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Deep Kernel Aalen-Johansen Estimator: An Interpretable and Flexible Neural Net Framework for Competing Risks

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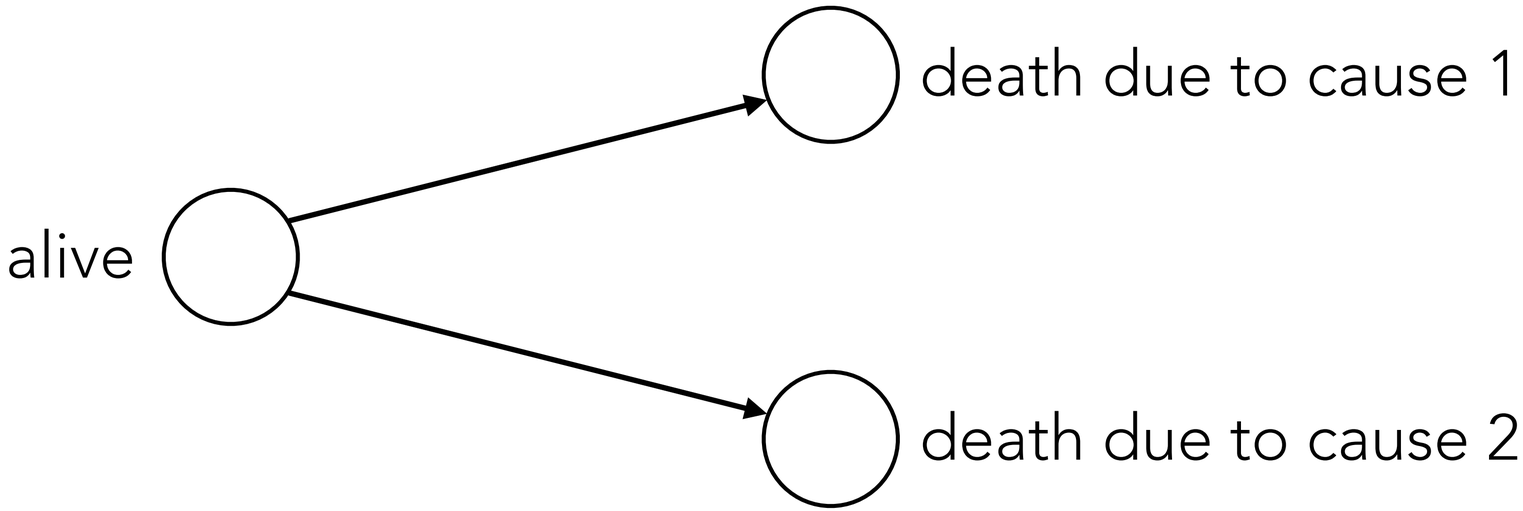

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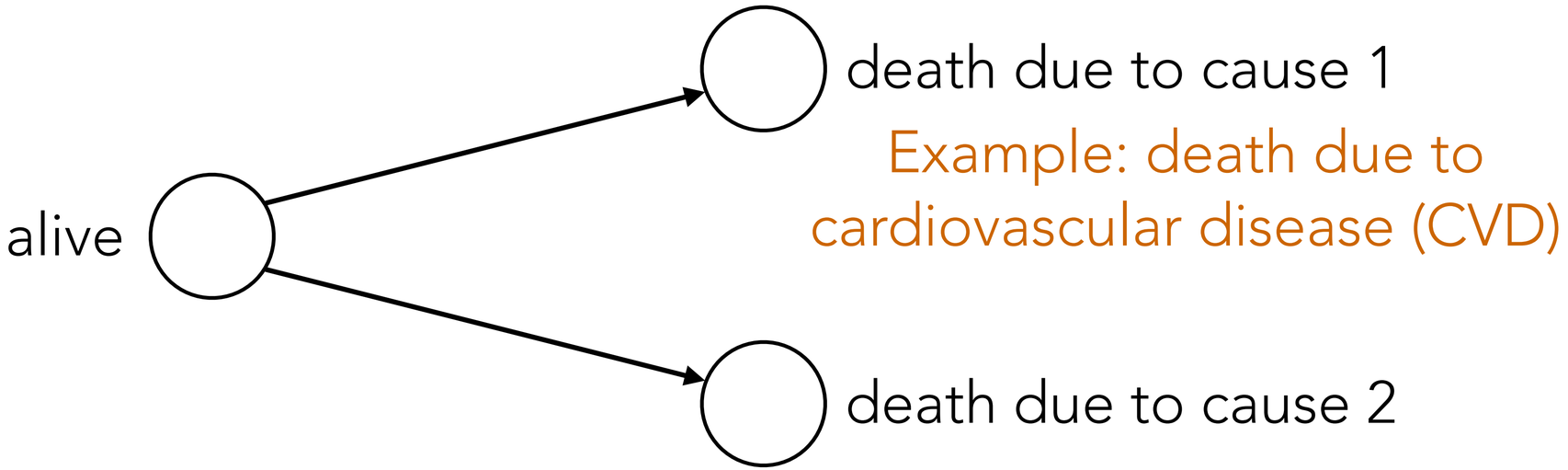

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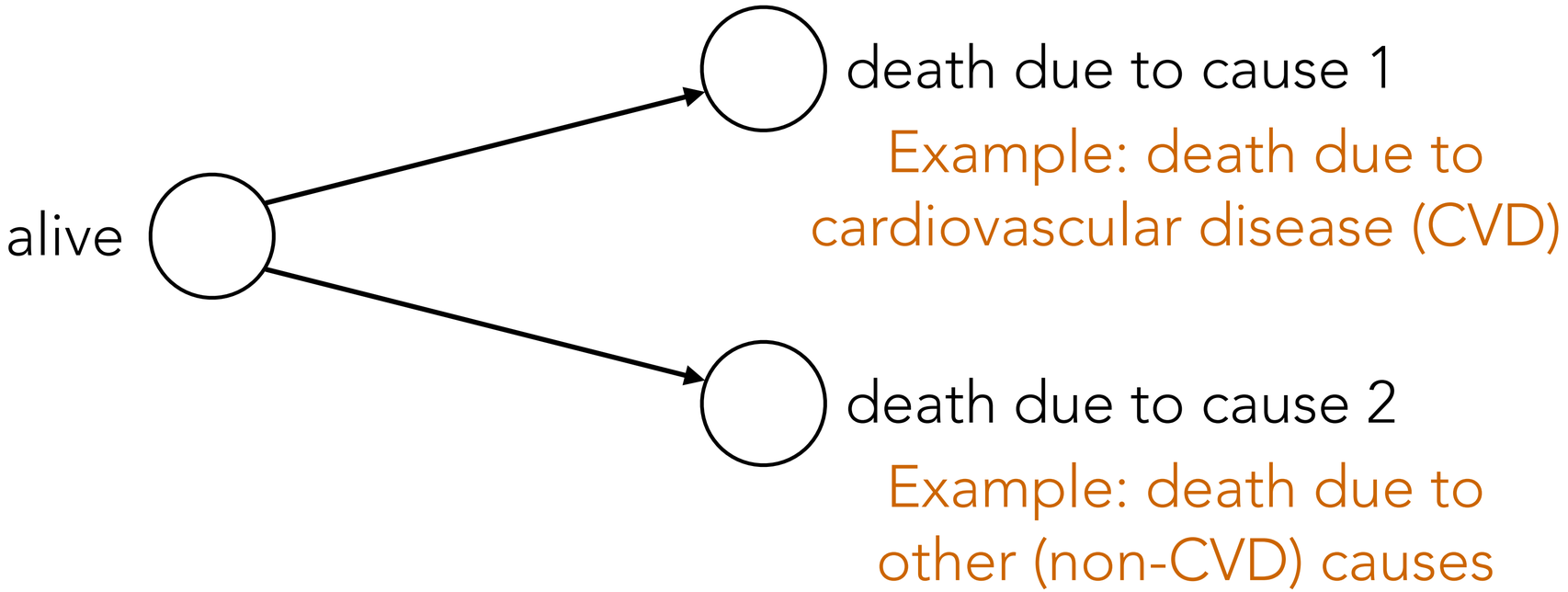

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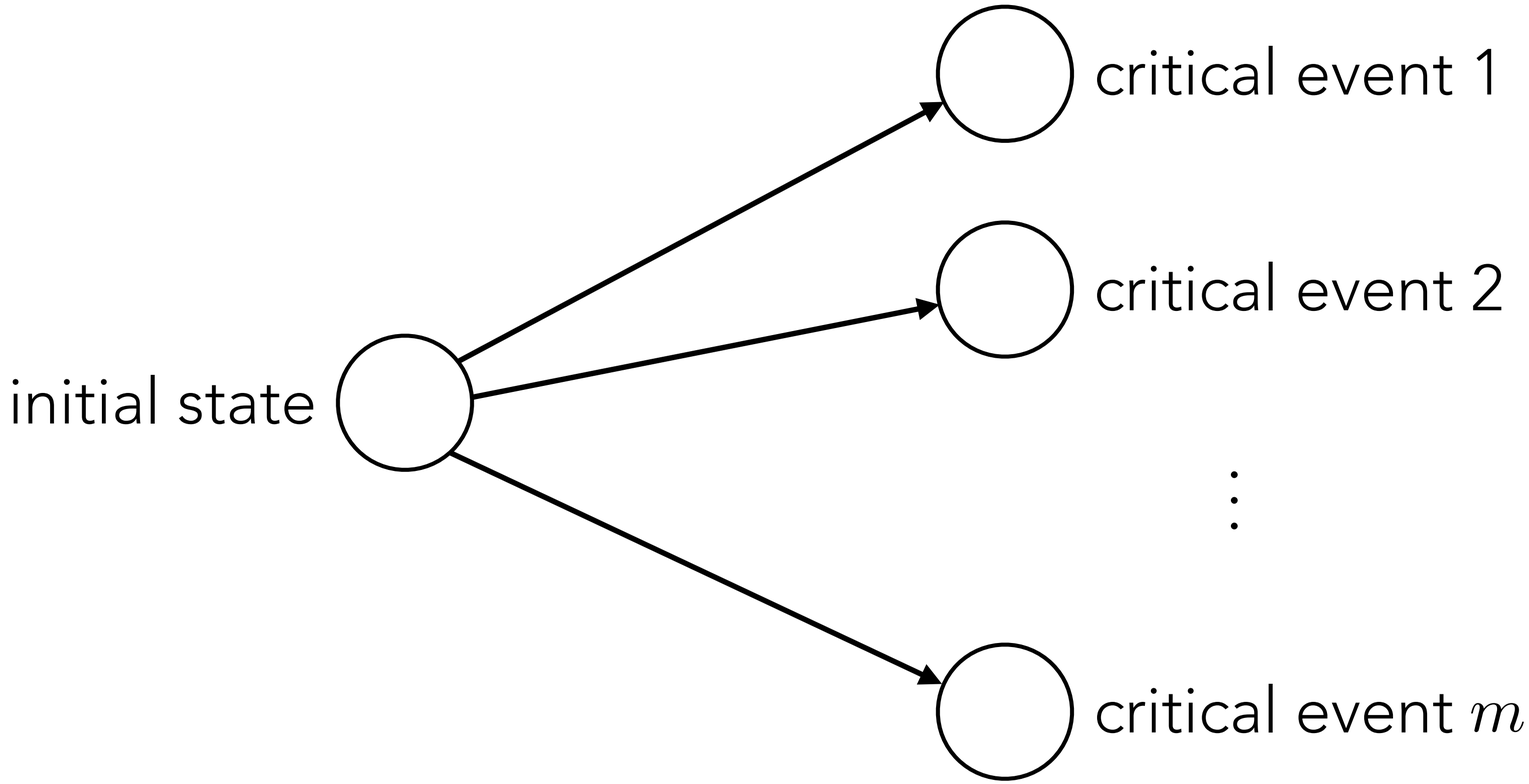

