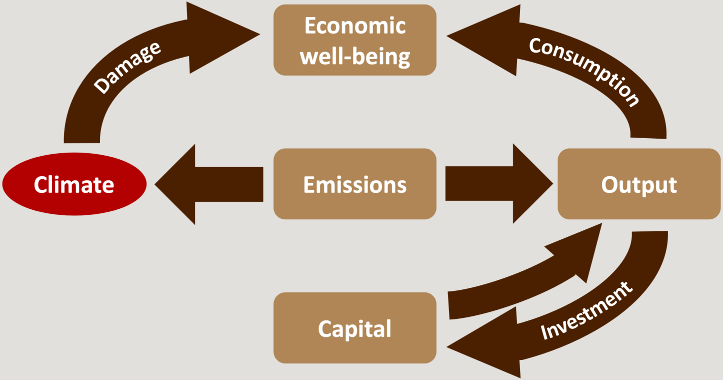
- There are many calls for immediate climate policy implementation.
- Existing limits to our understanding of the timing and magnitude of climate change impacts have led to apprehension by some.
- We study how a decision-maker confronts uncertainty in a setting where:
    - there will be future information about damage severity
    - but the value of further empiricism in the near term is limited
    - research and development can hasten the uncertain discovery a green technology

Modeling Framework
(without climate change)

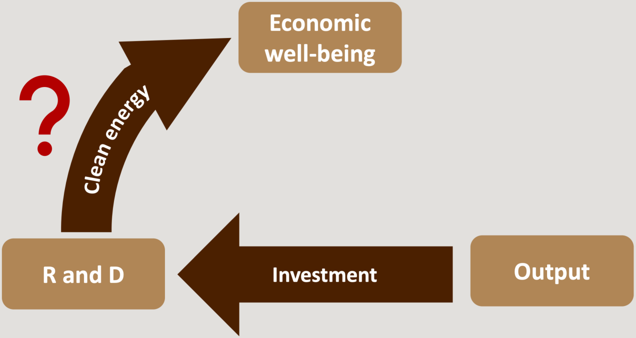


Modeling Framework

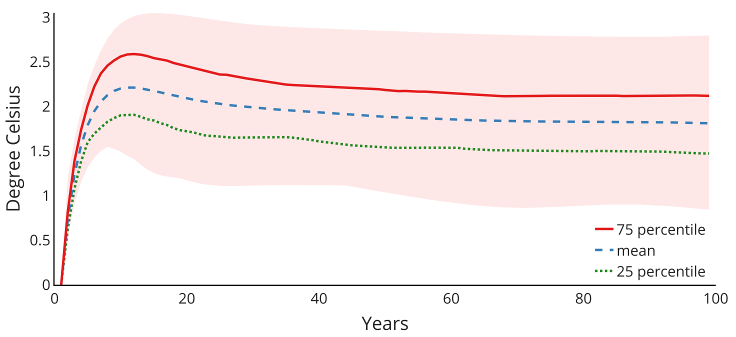
(including climate change)



Modeling Framework
(including research and development)

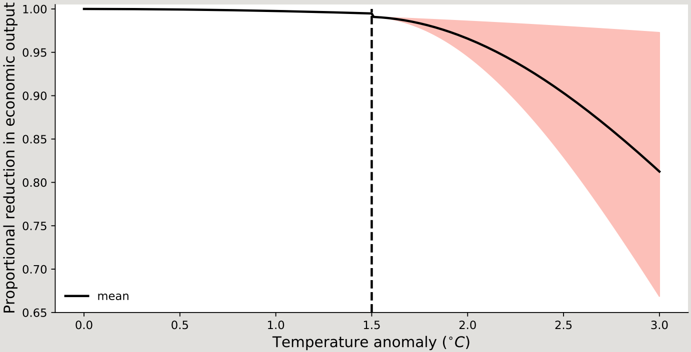


Divergent climate model predictions



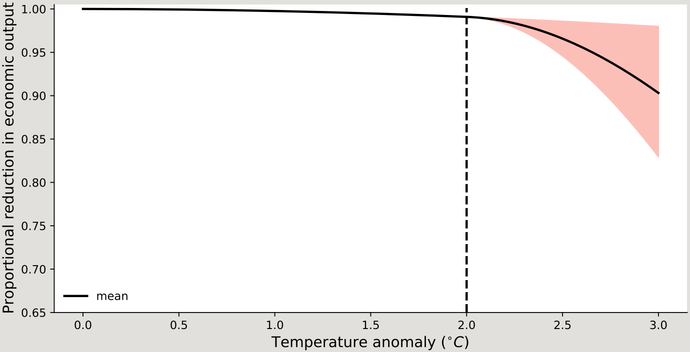
Percentiles for temperature responses to emission impulses. The emission pulse was 100 gigatons of carbon (GtC) spread over the first year. The temperature units for the vertical axis have been multiplied by ten. The boundaries of the shaded regions are the upper and lower envelopes based on 144 models.

