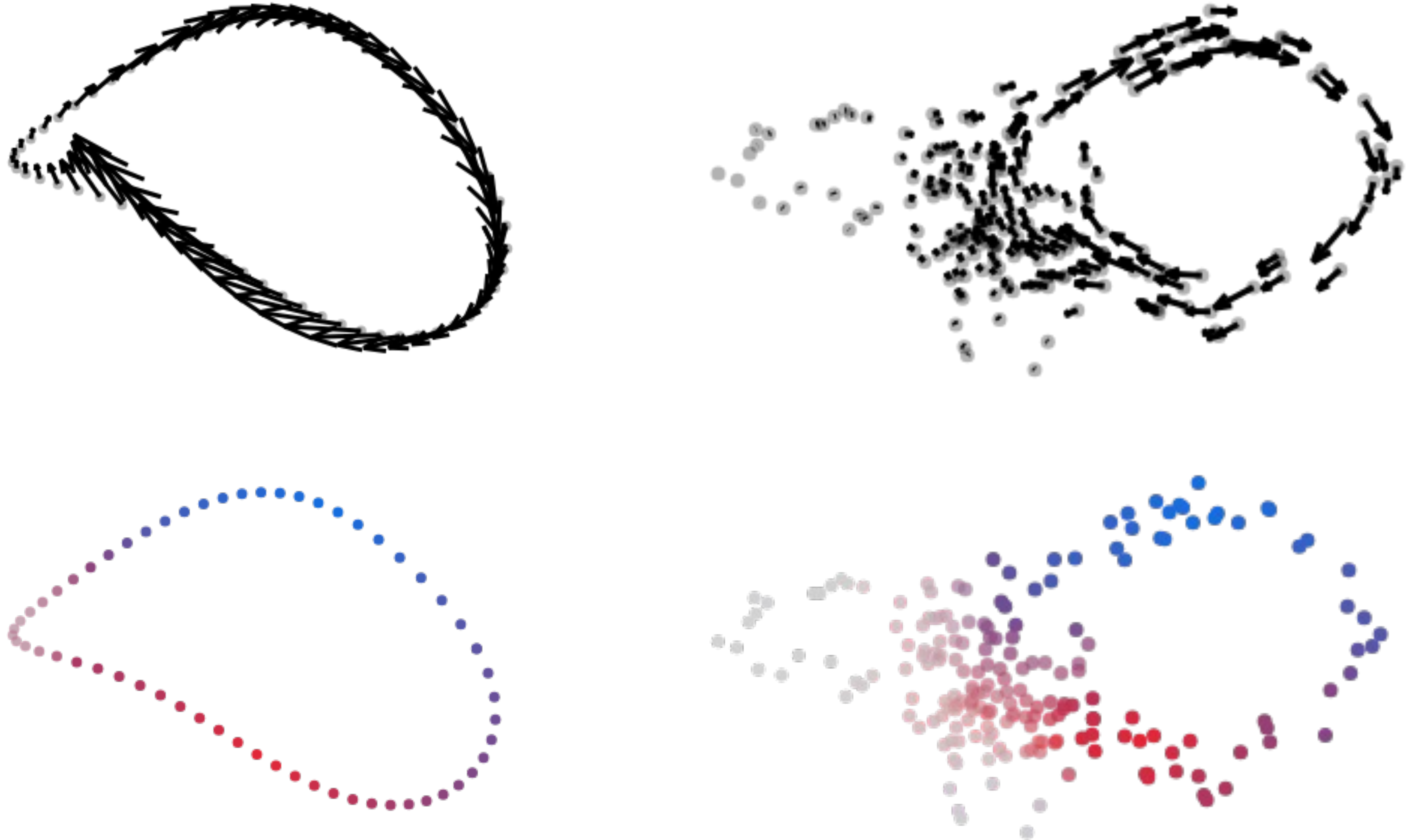
computing diffusion geometry

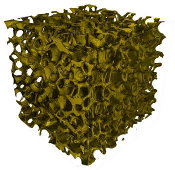
Iolo Jones & David Lanners, Durham University



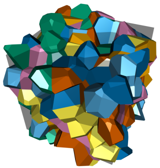
A Persistent Homology Approach for Characterizing Random Spatial Structures

Chiara Fend and Claudia Redenbach

Rheinland-Pfälzische Technische Universität Kaiserslautern Landau

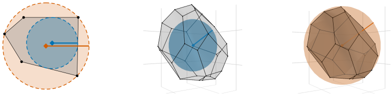


materials data

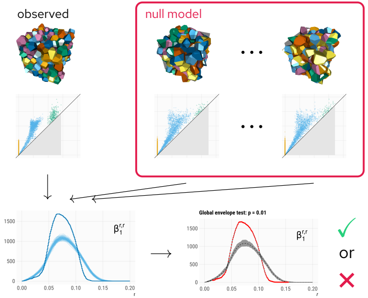


stochastic model

- new **polyhedral complex filtrations** of random tessellations using geometric characteristics



- graphical model validation via **goodness-of-fit testing**
- (functional) CLTs for the **persistent Betti numbers**
- many other interesting stochastic structures

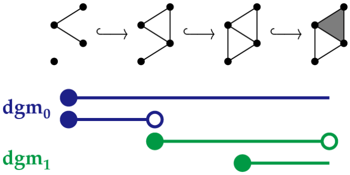


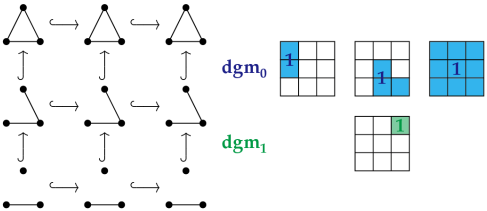
Super-Polynomial Growth of the Generalized Persistence Diagram

Donghan Kim, Woojin Kim, **Wonjun Lee**, Dept. of Mathematical Sciences, KAIST

Persistence Diagram v. Generalized Persistence Diagram (GPD)

: A generalization for the multi-parameter setting





Remark

The number of simplices in a given (1-parameter) filtration bounds the **size of** its persistence diagram.

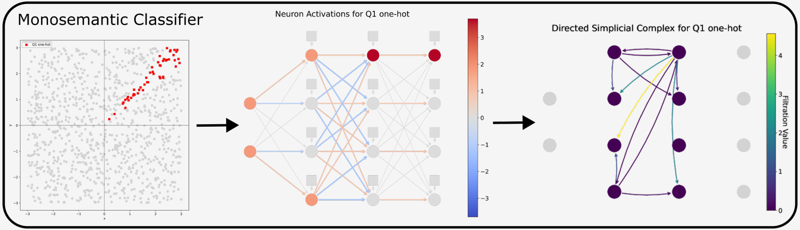
The **size of the GPD/PD** is the cardinality of its support.