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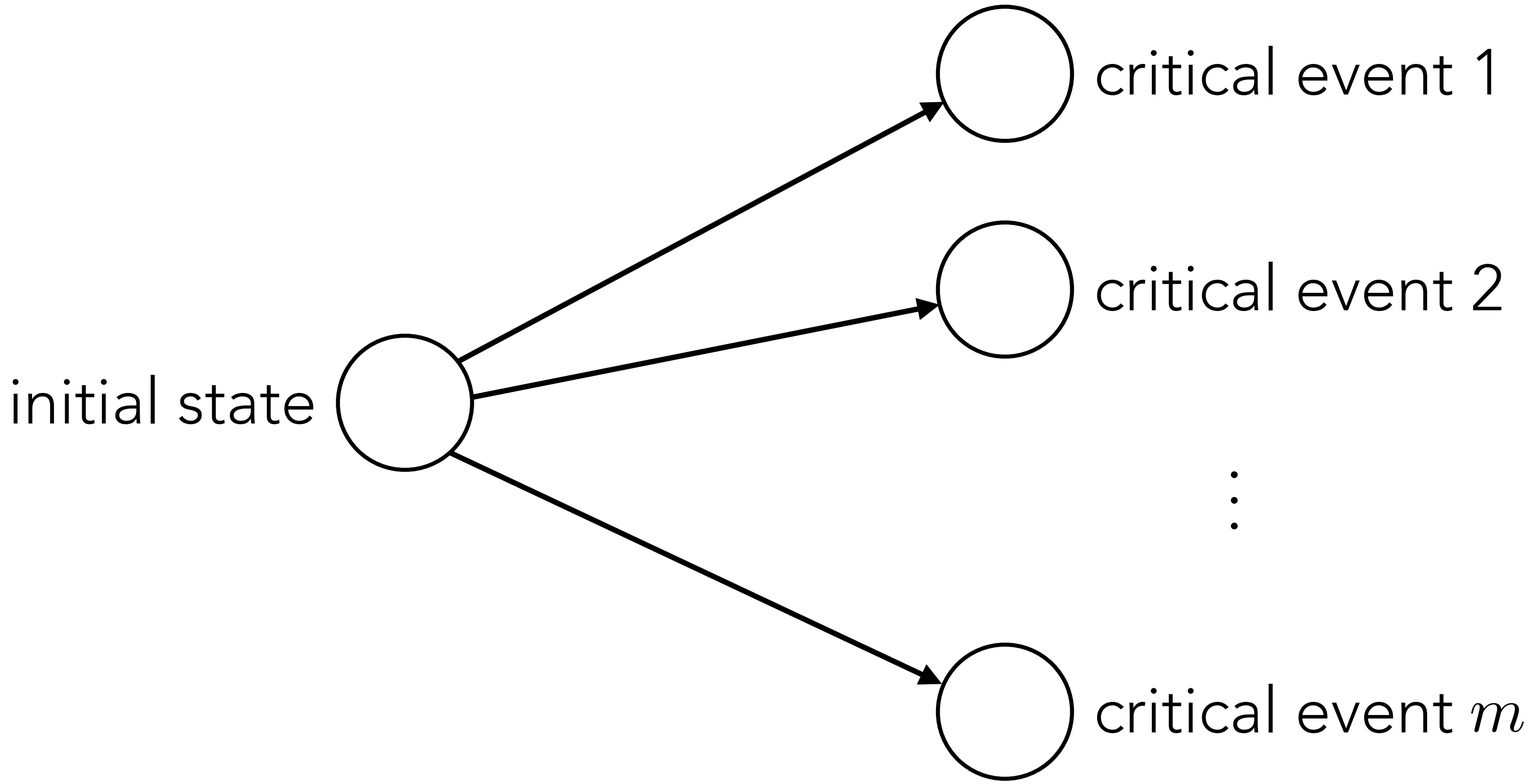

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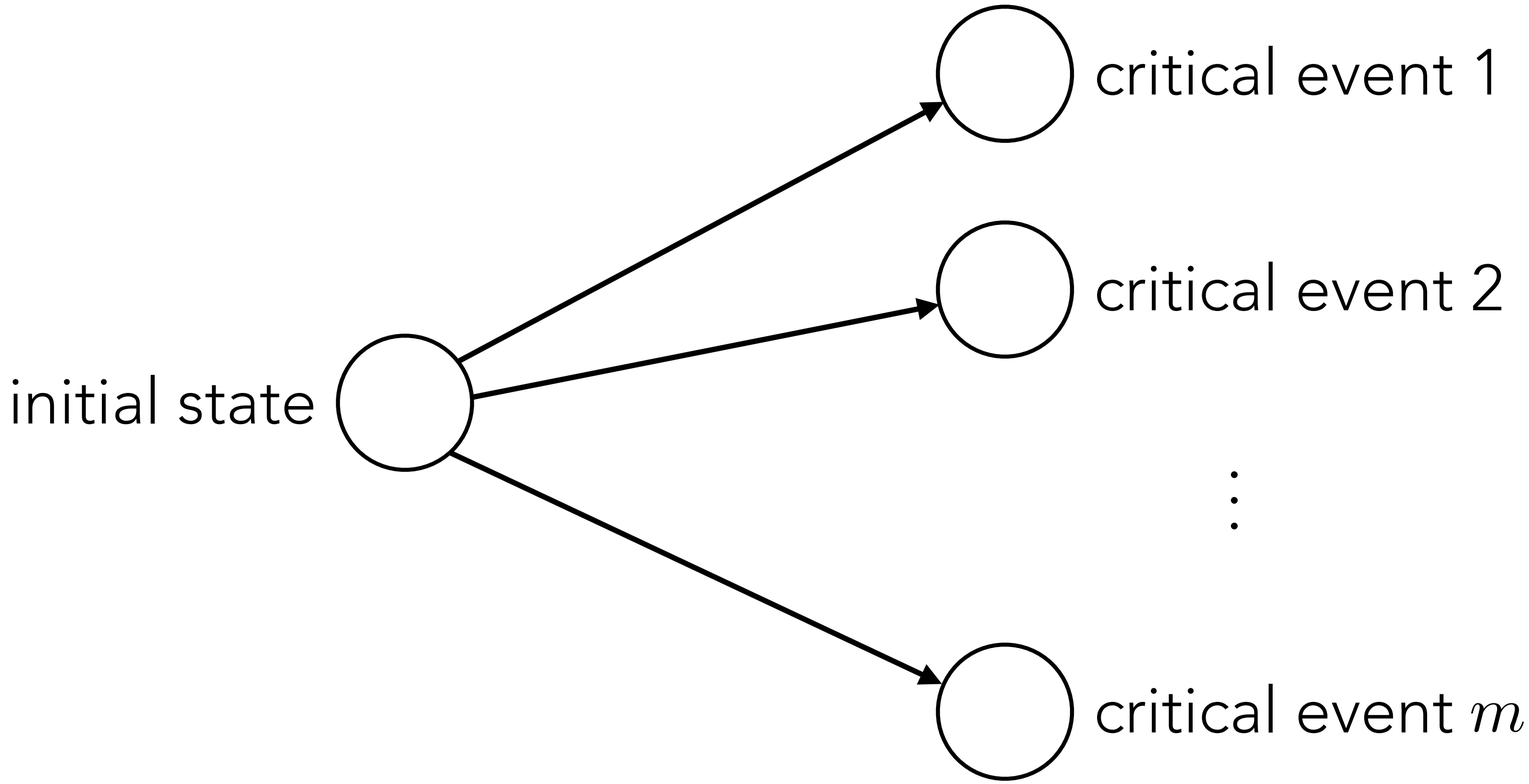

Want to reason about <u>which critical event will happen earliest</u> and <u>when</u>

Note: standard survival analysis is a special case where $m = 1$

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- The ML community has recently developed many competing risks models (**DeepHit** [Lee et al 2018], **DSM** [Nagpal et al 2021], **DeSurv** [Danks & Yau 2022], **SurvivalBoost** [Alberge et al 2025], …)

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each data point is represented as a weighted combination of clusters

If a data point only has nonzero weight for 1 cluster
⇒ prediction corresponds to AJ estimator fitted to only data points from that cluster

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- Numerical experiments

Competing Risks Problem Setup

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Training data: $(X_1, Y_1, \Delta_1), (X_2, Y_2, \Delta_2), \ldots, (X_n, Y_n, \Delta_n)$

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feature vector

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AJ estimator estimates population-level version

$$F_\delta^{\text{pop}}(t) \triangleq \mathbb{P}\big(\Delta^* = \delta, T \leq t\big)$$

AJ Estimator for Competing Risks [Aalen-Johansen 1978]

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# times event 1 happened					

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# at risk	n_1	n_2	n_3	$\cdots$	n_L

Intuition: $$F_\delta^{\text{pop}}(t) \approx \sum_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \le t}} \mathbb{P}\left(\begin{smallmatrix}\text{earliest event} \\ \text{after time } t_{\ell-1}\end{smallmatrix}\right) \mathbb{P}\left(\begin{smallmatrix}\text{experience event type } \delta \\ \text{at time } t_\ell\end{smallmatrix} \;\middle|\; \begin{smallmatrix}\text{earliest event} \\ \text{after time } t_{\ell-1}\end{smallmatrix}\right)$$

Training data: $(X_1, Y_1, \Delta_1), (X_2, Y_2, \Delta_2), \ldots, (X_n, Y_n, \Delta_n)$

AJ Estimator for Competing Risks [Aalen-Johansen 1978]

1. Find all unique times in which any critical event occurred

$$0 < t_1 < t_2 < \cdots < t_L \qquad L = \text{\# unique times of any critical event}$$

2. Build table below

	t_1	t_2	t_3	$\cdots$	t_L
# times event 1 happened	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$\cdots$	$d_{1,L}$
# times event 2 happened	$d_{2,1}$	$d_{2,2}$	$d_{2,3}$	$\cdots$	$d_{2,L}$
$\vdots$					
# times event m happened	$d_{m,1}$	$d_{m,2}$	$d_{m,3}$	$\cdots$	$d_{m,L}$
# at risk	n_1	n_2	n_3	$\cdots$	n_L

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$\approx \widehat{S}^{\text{KM}}(t_{\ell-1})$

classical Kaplan-Meier estimator [1958]

Training data: $(X_1, Y_1, \Delta_1), (X_2, Y_2, \Delta_2), \ldots, (X_n, Y_n, \Delta_n)$

AJ Estimator for Competing Risks [Aalen-Johansen 1978]

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# at risk	n_1	n_2	n_3	$\cdots$	n_L

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$\approx \widehat{S}^{\text{KM}}(t_{\ell-1})$

classical Kaplan-Meier estimator [1958]

$\approx \frac{d_{\delta,\ell}}{n_\ell}$

Training data: $(X_1, Y_1, \Delta_1), (X_2, Y_2, \Delta_2), \ldots, (X_n, Y_n, \Delta_n)$

AJ Estimator for Competing Risks [Aalen-Johansen 1978]

1. Find all unique times in which any critical event occurred

$$0 < t_1 < t_2 < \cdots < t_L \qquad L = \text{\# unique times of any critical event}$$

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# times event 1 happened	$d_{1,1}$	$d_{1,2}$	$d_{1,3}$	$\cdots$	$d_{1,L}$
# times event 2 happened	$d_{2,1}$	$d_{2,2}$	$d_{2,3}$	$\cdots$	$d_{2,L}$
$\vdots$					
# times event m happened	$d_{m,1}$	$d_{m,2}$	$d_{m,3}$	$\cdots$	$d_{m,L}$
# at risk	n_1	n_2	n_3	$\cdots$	n_L

3. Output: $\widehat{F}_{\delta}^{\mathrm{AJ}}(t) \triangleq \sum_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \leq t}} \widehat{S}^{\mathrm{KM}}(t_{\ell-1}) \frac{d_{\delta,\ell}}{n_\ell}$

Training data: $(X_1, Y_1, \Delta_1), (X_2, Y_2, \Delta_2), \ldots, (X_n, Y_n, \Delta_n)$

AJ Estimator for Competing Risks [Aalen-Johansen 1978]

1. Find all unique times in which any critical event occurred

$$0 < t_1 < t_2 < \cdots < t_L \qquad L = \text{\# unique times of any critical event}$$

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# at risk	n_1	n_2	n_3	$\cdots$	n_L

3. Output: $\widehat{F}_\delta^{\mathrm{AJ}}(t) \triangleq \sum_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \le t}} \widehat{S}^{\mathrm{KM}}(t_{\ell-1}) \frac{d_{\delta,\ell}}{n_\ell}$

$$\widehat{S}^{\mathrm{KM}}(t) \triangleq \prod_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \le t}} \left(1 - \frac{\sum_{\delta=1}^{m} d_{\delta,\ell}}{n_\ell}\right)$$

Training data: $(X_1, Y_1, \Delta_1), (X_2, Y_2, \Delta_2), \ldots, (X_n, Y_n, \Delta_n)$

Kernel Aalen-Johansen Estimator

1. Find all unique times in which any critical event occurred

$$0 < t_1 < t_2 < \cdots < t_L \qquad L = \text{\# unique times of any critical event}$$

2. Build table below

	t_1	t_2	t_3	$\cdots$	t_L
# times event 1 happened	$d_{1,1}(x)$	$d_{1,2}(x)$	$d_{1,3}(x)$	$\cdots$	$d_{1,L}(x)$
# times event 2 happened	$d_{2,1}(x)$	$d_{2,2}(x)$	$d_{2,3}(x)$	$\cdots$	$d_{2,L}(x)$
$\vdots$					
# times event m happened	$d_{m,1}(x)$	$d_{m,2}(x)$	$d_{m,3}(x)$	$\cdots$	$d_{m,L}(x)$
# at risk	$n_1(x)$	$n_2(x)$	$n_3(x)$	$\cdots$	$n_L(x)$