A stochastic model of damages



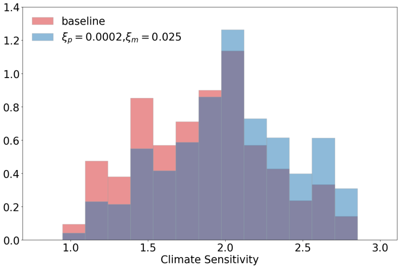
Possible proportional reductions of the productive capacity of the economy. Temperature anomaly threshold is 1.5 degrees celsius.

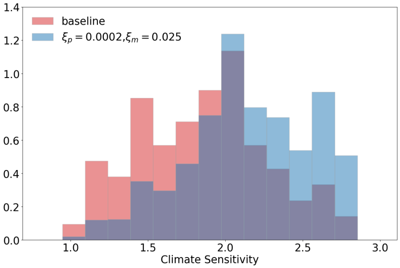
A stochastic model of damages



Possible proportional reductions of the productive capacity of the economy. Temperature anomaly threshold is 2.0 degrees celsius.

Ambiguity-adjusted climate model probabilities





Left plot is for the initial year 2020 and the right plot is for 25 years in the future (2045)

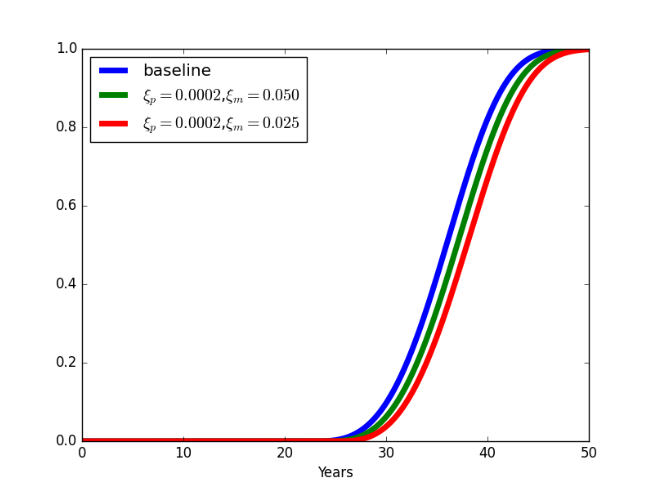
Jump process misspecification

We treat damages as a jump process:

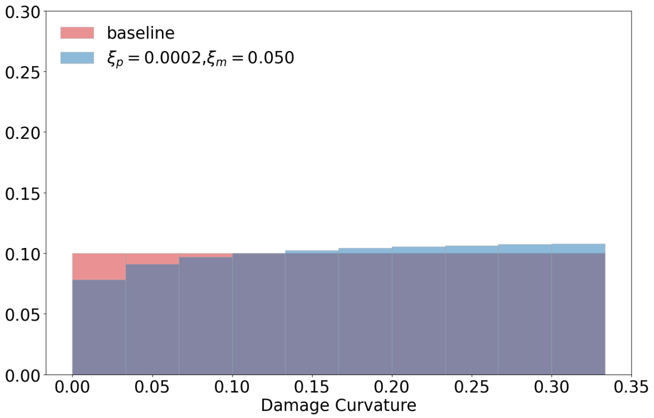
- a temperature dependent probability of a single jump at which point damage curvature is revealed
- a jump distribution that assigns probabilities to each of the possible damage curvature parameters

Misspecification is entertained for both the jump intensity and the jump distribution

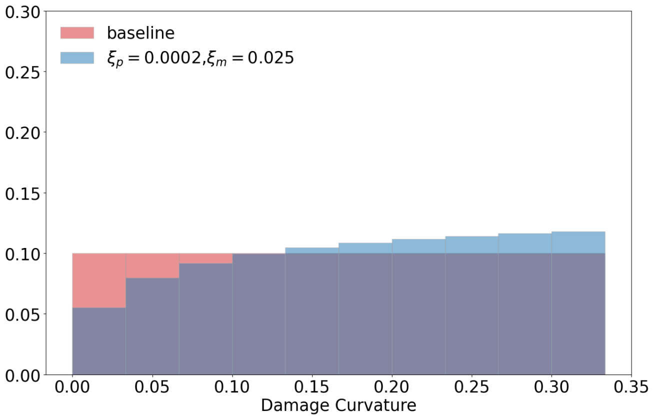
Robust-adjusted jump probabilities for damages