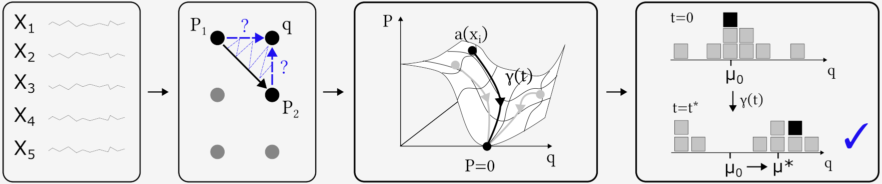
Question

What about multi-parameter filtrations?
Is there an analogy with the 1-parameter setting?

Our answer is **no**.

The Tangled Web they Weave: Exploring Neural Networks with Directed Topology



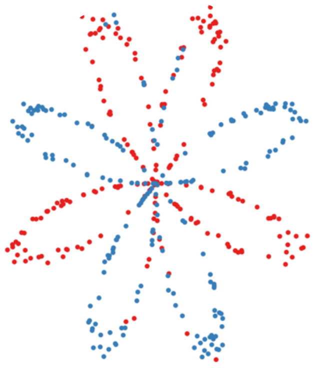


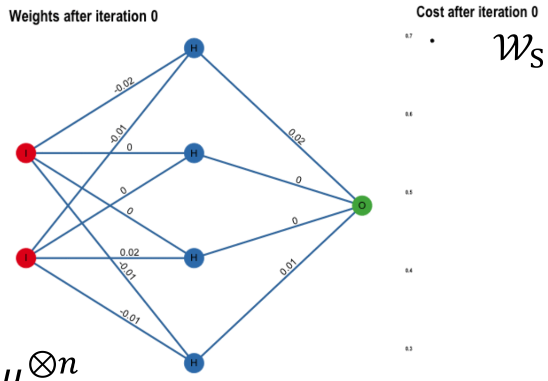
Fix Context **Candidate Simplex** **Coherent Counterfactual Ablation** **Check for Collective Response**

On the Limitations of Fractal Dimension as a Measure of Generalization

Charlie B. Tan, ***Inés García-Redondo***, Qiquan Wang, Michael M. Bronstein and Anthea Monod

Supervised Learning





$\mathcal{W}_S := \{\omega_1, \dots, \omega_n\}$

$S = \{z_1, \dots, z_n\} \sim \mu^{\otimes n}$

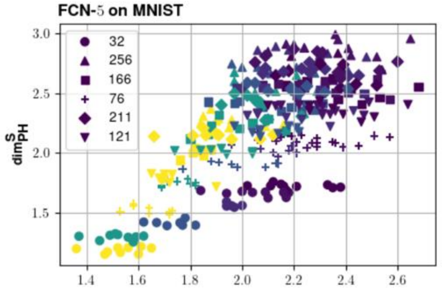
Generalization gap

$\mathcal{G}(\omega) := |training\ cost - test\ cost|$

Linking Generalization and PH

Birdal *et al*. (2021) and Dupuis *et al*. (2023):

There exists a **positive correlation** between the **generalization gap** and the **PH dimension** of the optimization trajectory $\mathcal{W}_S$ near the minimum



Our Contribution

1. **Statistically grounded analysis** of the correlation between PH dimension and the generalization error – hyperparameters have a confounding effect

2. Found **two counterexamples** for the proposed correlation

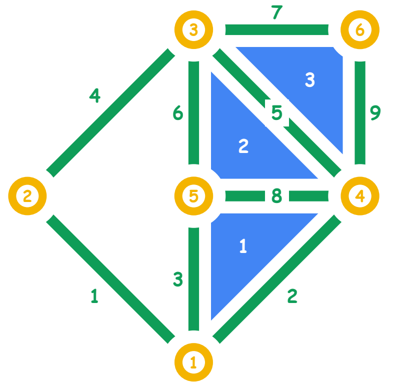
Come to ***poster 8*** *if you want to hear more about this!*

Pruning vineyards: Updating barcodes and representative cycles

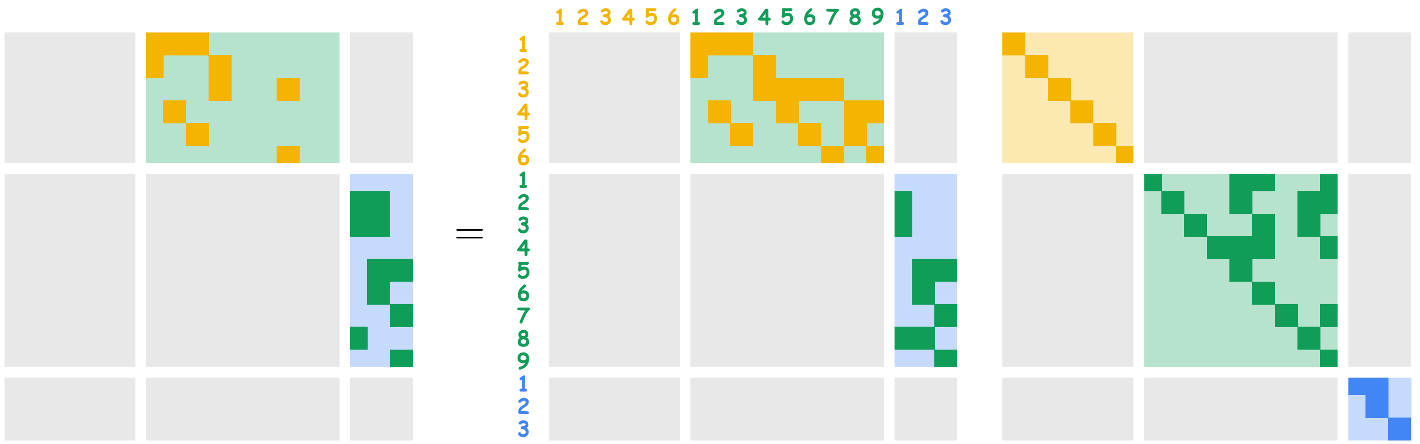
IMSI AATRN / Chicago, IL, USA / August 19, 2025