3. Output: $\widehat{F}_{\delta}^{\mathrm{KAJ}}(t|x) \triangleq \sum_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \leq t}} \widehat{S}^{\mathrm{KKM}}(t_{\ell-1}|x) \frac{d_{\delta,\ell}(x)}{n_\ell(x)}$

$$\widehat{S}^{\mathrm{KKM}}(t|x) \triangleq \prod_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \leq t}} \left(1 - \frac{\sum_{\delta=1}^{m} d_{\delta,\ell}(x)}{n_\ell(x)}\right)$$

Training data: $(X_1, Y_1, \Delta_1), (X_2, Y_2, \Delta_2), \ldots, (X_n, Y_n, \Delta_n)$

Kernel Aalen-Johansen Estimator

1. Find all unique times in which any critical event occurred

$$0 < t_1 < t_2 < \cdots < t_L \qquad L = \text{\# unique times of any critical event}$$

2. Build table below

	t_1	t_2	t_3	$\cdots$	t_L
# times event 1 happened among patients who look like x	$d_{1,1}(x)$	$d_{1,2}(x)$	$d_{1,3}(x)$	$\cdots$	$d_{1,L}(x)$
# times event 2 happened among patients who look like x	$d_{2,1}(x)$	$d_{2,2}(x)$	$d_{2,3}(x)$	$\cdots$	$d_{2,L}(x)$
$\vdots$					
# times event m happened among patients who look like x	$d_{m,1}(x)$	$d_{m,2}(x)$	$d_{m,3}(x)$	$\cdots$	$d_{m,L}(x)$
# at risk among patients who look like x	$n_1(x)$	$n_2(x)$	$n_3(x)$	$\cdots$	$n_L(x)$

3. Output: $\widehat{F}_\delta^{\mathrm{KAJ}}(t|x) \triangleq \sum_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \le t}} \widehat{S}^{\mathrm{KKM}}(t_{\ell-1}|x) \frac{d_{\delta,\ell}(x)}{n_\ell(x)}$

$$\widehat{S}^{\mathrm{KKM}}(t|x) \triangleq \prod_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \le t}} \left(1 - \frac{\sum_{\delta=1}^{m} d_{\delta,\ell}(x)}{n_\ell(x)}\right)$$

$$d_{\delta,\ell}(x) \triangleq \sum_{j=1}^{n} \mathbb{1}\{\Delta_j = \delta, Y_j = t_\ell\} K(x, X_j)$$

times event δ occurs at time t_ℓ among those who look like x

2. Build table below

	t_1	t_2	t_3	$\cdots$	t_L
# times event 1 happened among patients who look like x	$d_{1,1}(x)$	$d_{1,2}(x)$	$d_{1,3}(x)$	$\cdots$	$d_{1,L}(x)$
# times event 2 happened among patients who look like x	$d_{2,1}(x)$	$d_{2,2}(x)$	$d_{2,3}(x)$	$\cdots$	$d_{2,L}(x)$
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$\widehat{S}^{\mathrm{KKM}}(t|x) \triangleq \prod_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \le t}} \left(1 - \frac{\sum_{\delta=1}^{m} d_{\delta,\ell}(x)}{n_\ell(x)}\right)$

measures how similar x and X_j are

$$d_{\delta,\ell}(x) \triangleq \sum_{j=1}^{n} \mathbb{1}\{\Delta_j = \delta, Y_j = t_\ell\} K(x, X_j)$$

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2. Build table below

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# times event 2 happened among patients who look like x	$d_{2,1}(x)$	$d_{2,2}(x)$	$d_{2,3}(x)$	$\cdots$	$d_{2,L}(x)$
$\vdots$					
# times event m happened among patients who look like x	$d_{m,1}(x)$	$d_{m,2}(x)$	$d_{m,3}(x)$	$\cdots$	$d_{m,L}(x)$
# at risk among patients who look like x	$n_1(x)$	$n_2(x)$	$n_3(x)$	$\cdots$	$n_L(x)$

3. Output: $\widehat{F}_\delta^{\mathrm{KAJ}}(t|x) \triangleq \sum_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \le t}} \widehat{S}^{\mathrm{KKM}}(t_{\ell-1}|x) \frac{d_{\delta,\ell}(x)}{n_\ell(x)}$

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times event δ occurs at time t_ℓ among those who look like x

$$n_\ell(x) \triangleq \sum_{j=1}^{n} \mathbb{1}\{Y_j \geq t_\ell\} K(x, X_j)$$

at risk at time t_ℓ among those who look like x

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	t_1	t_2	t_3	$\cdots$	t_L
# times event 1 happened among patients who look like x	$d_{1,1}(x)$	$d_{1,2}(x)$	$d_{1,3}(x)$	$\cdots$	$d_{1,L}(x)$
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at risk at time t_ℓ among those who look like x

Example kernel functions:

3. Output: $\widehat{F}_\delta^{\mathrm{KAJ}}(t|x) \triangleq \sum_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \leq t}} \widehat{S}^{\mathrm{KKM}}(t_{\ell-1}|x) \frac{d_{\delta,\ell}(x)}{n_\ell(x)}$

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at risk at time t_ℓ among those who look like x

Example kernel functions:

$K(x, x') = 1$ for all x, x'

3. Output: $\widehat{F}_\delta^{\mathrm{KAJ}}(t|x) \triangleq \sum_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \leq t}} \widehat{S}^{\mathrm{KKM}}(t_{\ell-1}|x) \frac{d_{\delta,\ell}(x)}{n_\ell(x)}$

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at risk at time t_ℓ among those who look like x

Example kernel functions:

$K(x, x') = 1$ for all x, x' $\Rightarrow$ recover classical (population-level) AJ estimator

3. Output: $\widehat{F}_\delta^{\mathrm{KAJ}}(t|x) \triangleq \sum_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \leq t}} \widehat{S}^{\mathrm{KKM}}(t_{\ell-1}|x) \frac{d_{\delta,\ell}(x)}{n_\ell(x)}$

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times event δ occurs at time t_ℓ among those who look like x

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at risk at time t_ℓ among those who look like x

Example kernel functions:

$K(x, x') = 1$ for all x, x' $\Rightarrow$ recover classical (population-level) AJ estimator

$K(x, x') = \exp(-\|f(x;\theta) - f(x';\theta)\|^2)$

kernel function parameterized by an "encoder" neural net $f(\cdot;\theta)$

3. Output: $\widehat{F}_\delta^{\mathrm{KAJ}}(t|x) \triangleq \sum_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \leq t}} \widehat{S}^{\mathrm{KKM}}(t_{\ell-1}|x) \frac{d_{\delta,\ell}(x)}{n_\ell(x)}$

$\widehat{S}^{\mathrm{KKM}}(t|x) \triangleq \prod_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \leq t}} \left(1 - \frac{\sum_{\delta=1}^{m} d_{\delta,\ell}(x)}{n_\ell(x)}\right)$

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$$d_{\delta,\ell}(x) \triangleq \sum_{j=1}^{n} \mathbb{1}\{\Delta_j = \delta, Y_j = t_\ell\} K(x, X_j)$$

times event δ occurs at time t_ℓ among those who look like x

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at risk at time t_ℓ among those who look like x

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times event δ occurs at time t_ℓ among those who look like x

$$n_\ell(x) \triangleq \sum_{j=1}^{n} \mathbb{1}\{Y_j \geq t_\ell\} K(x, X_j)$$

at risk at time t_ℓ among those who look like x

Can learn neural net parameters using maximum likelihood (train in minibatches via minibatch gradient descent)

$$K(x, x') = \exp(-\|f(x;\theta) - f(x';\theta)\|^2)$$

kernel function parameterized by an "encoder" neural net $f(\cdot;\theta)$

3. Output: $\widehat{F}_\delta^{\mathrm{KAJ}}(t|x) \triangleq \sum_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \leq t}} \widehat{S}^{\mathrm{KKM}}(t_{\ell-1}|x) \frac{d_{\delta,\ell}(x)}{n_\ell(x)}$

$$\widehat{S}^{\mathrm{KKM}}(t|x) \triangleq \prod_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \leq t}} \left(1 - \frac{\sum_{\delta=1}^{m} d_{\delta,\ell}(x)}{n_\ell(x)}\right)$$