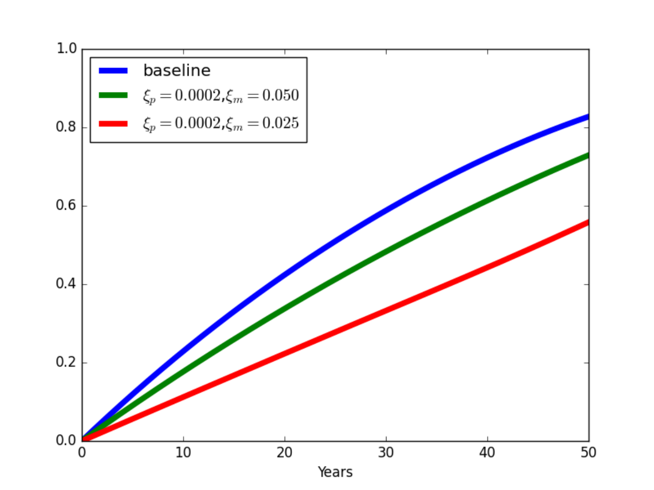
Robust-adjusted damage curvature probabilities I



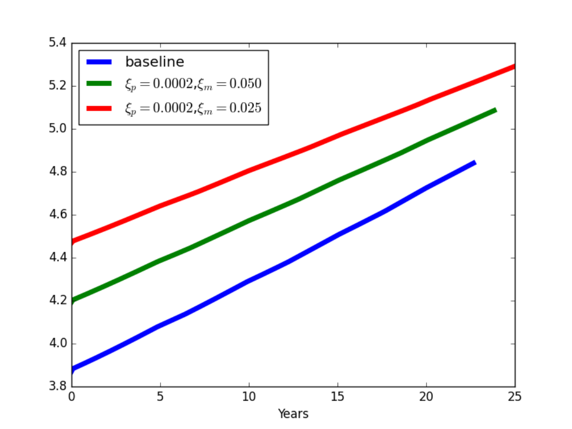
Robust-adjusted damage curvature probabalities II



Robust-adjusted jump probabilities for green technology

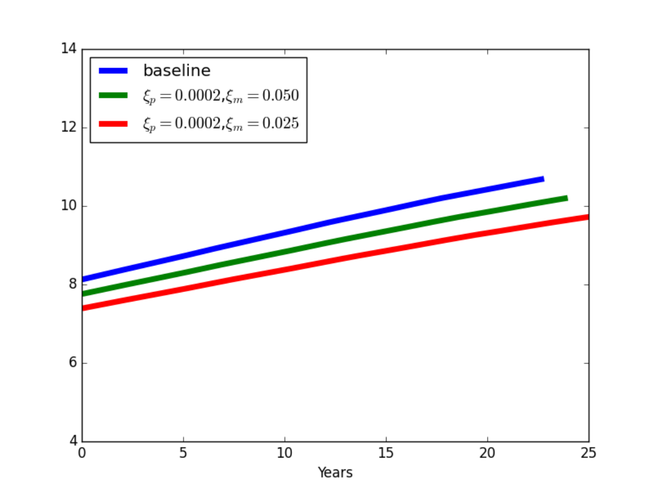


Social cost of carbon (logarithms)



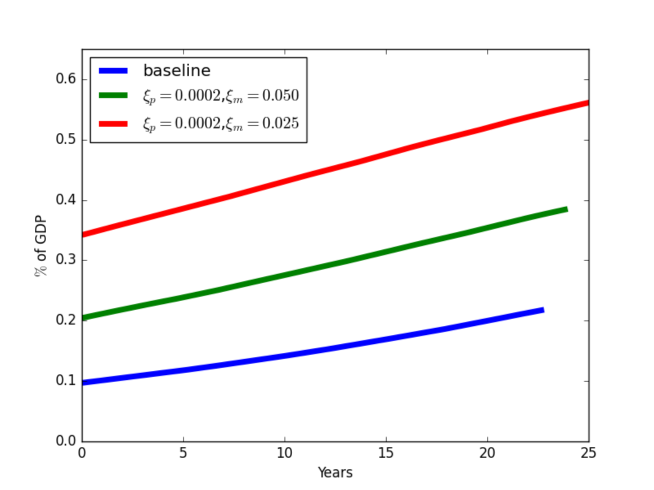
Conditioned on the technology jump not occurring.

Emissions



Conditioned on the technology jump not occurring

R and D investment



Conditioned on the technology jump not occurring

Active research on uncertainty and policy

- investigate uncertain nonlinearity in the carbon/climate dynamics that could produce "tipping point" like behavior
- study the allocation of land to various privately productive activities versus socially valuable carbon sequestration

Concluding remarks

- Uncertainty matters for policy tools like the social cost of carbon government subsidies of green research and development
- Understanding the sources of subjective uncertainty in models used by the private sector and by governments will make economic policy more effective