Barbara Giunti, University at Albany SUNY
Jānis Lazovskis, University of Latvia

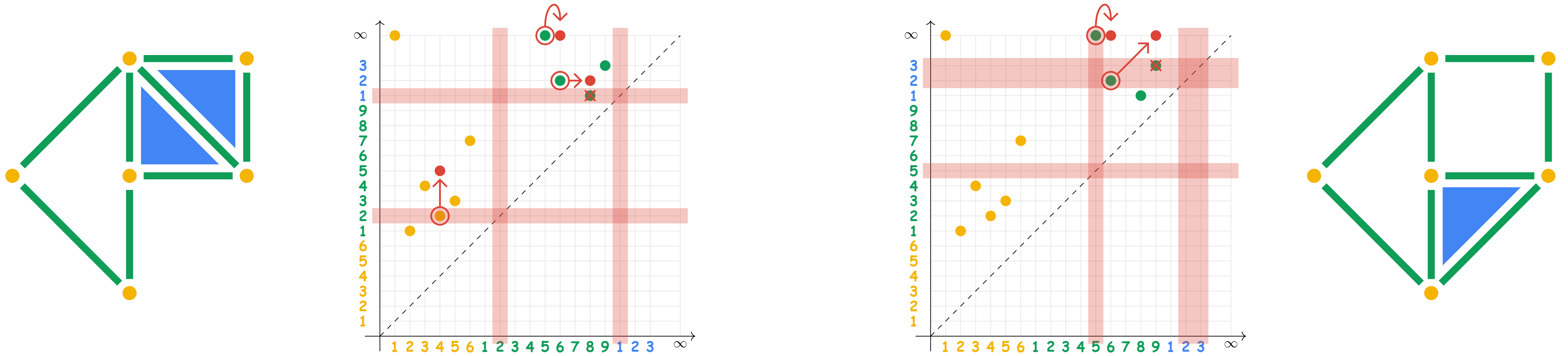
1. **Filter** the topological space



2. **Factor** the boundary matrix



3. **Update** the filtration



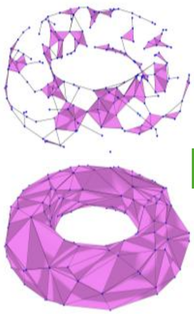
A Survey of Dimension Estimation Methods

James Binnie w/ Paweł Dłotko, John Harvey, Jakub Malinowski, Ka Man Yim

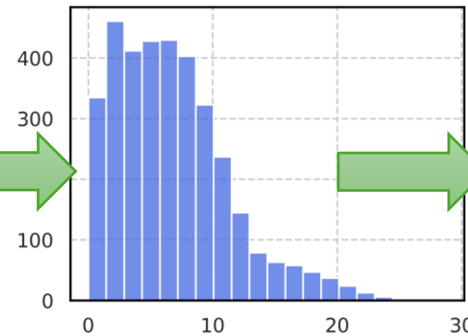
https://arxiv.org/abs/2507.13887v1

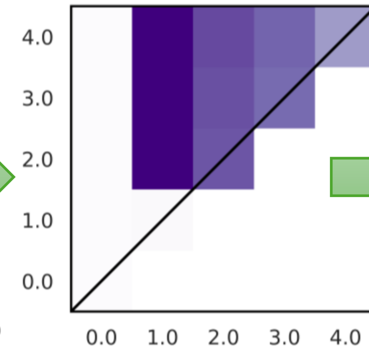
Efficient Computation of Persistent Laplacians

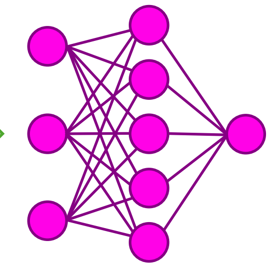
Ben Jones and Guo-Wei Wei



$$\Delta_n^{a,b}$$







Filtration

Persistent Laplacians

Eigenvalues

Summaries

Analysis

Persistent Topological Laplacians (PTLs)

$$\Delta_n^{a,b} : C_n^a \to C_n^a$$

$$\ker \Delta_n^{a,b} \cong H_n^{a,b}(K;\mathbb{R})$$

Generalize:

Graph Laplacian
Combinatorial Laplacian
Persistent Homology*

What are they?

What do they mean?

Are they easy to compute?

Faster via topology

Faster via linear algebra

How can I use them?

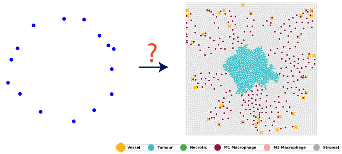
PErsistent
Topological
Laplacian
Software
> pip install petls

Topological graph learning for spatial data from the tumour microenvironment

Jérémy JP Baffou, Vaishnavi Subramanian, Heather Dawson, Inti Zlobec, Dorina Thanou, and Bernadette J Stolz

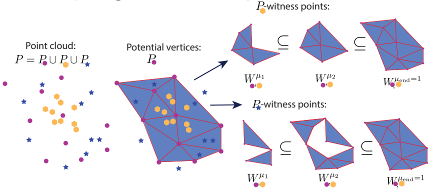
Question

Single-system Multi-system



Method

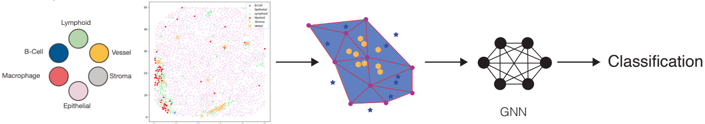
Relational topological data analysis



Stolz *et al.*, Bulletin of Mathematical Biology 86(11), 2024.

Application

Topology encodes spatial architecture in real-world data



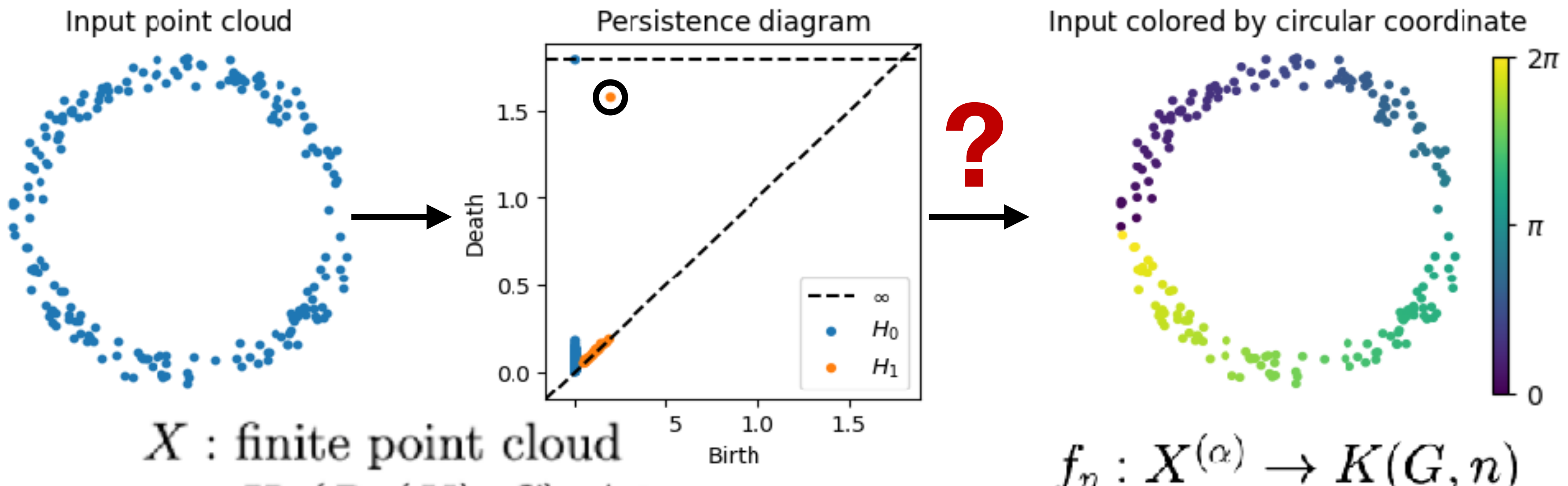
Baffou, [...], Stolz, in preparation, 2025.