DKAJ High-Level Idea

DKAJ High-Level Idea

$$f(\cdot;\theta)$$

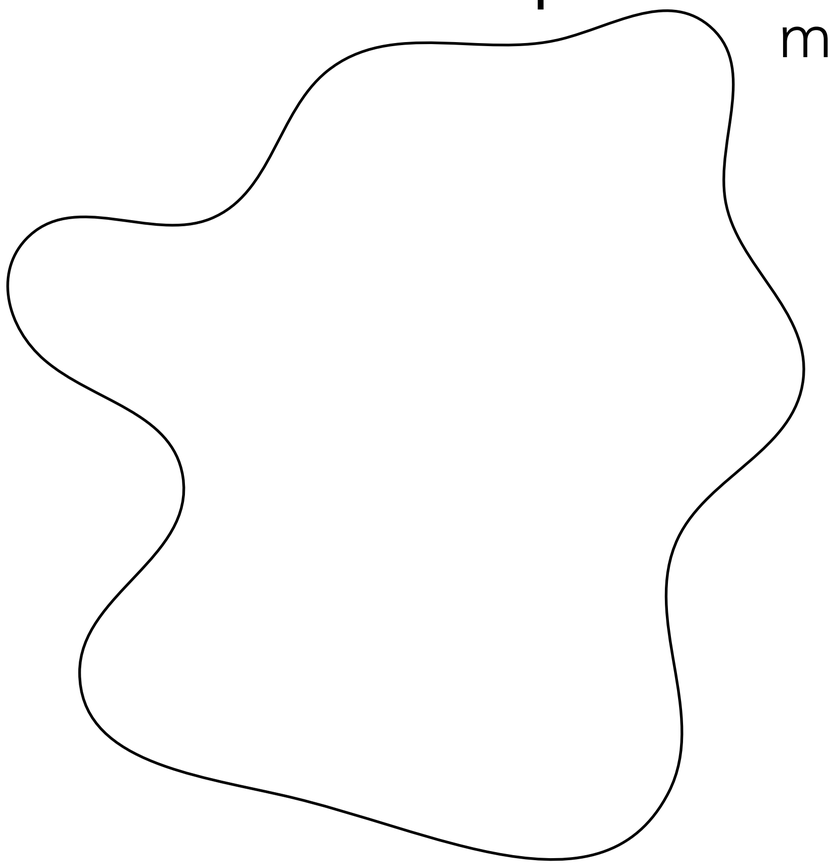
DKAJ High-Level Idea

encoder learned via
maximum likelihood

$f(\cdot;\widehat{\theta})$

DKAJ High-Level Idea

Raw feature space

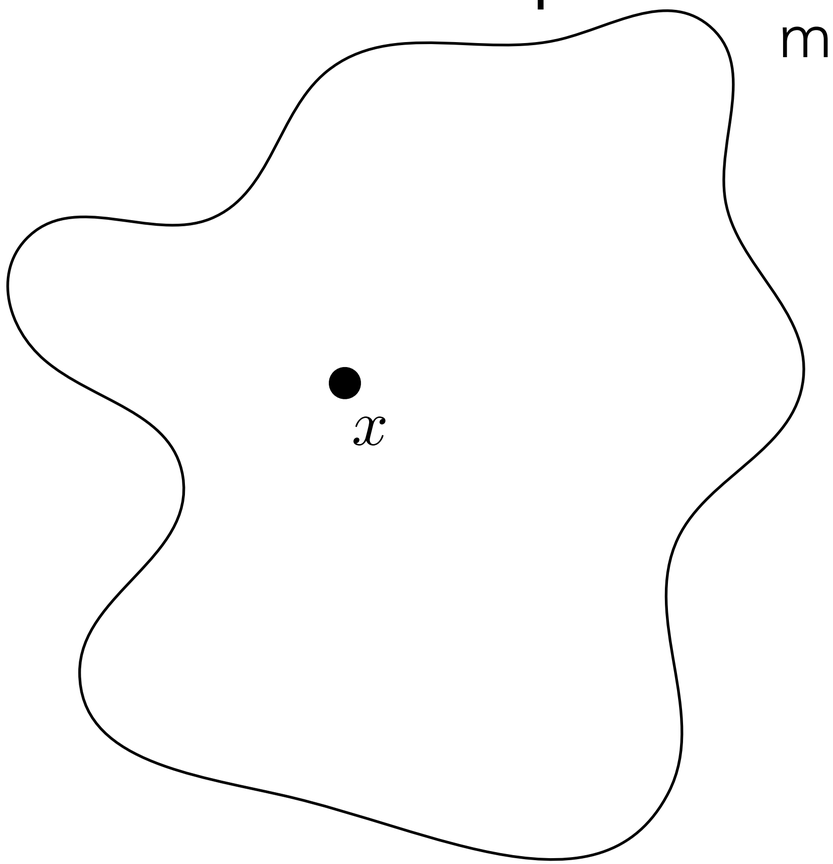


encoder learned via maximum likelihood

$$f(\cdot;\widehat{\theta})$$

DKAJ High-Level Idea

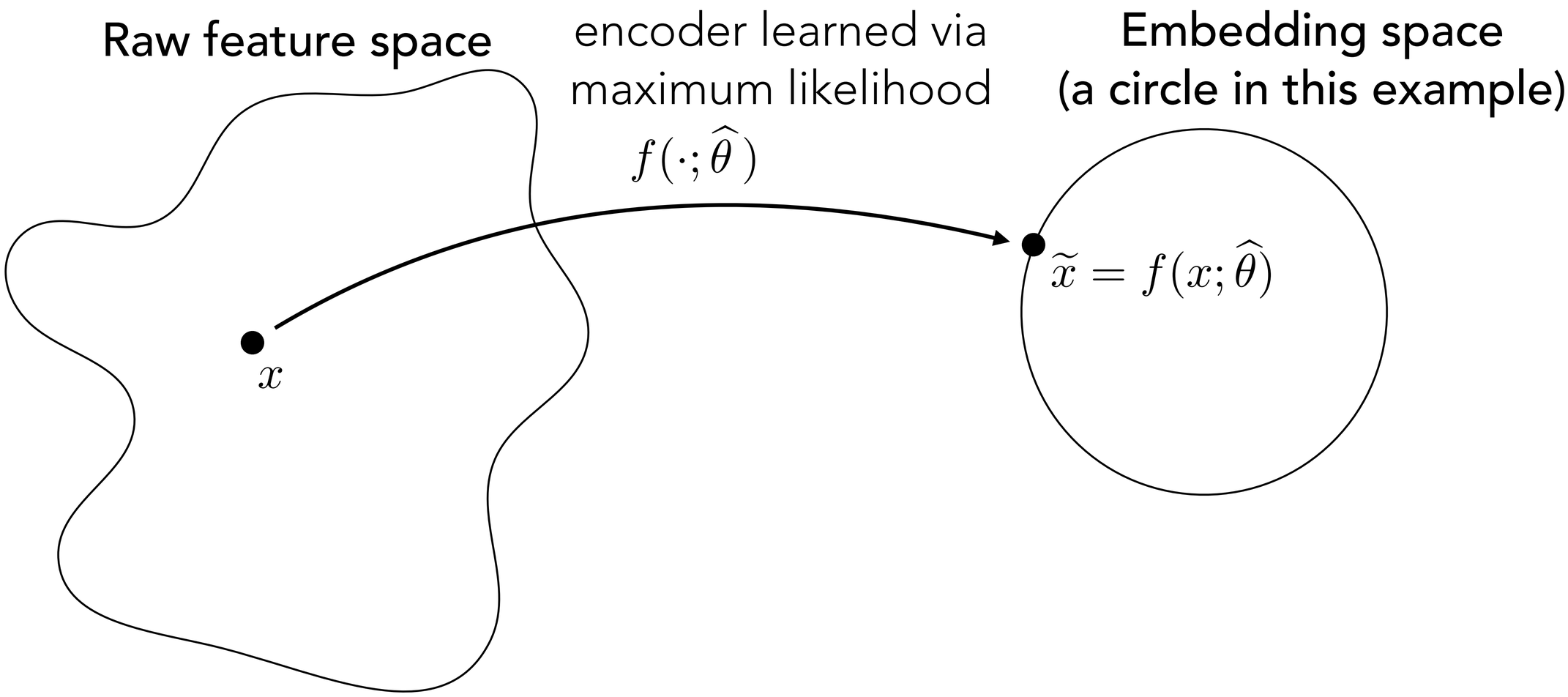
Raw feature space



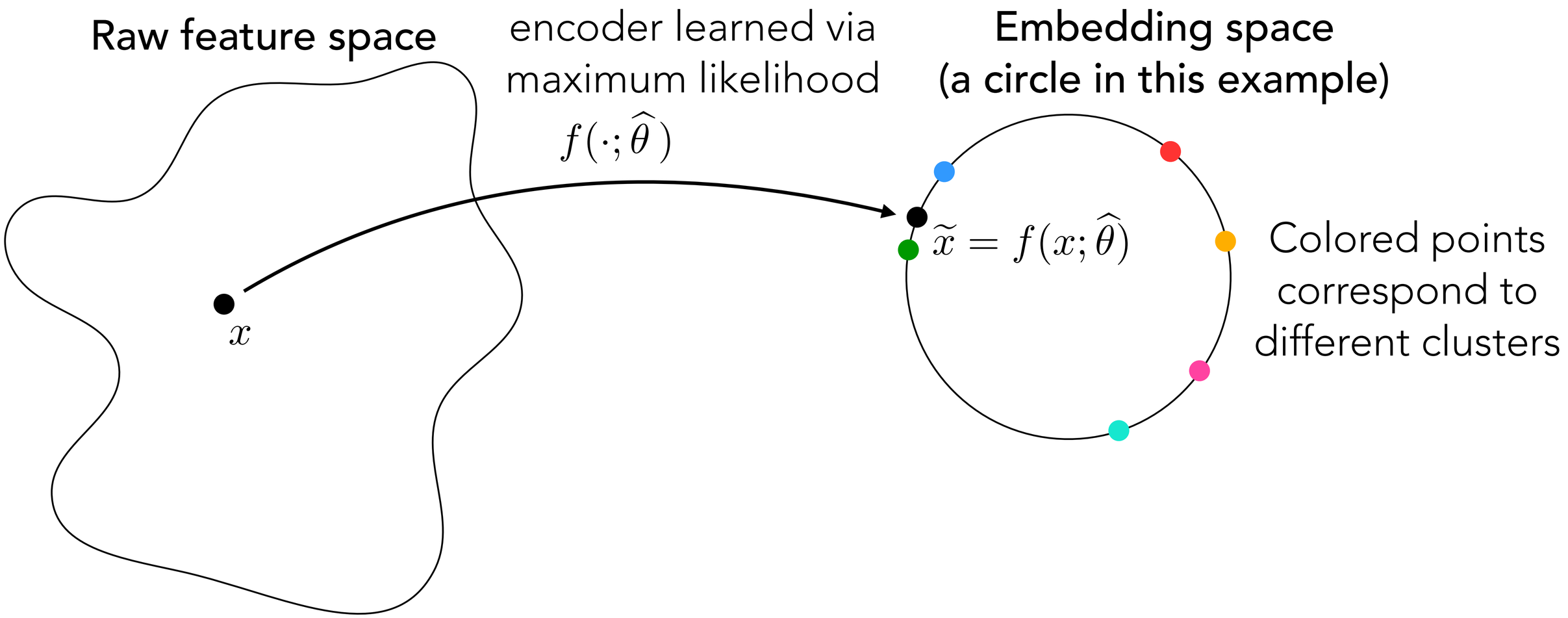