MAX PLANCK INSTITUTE OF BIOCHEMISTRY

Extracting Sparse Eilenberg-MacLane Coordinates via Principal Bundles

Tony Xiaochen Xiao (Northeastern University)
Advisor: Jose A. Perea

The Problem



X : finite point cloud
$\eta \in H^n(R_\alpha(X); G) \neq 0$

$f_\eta : X^{(\alpha)} \to K(G, n)$

Examples

Circular coordinates	$G = \mathbb{Z}, n = 1$	$f_\eta : X^{(\alpha)} \to S^1$
Toroidal coordinates	$G = \mathbb{Z}^l, n = 1$	$f_{\eta_1,\ldots,\eta_l} : X^{(\alpha)} \to T^l$
Projective coordinates	$G = \mathbb{Z}_2, n = 1$	$f_\theta : X^{(\alpha)} \to \mathbb{RP}^n$
	$G = \mathbb{Z}, n = 2$	$f_\nu : X^{(\alpha)} \to \mathbb{CP}^n$
Lens coordinates	$G = \mathbb{Z}_q, n = 1$	$f_\mu : X^{(\alpha)} \to S^{2n-1}/(\mathbb{Z}_q)$

Each generator from the persistence diagram gives an "sparse Eilenberg-MacLane Coordinate"

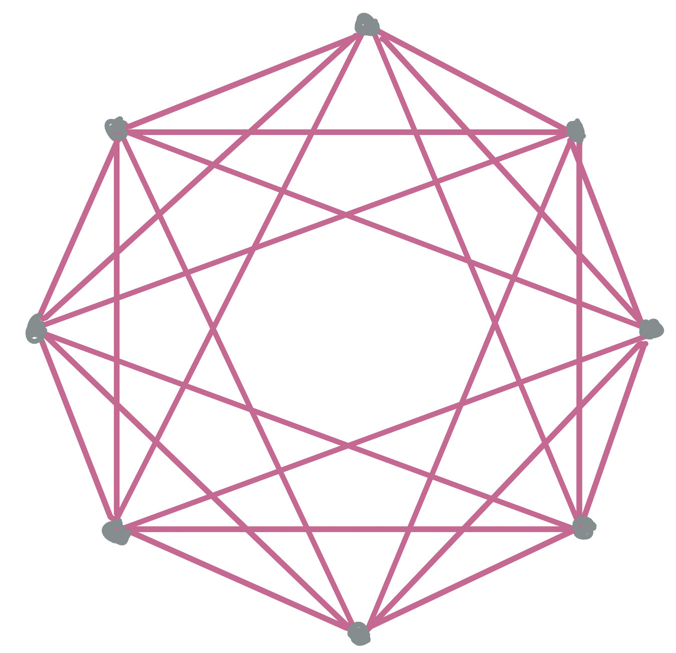
Anti-Vietoris-Rips metric thickenings and Borsuk graphs

Sucharita Mallick, Joint work with Henry Adams, Alex Elchesen and, Michael Moy

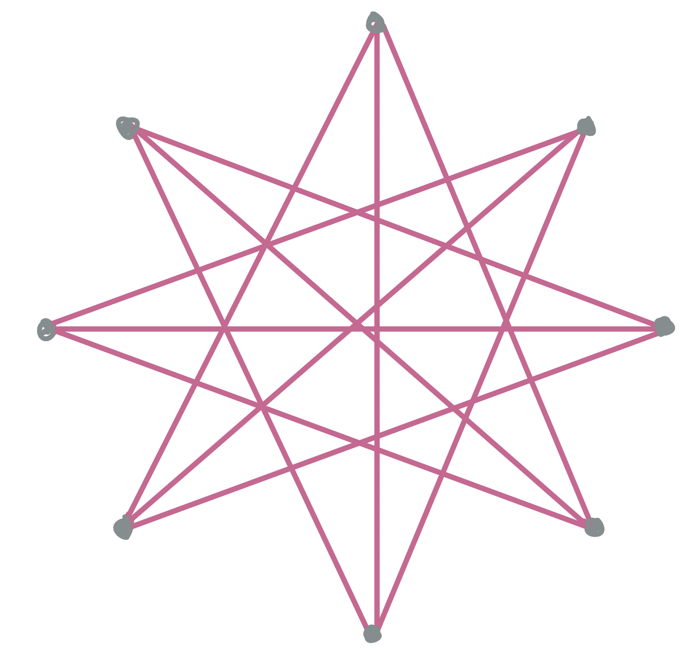
- Vietoris-Rips (VR) complex vs Anti-Vietoris-Rips (Anti-VR) complex.