encoder learned via maximum likelihood

$f(\cdot;\widehat{\theta})$

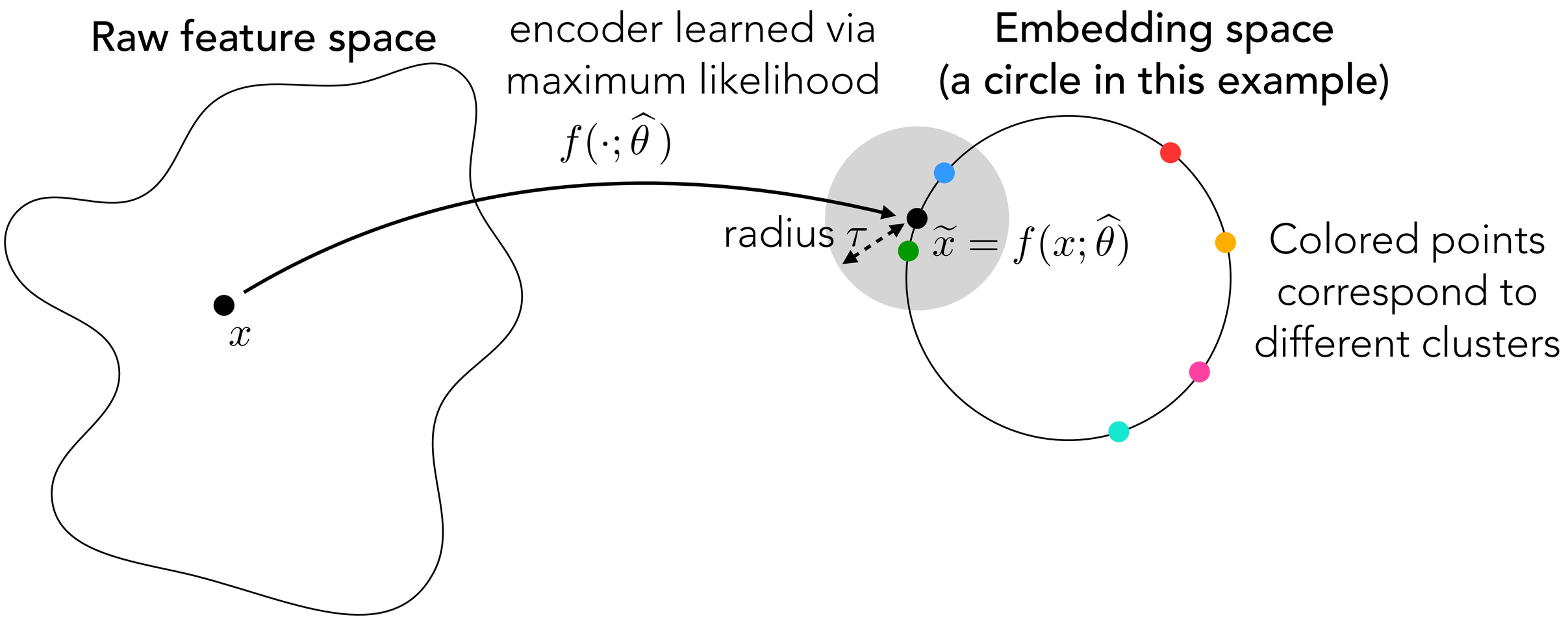
DKAJ High-Level Idea



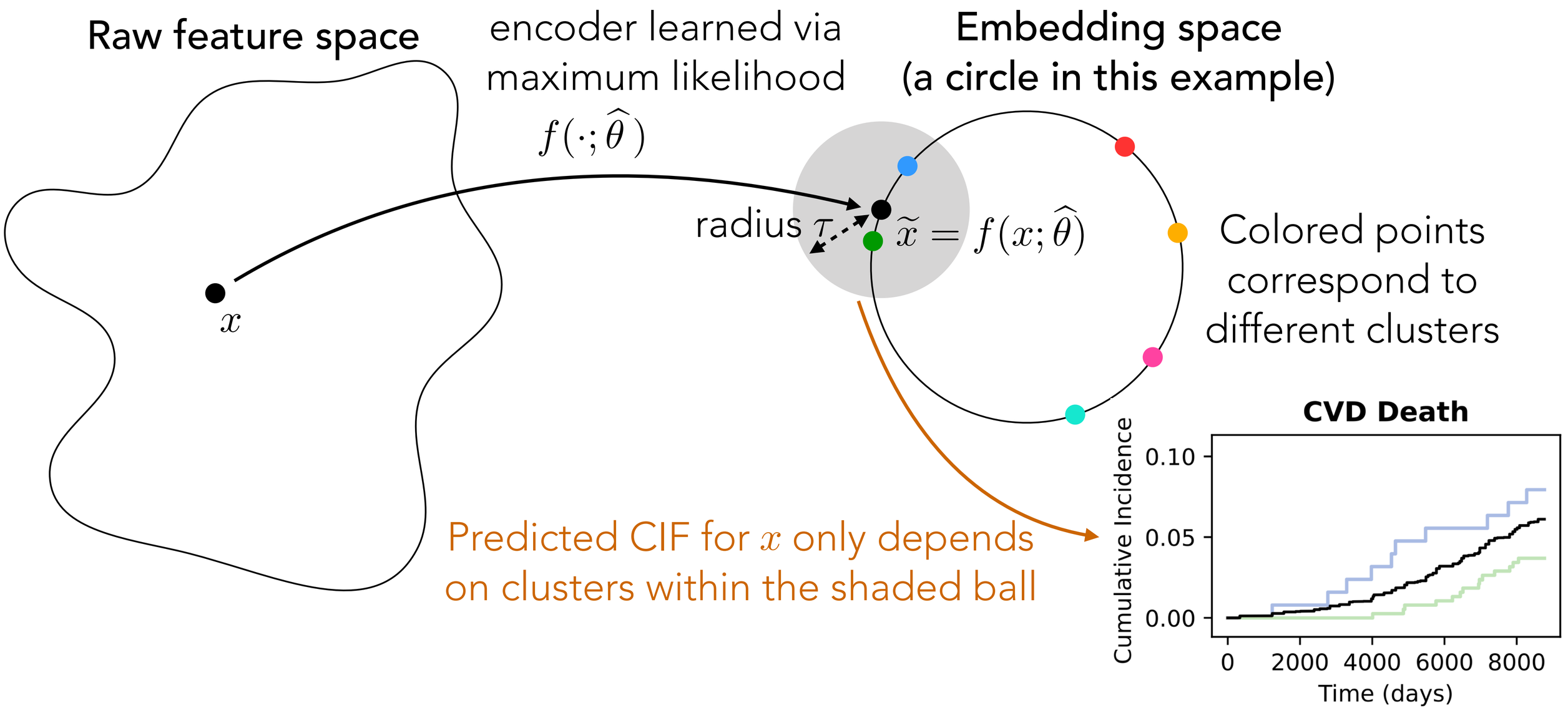
DKAJ High-Level Idea



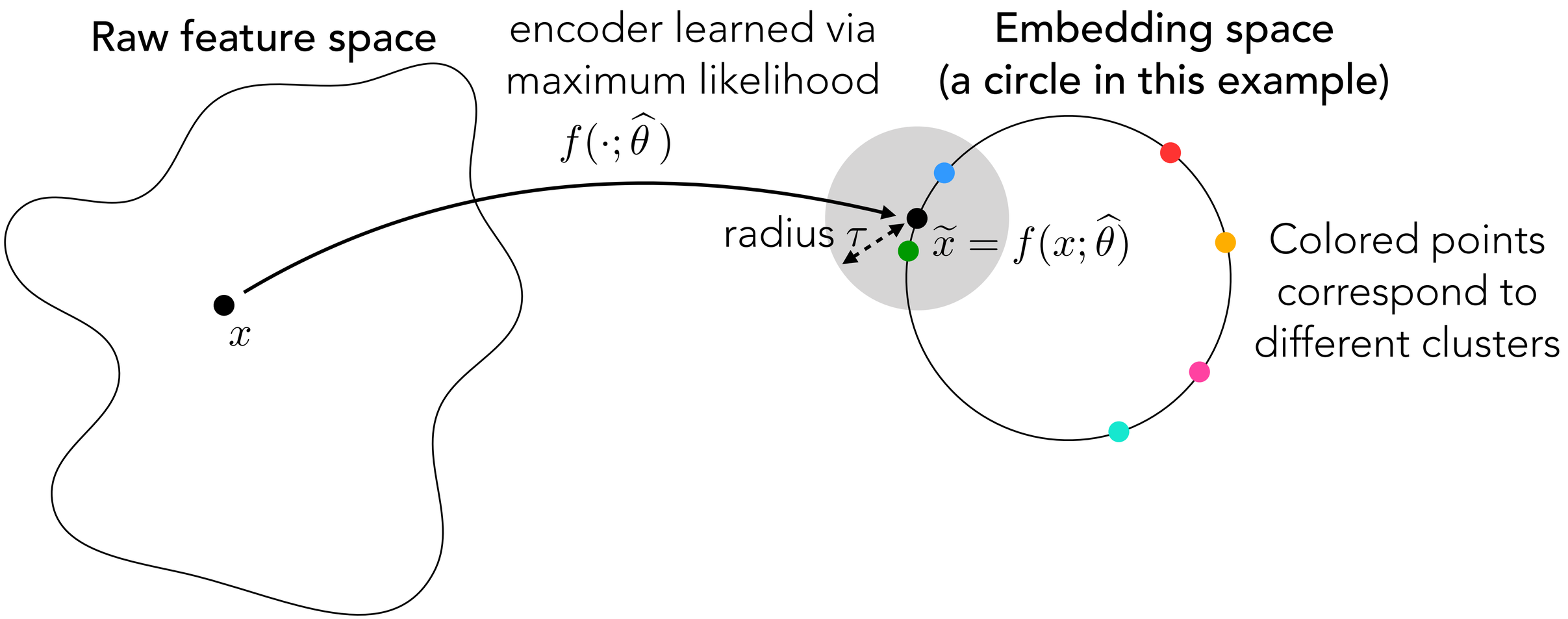
DKAJ High-Level Idea



DKAJ High-Level Idea



DKAJ High-Level Idea



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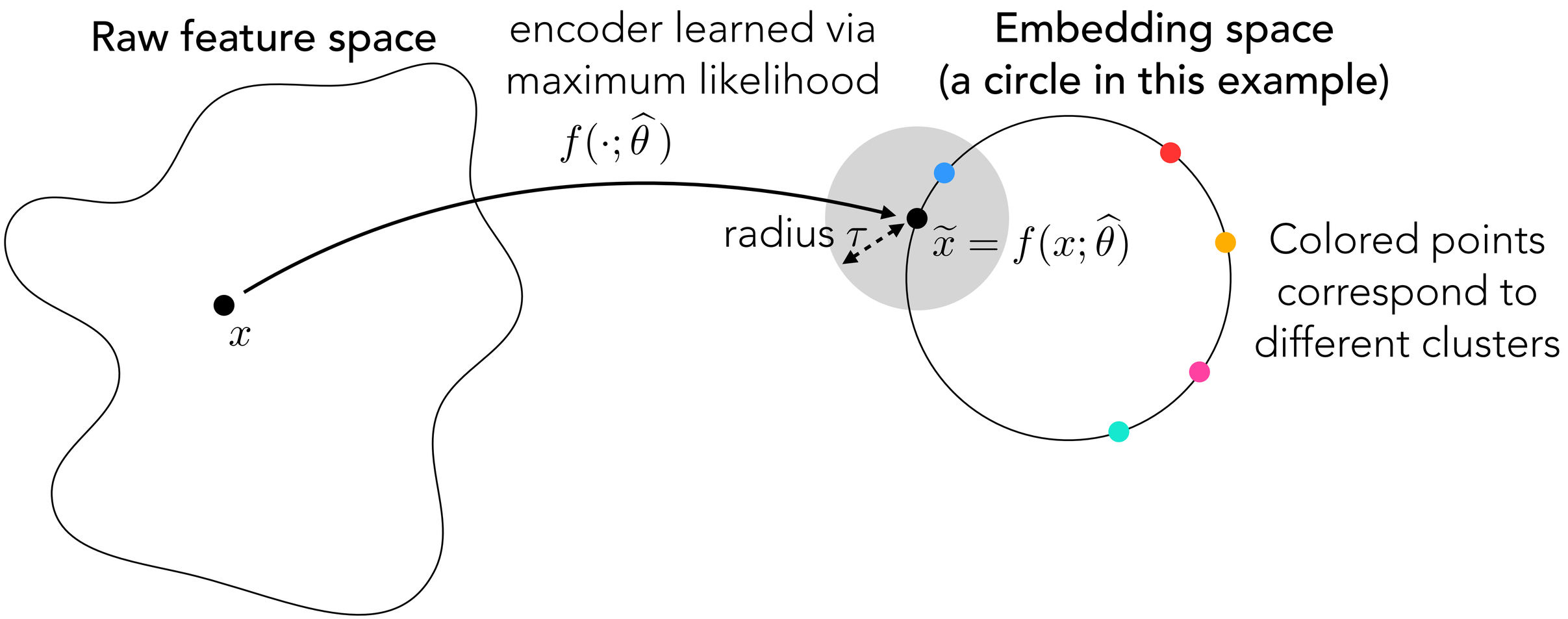

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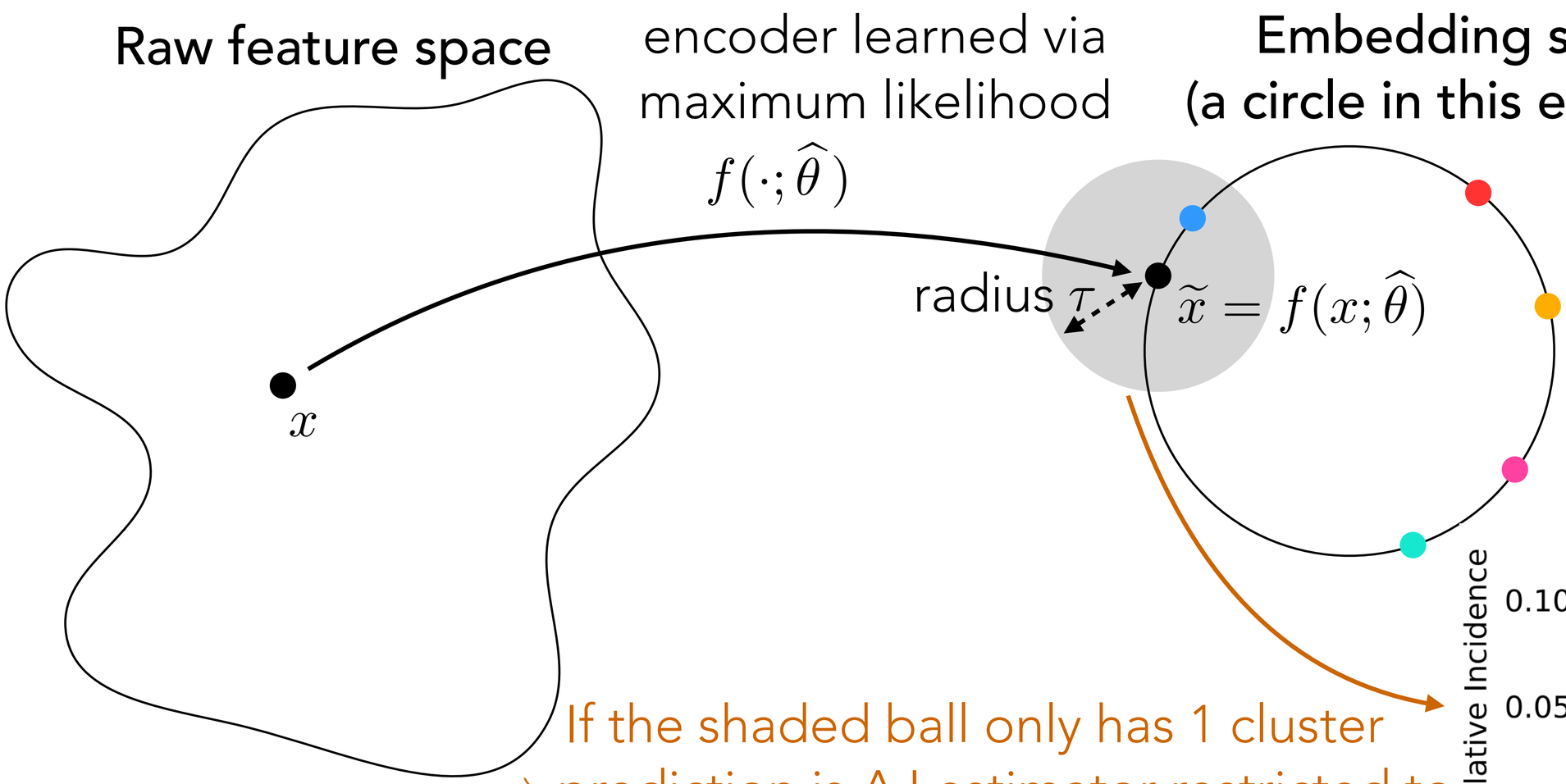