Vietoris-Rips (VR) complex → Points are close by

Anti-Vietoris-Rips → Points are far apart



$VR(C_8;3)$



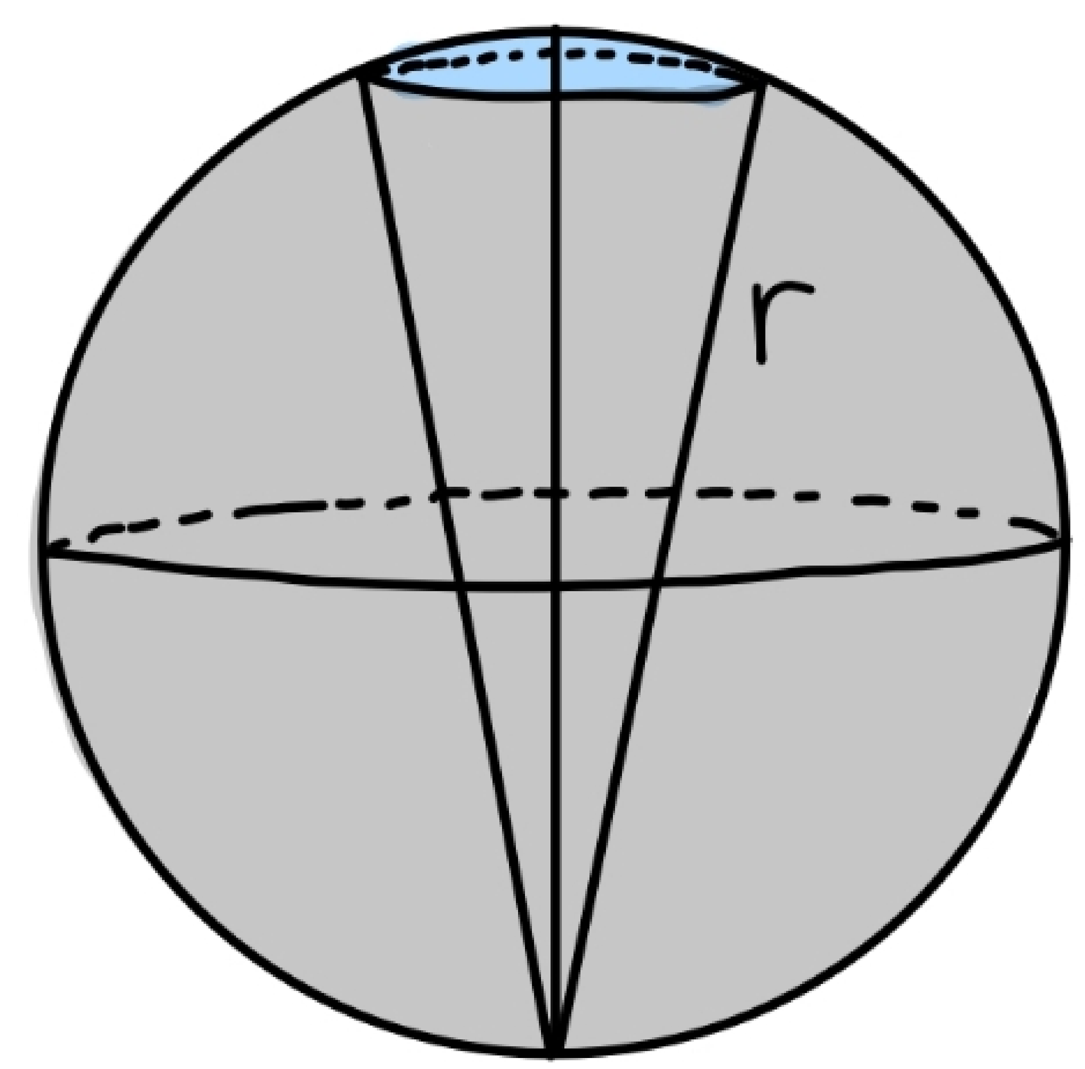
$AVR(C_8;3)$

- Similarity matrix : higher values correspond to similar data points.

VR complex from distance matrix ✗ Anti-VR complex from similarity matrix ✓

- Let S^n be the n-sphere with geodesic metric and diameter π.

- $$AVR^m(S^n;r) = \begin{cases} S^n & , r > \pi \\ \mathbb{RP}^n & , \frac{2\pi}{3} < r \leq \pi \\ * & , r = 0 \end{cases}.$$



S^n

$Bor(S^n;r)$

- Certain homotopy types are used as a topological obstruction to the existence of certain graph homorphisms between Borsuk graphs.

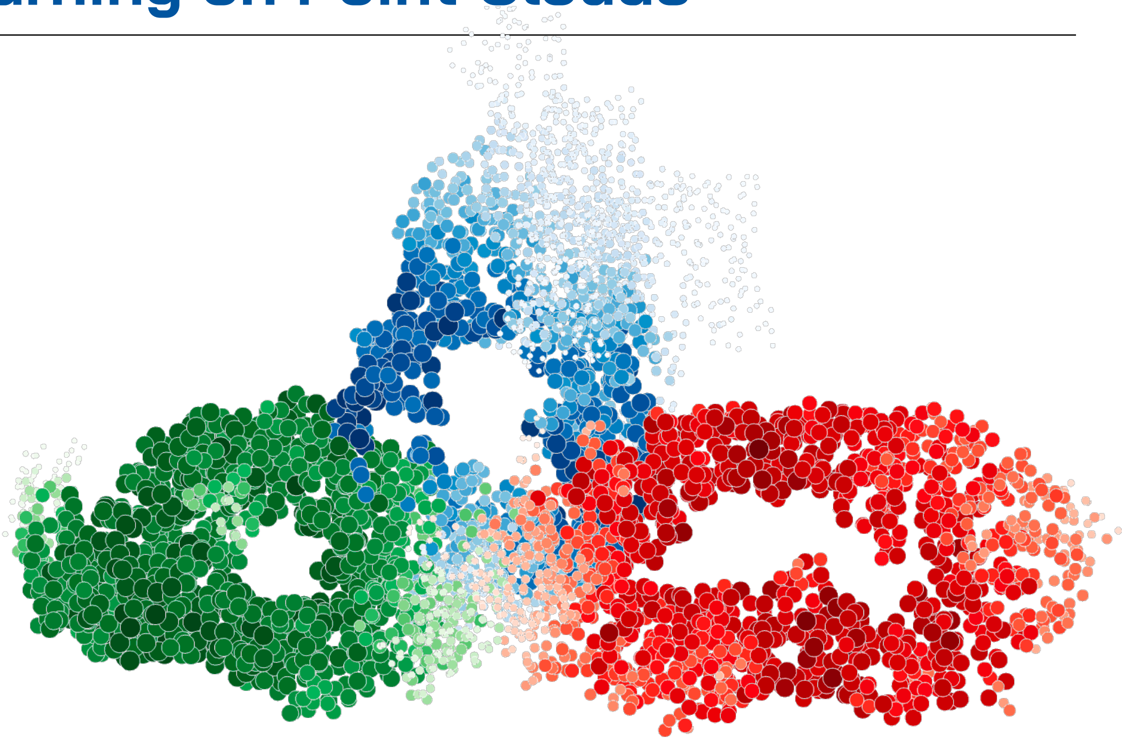
Point-Level Topological Representation Learning on Point Clouds

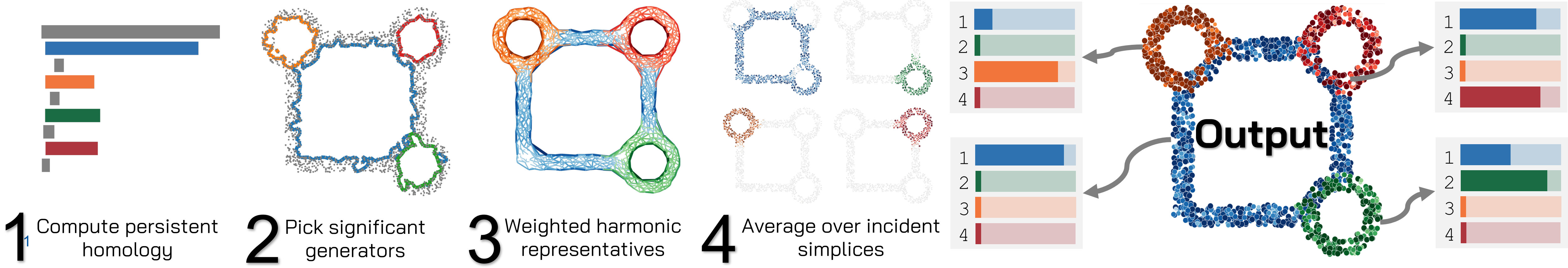
Vincent P. Grande, Michael T. Schaub, ICML 2025