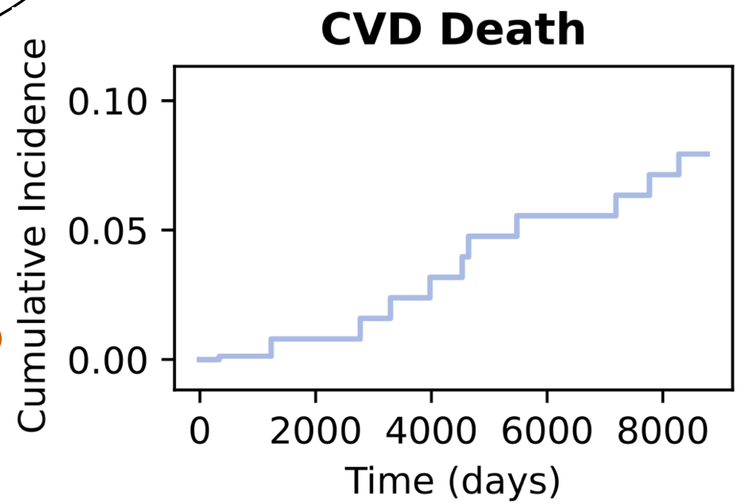

Recall: kernel function is $K(x, x') = \exp(-\|f(x;\theta) - f(x';\theta)\|^2)$

DKAJ Training

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DKAJ Training

Training data

$(X_1, Y_1, \Delta_1), \ldots, (X_n, Y_n, \Delta_n)$

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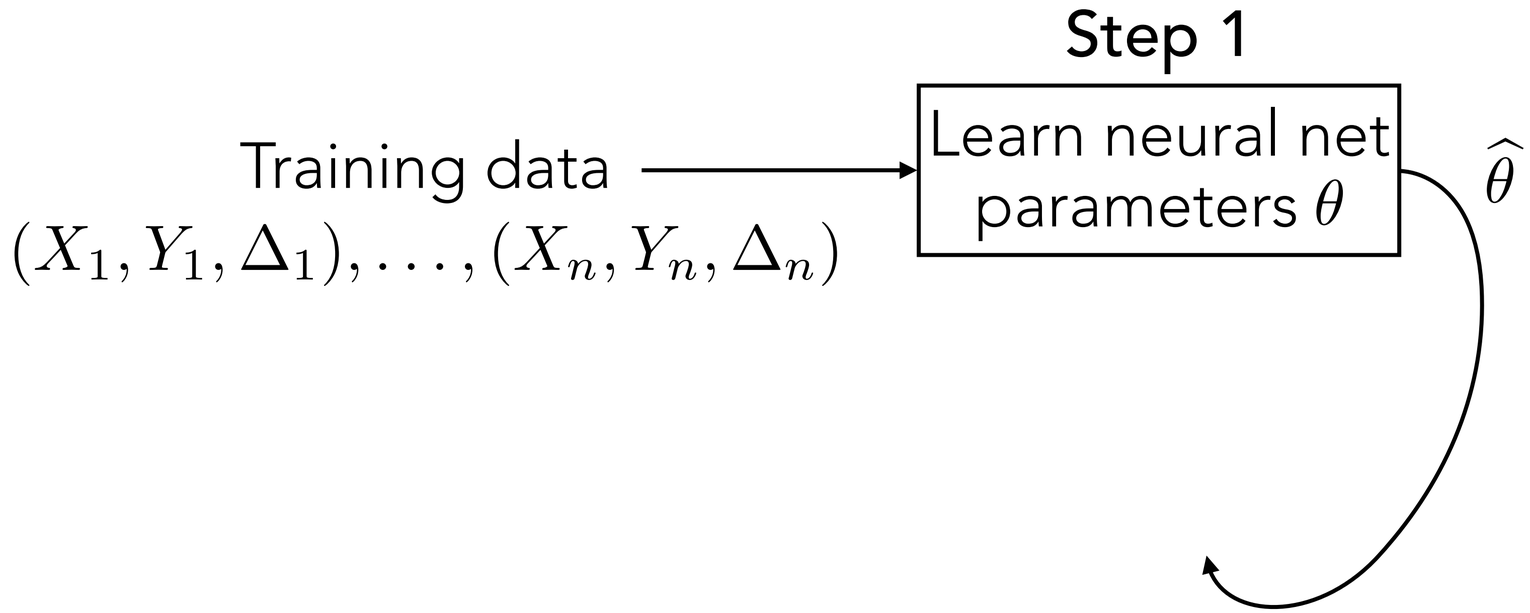

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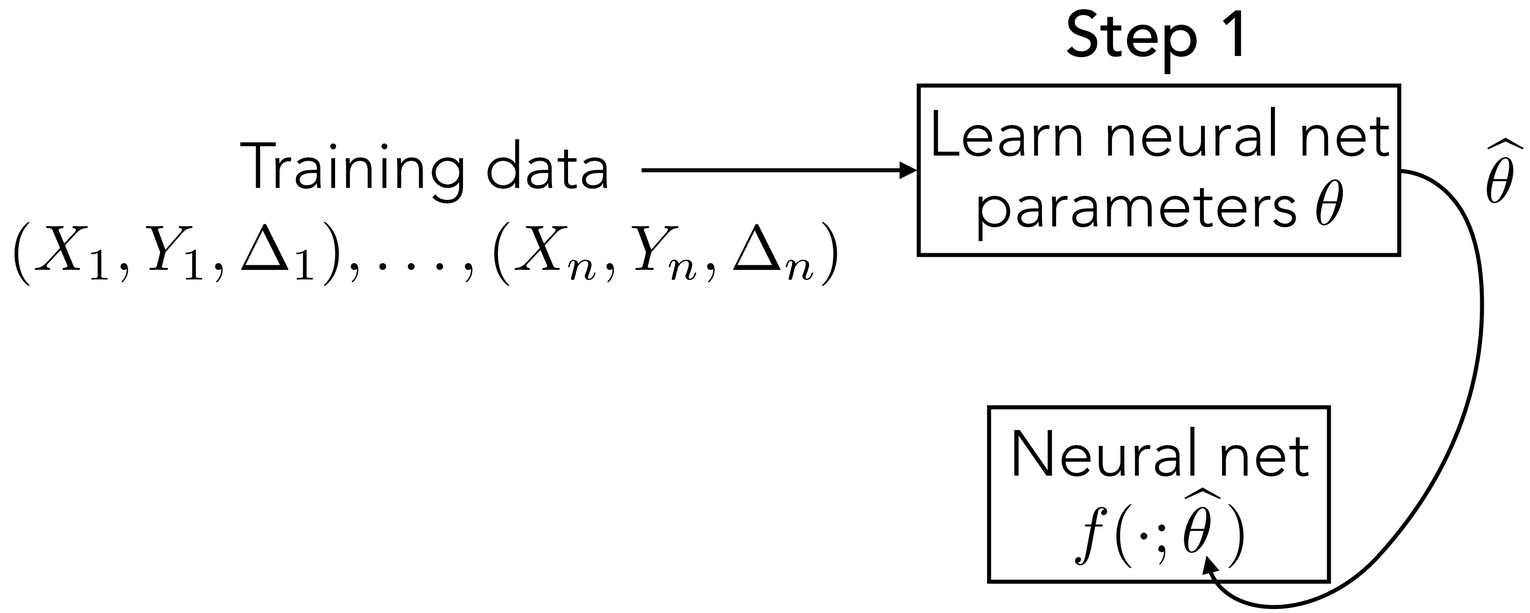

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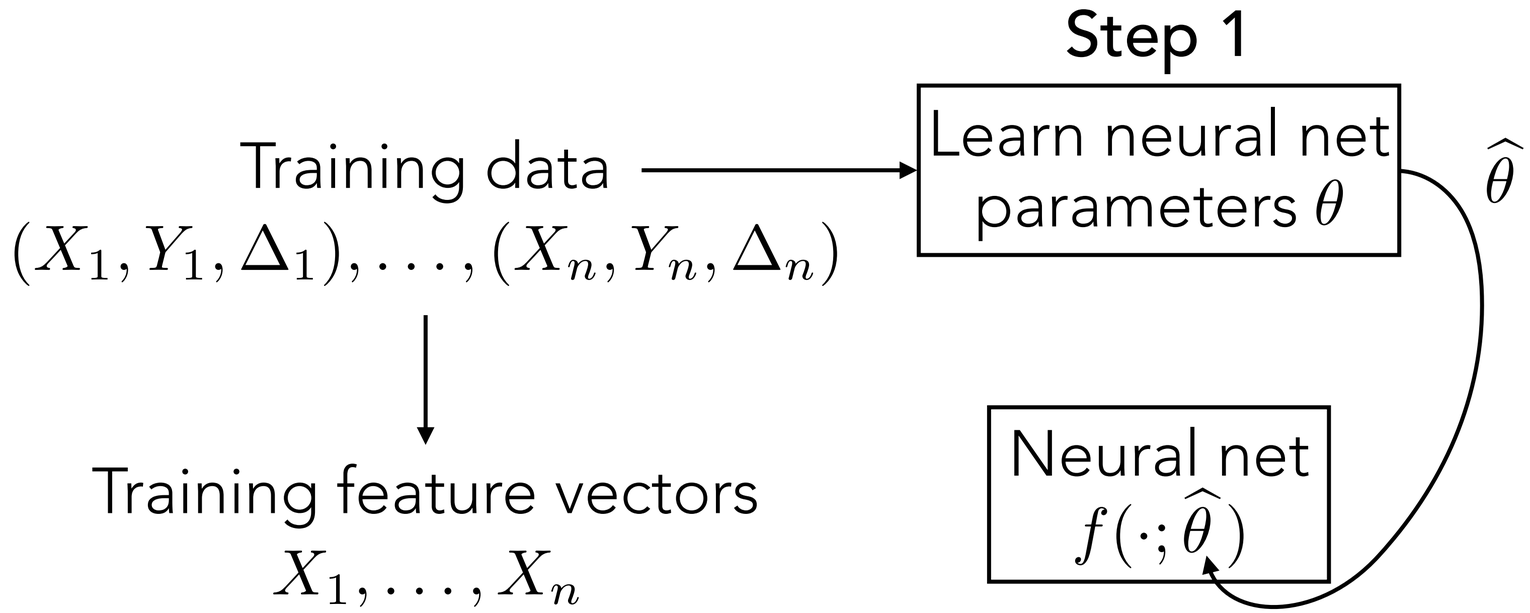

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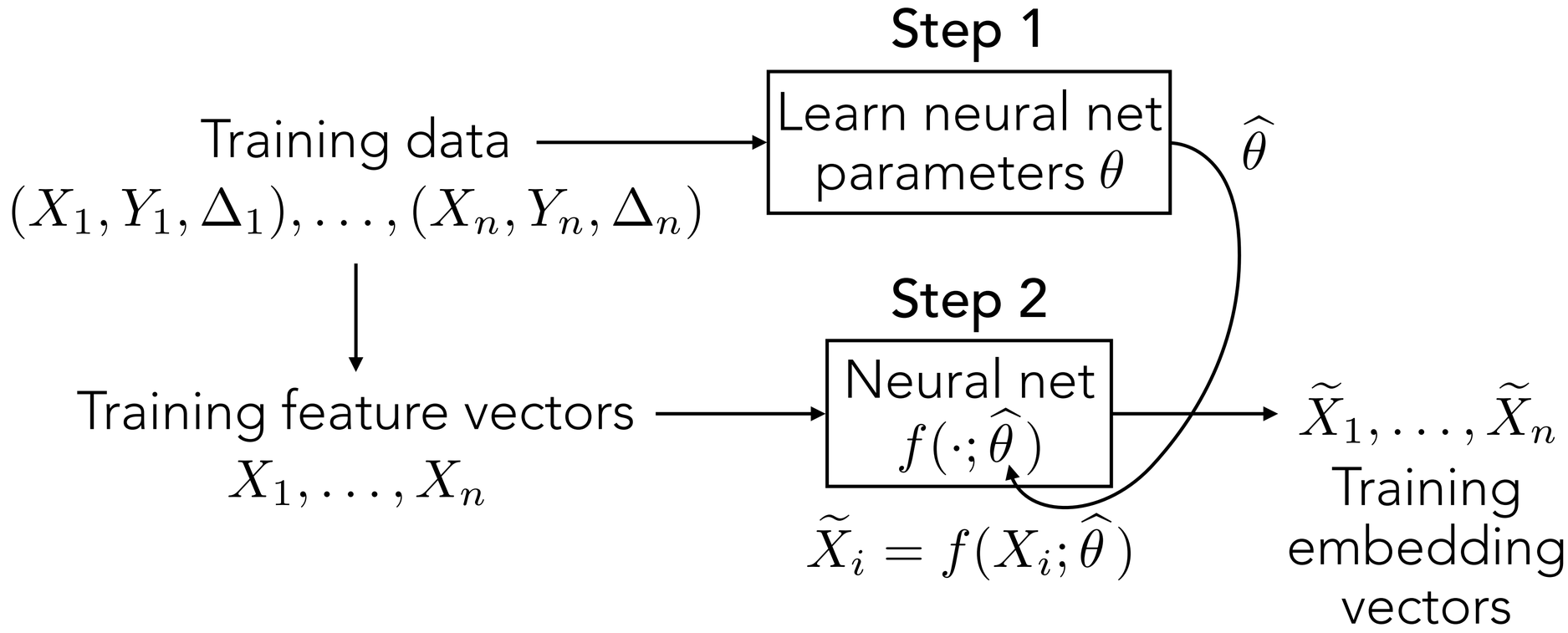

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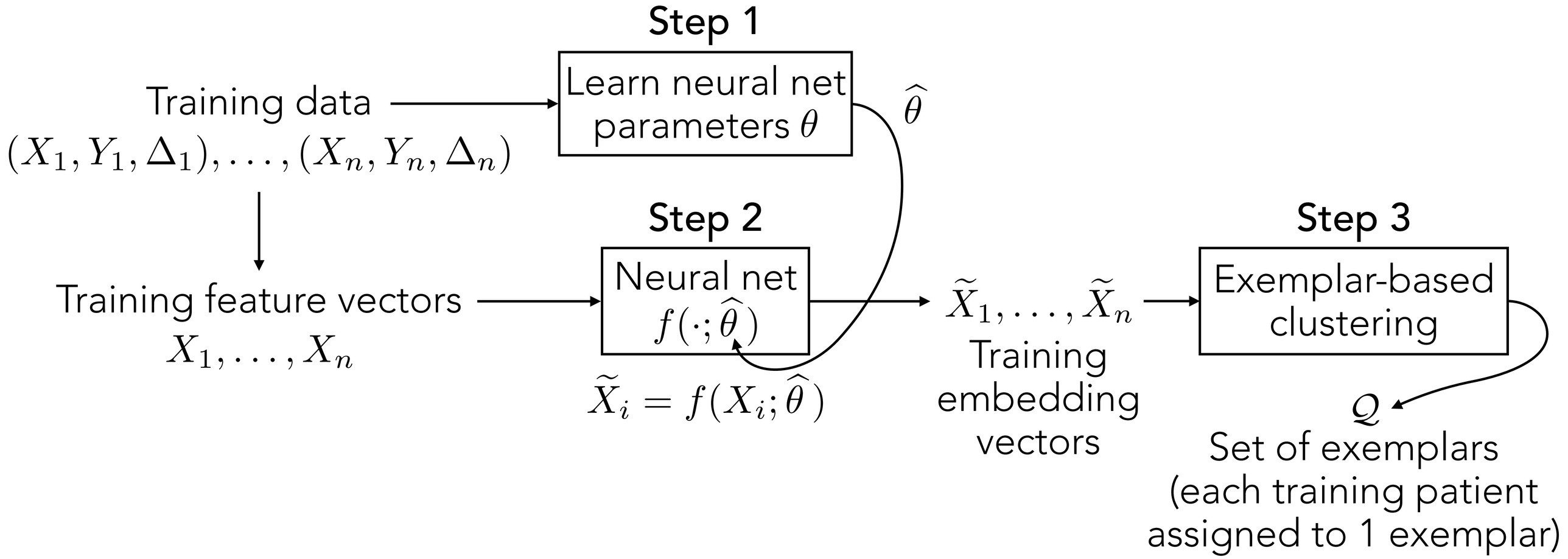

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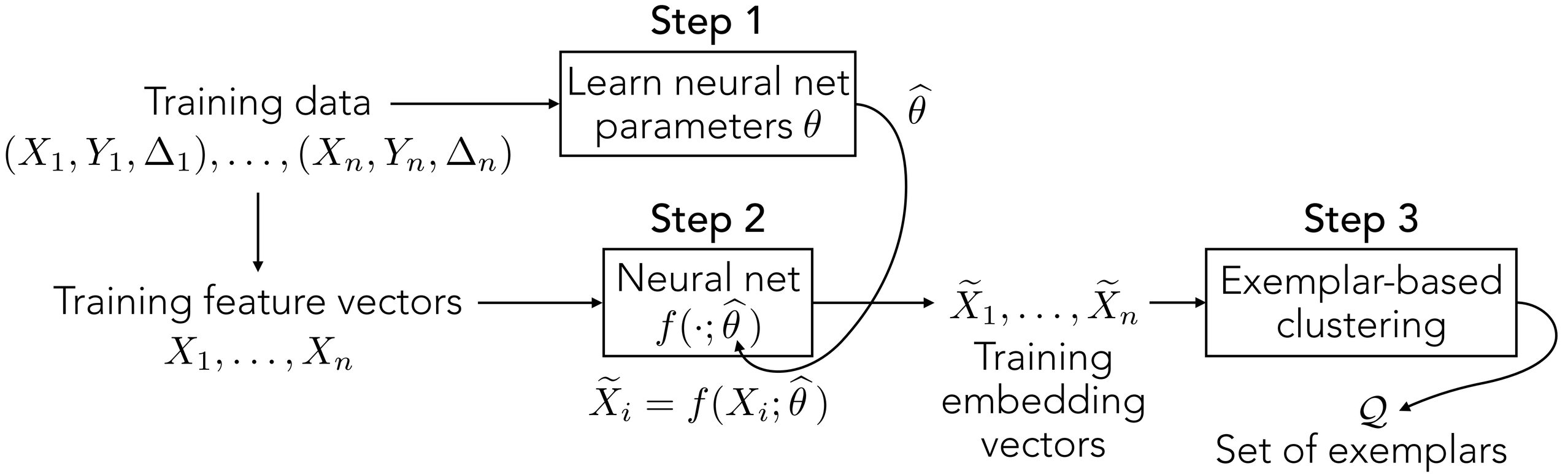

Step 4

For each exemplar $q \in \mathcal{Q}$, compute summary functions:

$d_{\delta,\ell}^{\text{cluster}}(q)$ = # times event δ happened at time t_ℓ among patients in q's cluster

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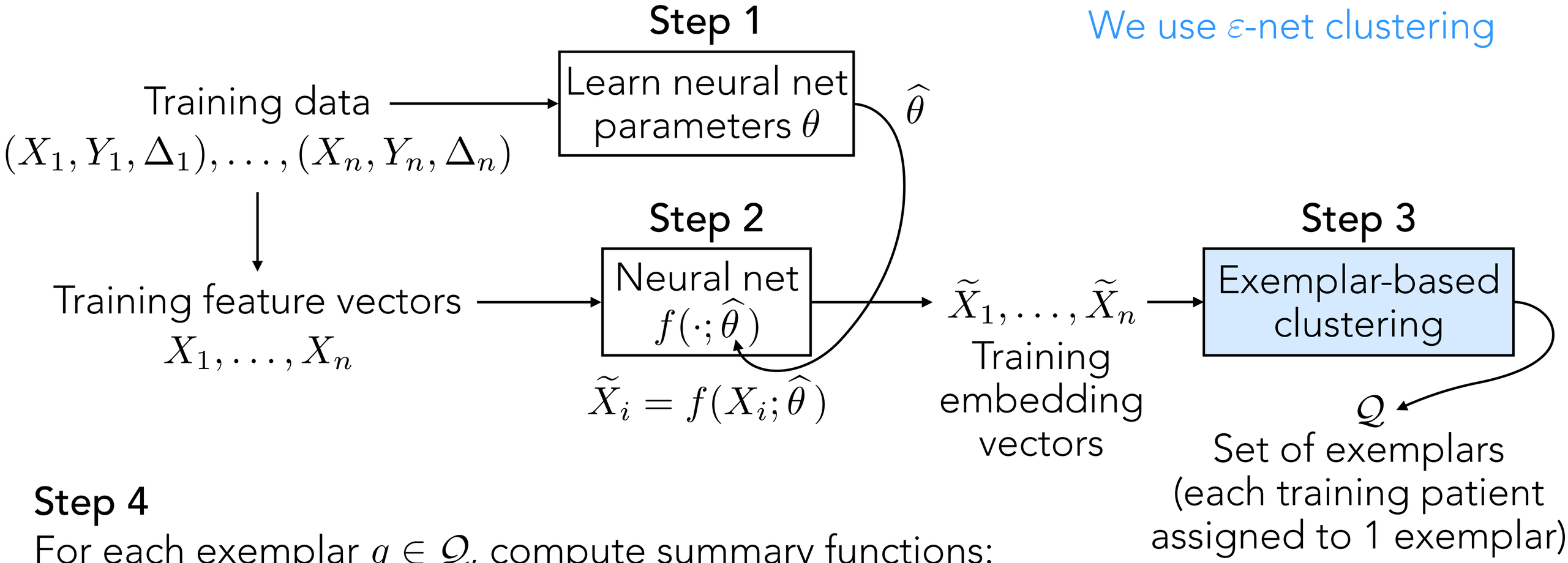

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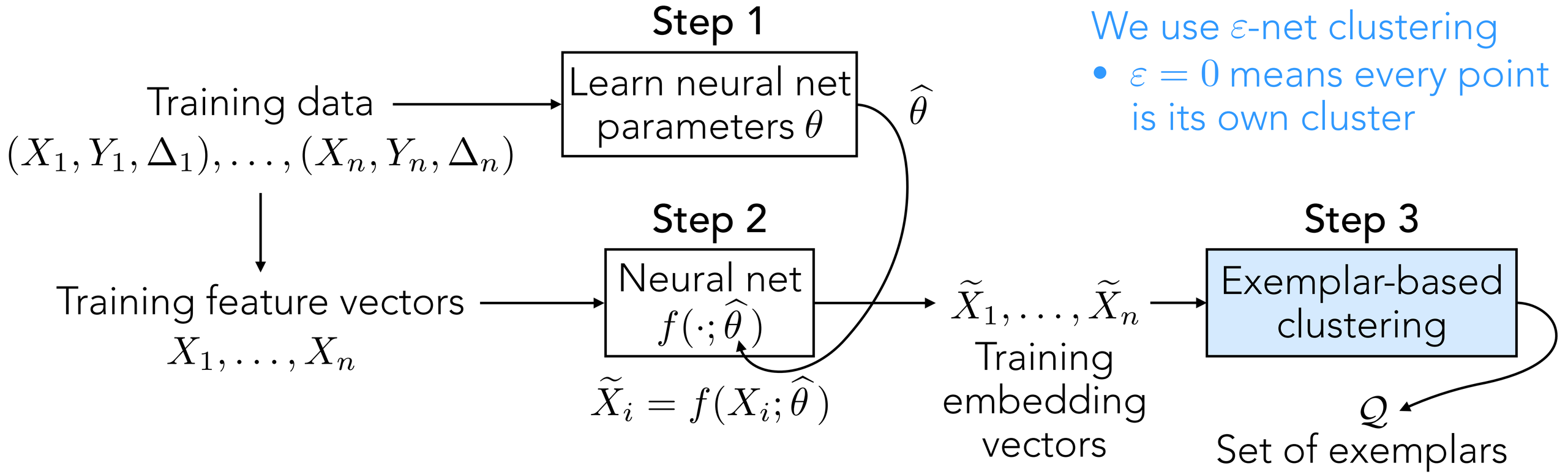

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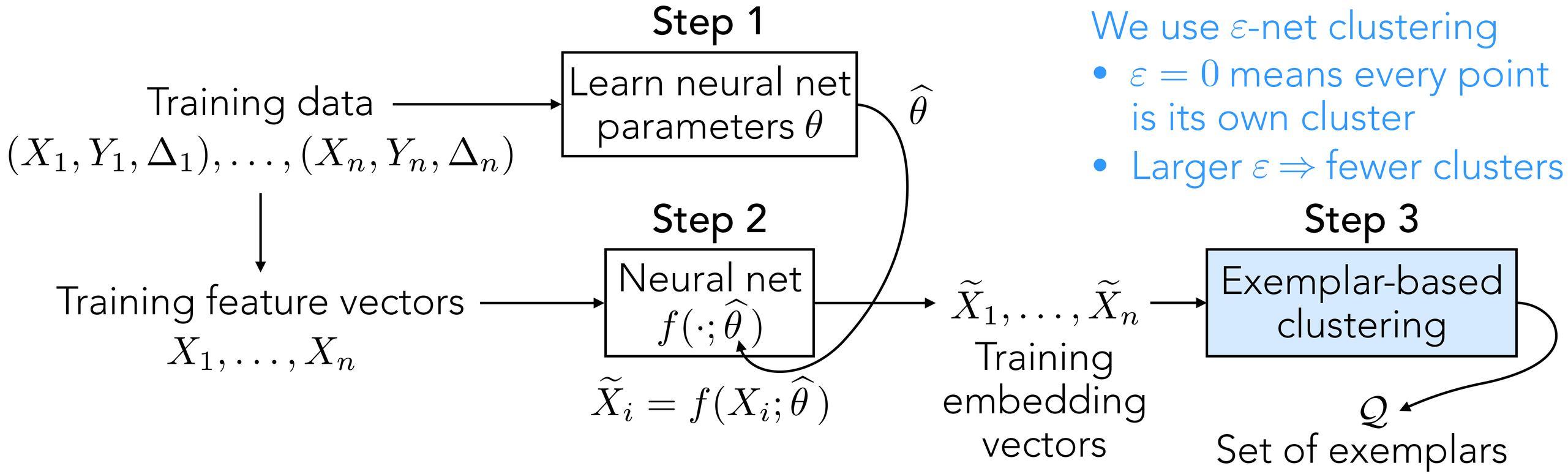

We use ε-net clustering

- $\varepsilon = 0$ means every point is its own cluster
- Larger $\varepsilon \Rightarrow$ fewer clusters