Motivation

- Point cloud descriptors like **persistent homology** provide one powerful **global descriptor** describing the **overall shape** of the point cloud
- However, often we are interested in how **individual points** relate to **global topology**

Our method (TOPF) turns **global topological features** from persistent homology into **local point-level features** using **harmonic representatives**





Latent Space Topology Evolution in Multilayer Perceptrons

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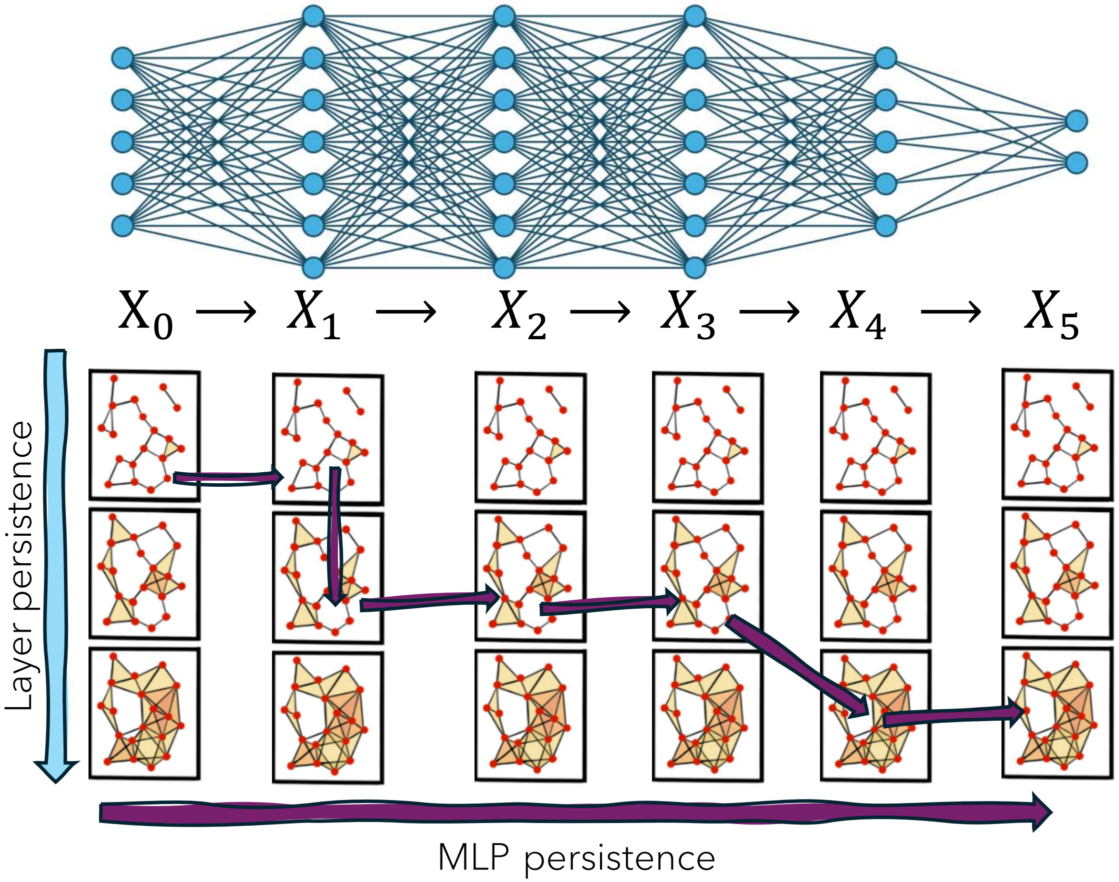


Problem:

MLPs are **black boxes** - we lack tools to understand their internal representations

Approach:

Construct **simplicial towers** (sequences of simplicial complexes + simplicial maps) that capture topological evolution across network layers.

Use cases:

- Identify redundant layers
- Optimize layer widths
- Detect overparameterization
- Visualize decision boundaries
- Track data clusters
- Understand misclassifications

Keywords: Neural network interpretability, pullback covers, Multiscale mapper, Bi-persistence

ArXiv.2506.01569

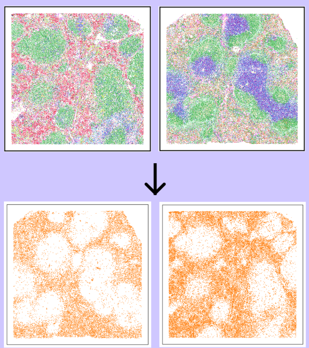
Topology across scales on heterogeneous cell data

Maria Torras-Pérez, Iris H.R. Yoon, Praveen Weeratunga, Ling-Pei Ho, Helen M. Byrne, Ulrike Tillmann, Heather A. Harrington

UNIVERSITY OF OXFORD
Mathematical Institute

Multiplexed images point cloud data

- Lupus murine spleen
- COVID-19 human lungs



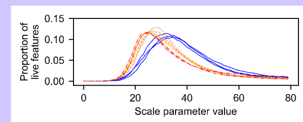
Filtered simplicial complexes

- Alpha complexes
- Witness complexes for large point clouds

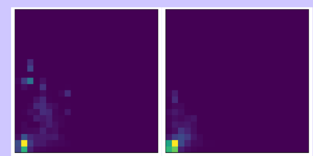
Persistent homology

Vectorisations

- Betti curves



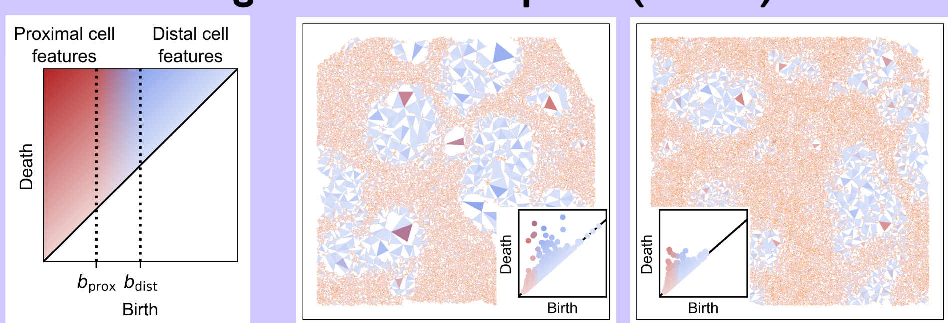
- Persistence images with different weights



- Elementary statistics

Clustering

Persistence Weighted Death Simplices (PWDS) Visualisation



Wasserstein Stability for Barcodes and Persistence Landscapes

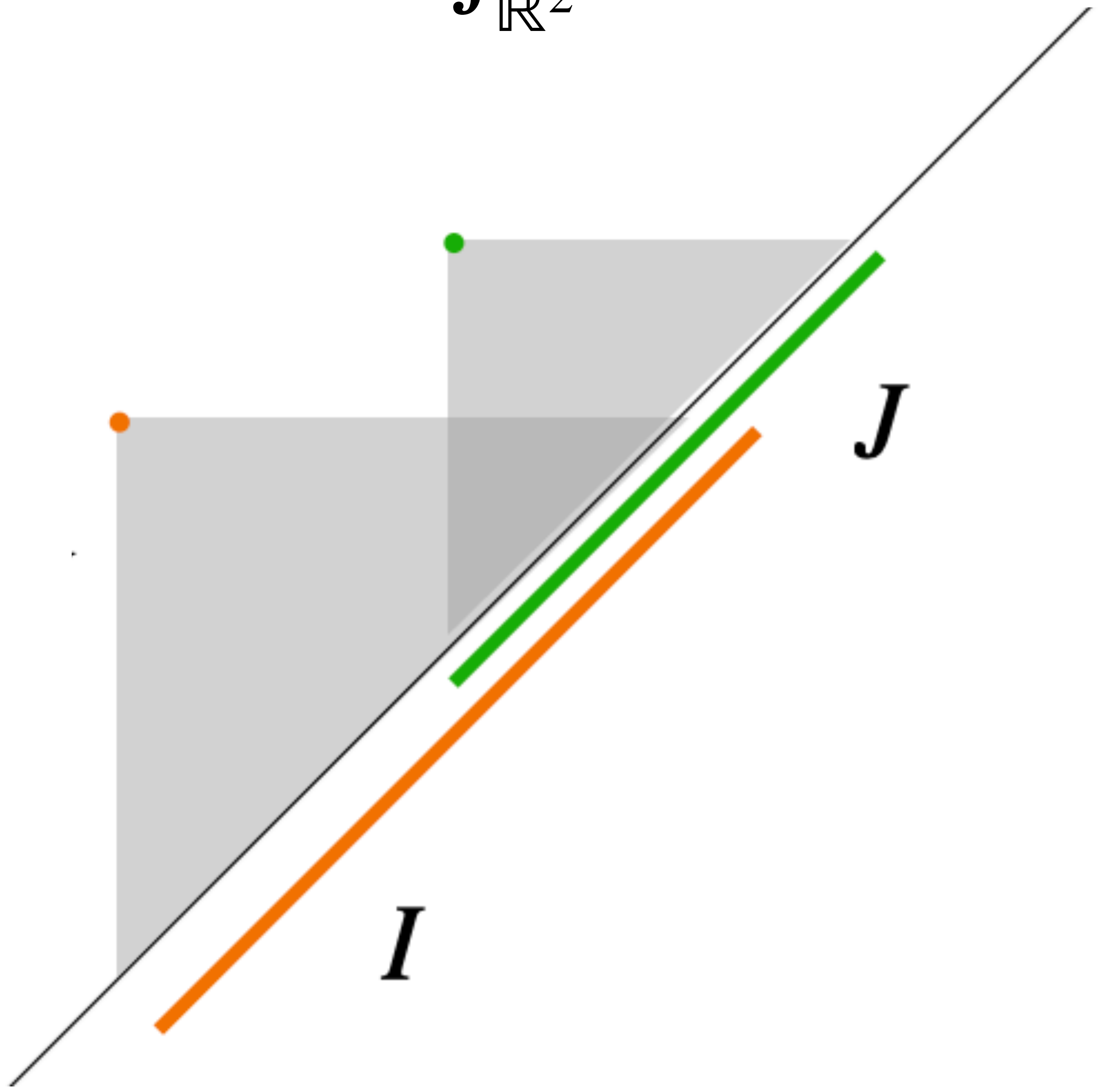
Wanchen Zhao   Advisor: Peter Bubenik

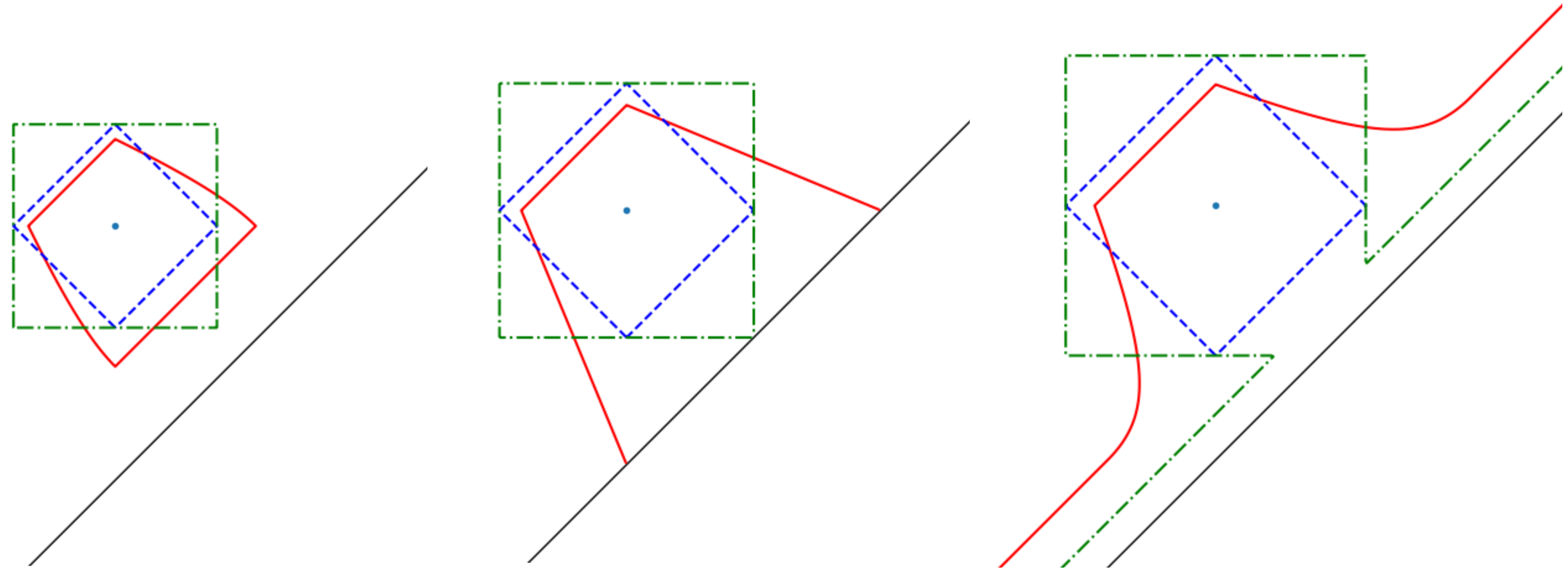
Filtered Chain Complexes → **Barcodes** → **Persistence Landscapes**

$$\|\{d_{\text{rank}}(I_{w(\sigma)}, I_{w'(\sigma)})\}_{\sigma\in K}\|_1 \quad \geq \quad W_1^{\text{rank}}(\text{Dgm(w)}, \text{Dgm(w}')) \quad \geq \quad 2\|\lambda - \lambda'\|_1$$

Metric on the set of interval modules:

$$d_{rank}(I, J) := \int_{\mathbb{R}^2} |\, rank\ I - rank\ J\,|$$





Persistent homology via ellipsoids

Niklas Canova
ETH Zurich

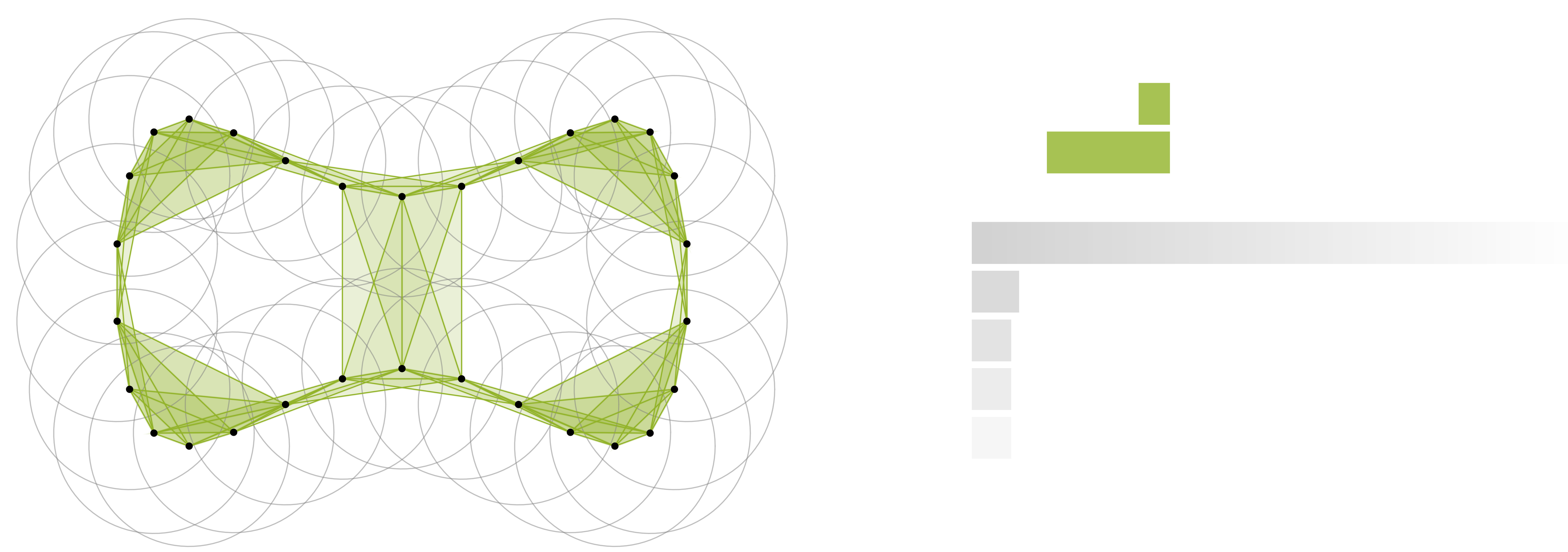
Sara Kališnik
Penn State University

Aaron Moser
ETH Zurich

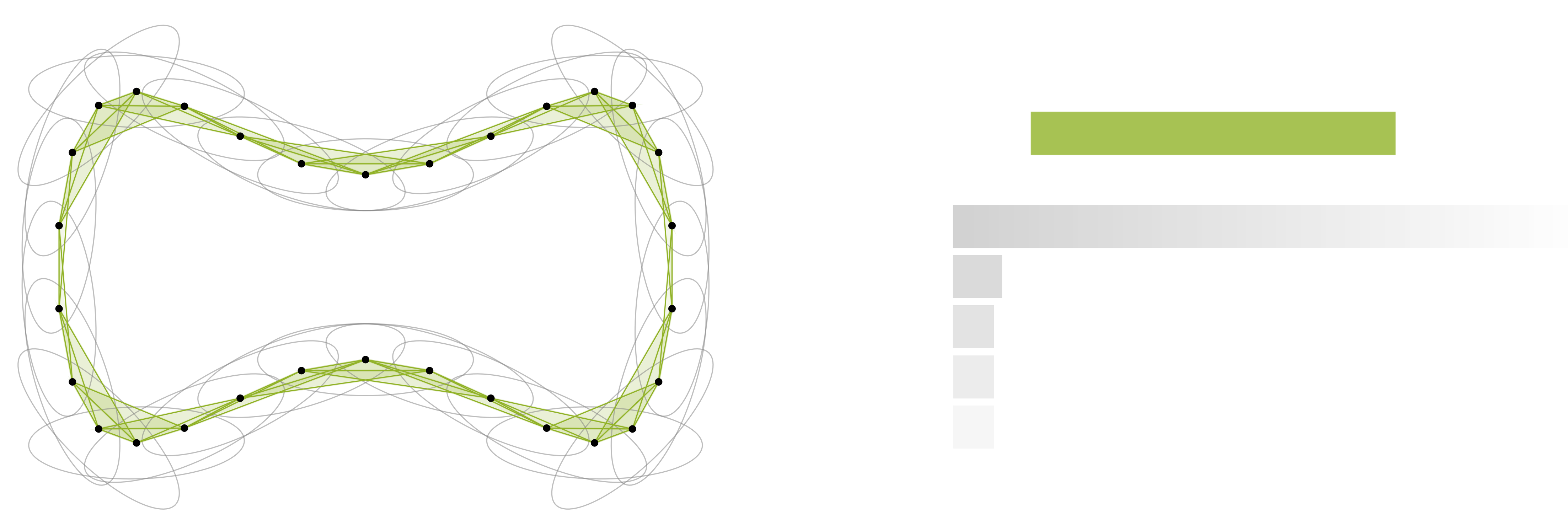
Bastian Rieck
University of Fribourg

Ana Žegarac
ETH Zurich

Rips



Ellipsoids



Better captures homology of the underlying manifold

Poster

- Construction details and properties
- Experimental results