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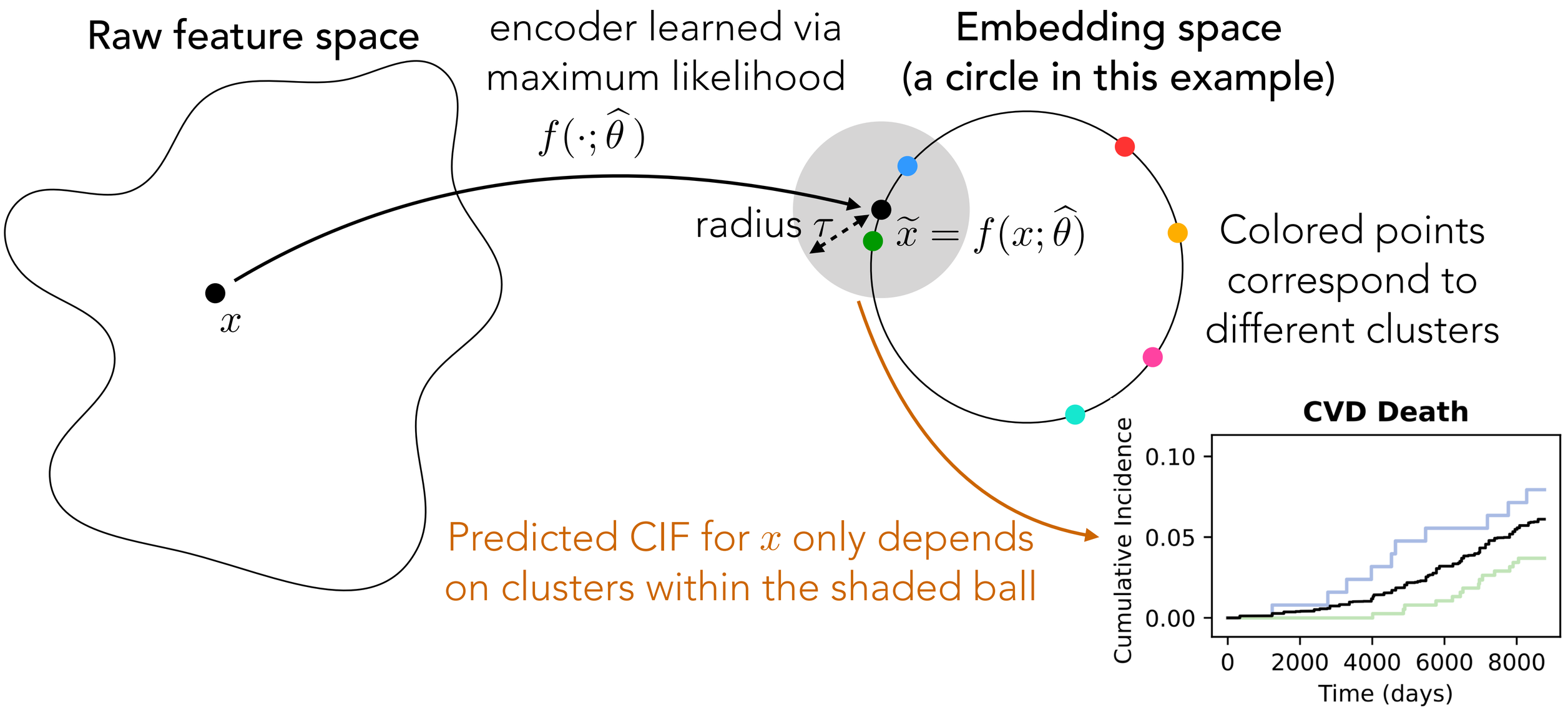
Reminder: DKAJ High-Level Idea



DKAJ Prediction

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Takeaway: CIFs for x are predicted using only close-enough exemplars (in embedding space)

Numerical Experiments

Benchmark

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- 4 standard datasets each with 56/14/30 train/val/test split

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Test set C^{td} index (mean±std dev across 10 experimental repeats) for 4 datasets (each with 2 event types): **bold** = best, blue = 2nd best

Dataset	Method	Critical Event 1	Critical Event 2
PBC [Fleming & Harrington 1991] N=1945, # features=15	Fine & Gray	0.8212±0.0119	0.8769±0.0203
	Cause-specific Cox	0.8292±0.0100	0.9091±0.0134
	DeepHit	0.8407±0.0131	0.9060±0.0130
	DSM	0.8319±0.0115	0.9039±0.0189
	NeuralFG	0.8363±0.0150	0.9120±0.0149
	SurvivalBoost	**0.8719±0.0107**	**0.9360±0.0114**
	DKAJ (ours)	0.8351±0.0106	0.8782±0.0412
Framingham [Kannel & McGee 1979] N=4434, # features=18	Fine & Gray	0.7733±0.0106	0.7144±0.0182
	Cause-specific Cox	**0.7751±0.0107**	**0.7160±0.0174**
	DeepHit	0.7423±0.0198	0.6957±0.0195
	DSM	0.7664±0.0170	0.7095±0.0184
	NeuralFG	0.6833±0.0619	0.6978±0.0287
	SurvivalBoost	0.7645±0.0152	0.7031±0.0156
	DKAJ (ours)	0.7656±0.0147	0.7034±0.0185
SEER [https://seer.cancer.gov] N=24907, # features=22	Fine & Gray	0.8151±0.0021	0.8478±0.0099
	Cause-specific Cox	0.7630±0.0085	0.8413±0.0132
	DeepHit	0.8239±0.0028	0.8548±0.0120
	DSM	0.7807±0.0088	0.8422±0.0119
	NeuralFG	0.7743±0.0090	0.8362±0.0138
	SurvivalBoost	**0.8418±0.0038**	**0.8583±0.0081**
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Synthetic [Lee et al 2018] N=30000, # features=12	Fine & Gray	0.5823±0.0051	0.5917±0.0071
	Cause-specific Cox	0.5808±0.0052	0.5903±0.0072
	DeepHit	0.7401±0.0067	0.7437±0.0043
	DSM	0.7280±0.0055	0.7320±0.0042
	NeuralFG	**0.7481±0.0073**	**0.7513±0.0045**
	SurvivalBoost	0.7160±0.0083	0.7190±0.0039
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	DeepHit	0.7401±0.0067	0.7437±0.0043
	DSM	0.7280±0.0055	0.7320±0.0042
	NeuralFG	**0.7481±0.0073**	**0.7513±0.0045**
	SurvivalBoost	0.7160±0.0083	0.7190±0.0039
	DKAJ (ours)	0.7399±0.0062	0.7446±0.0048

Illustration of DKAJ Model Interpretation: Framingham

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2 critical events:
CVD (cardiovascular disease) death,
Non-CVD death

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Concrete example:
focus on the 5 largest ones
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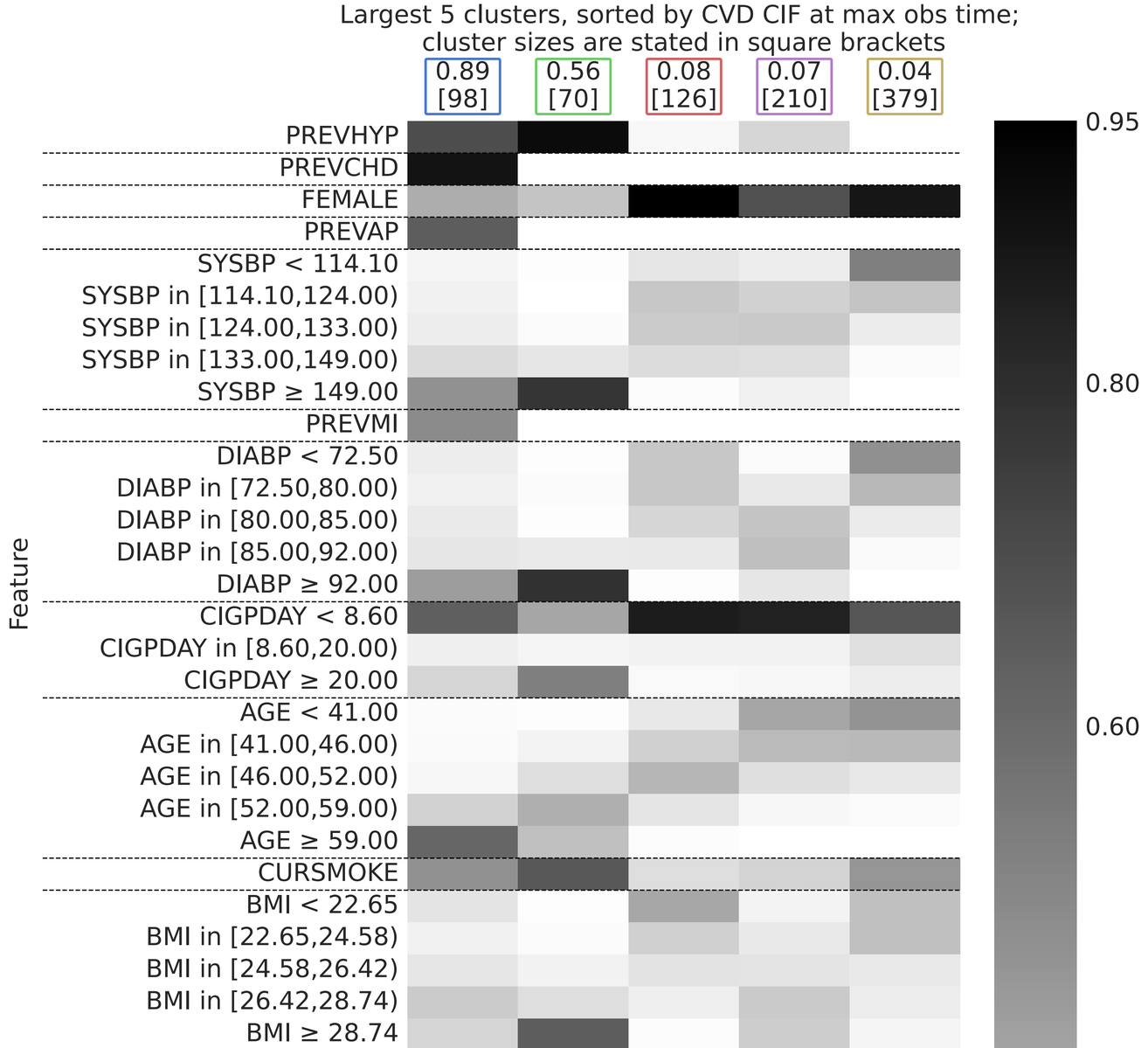

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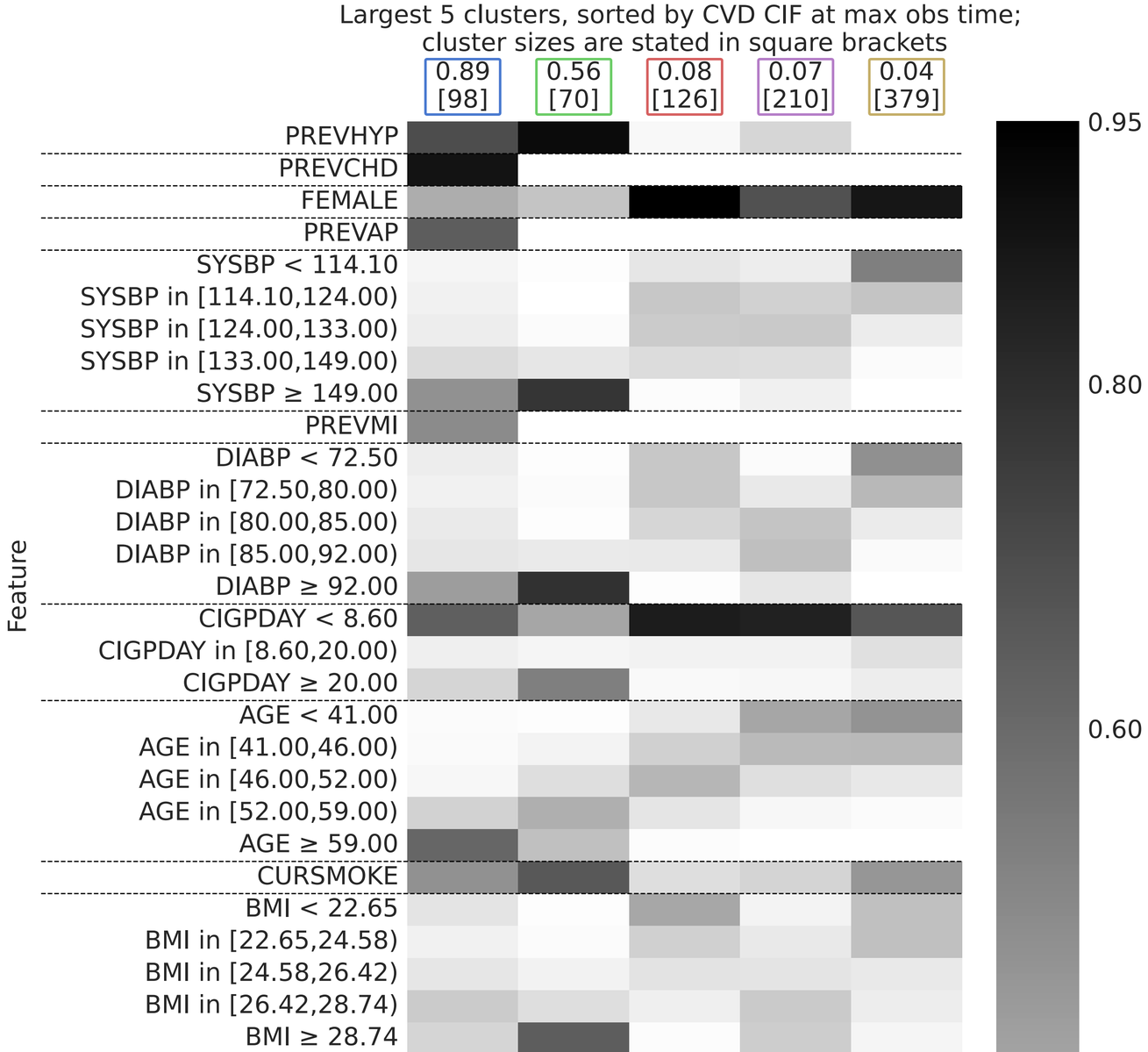

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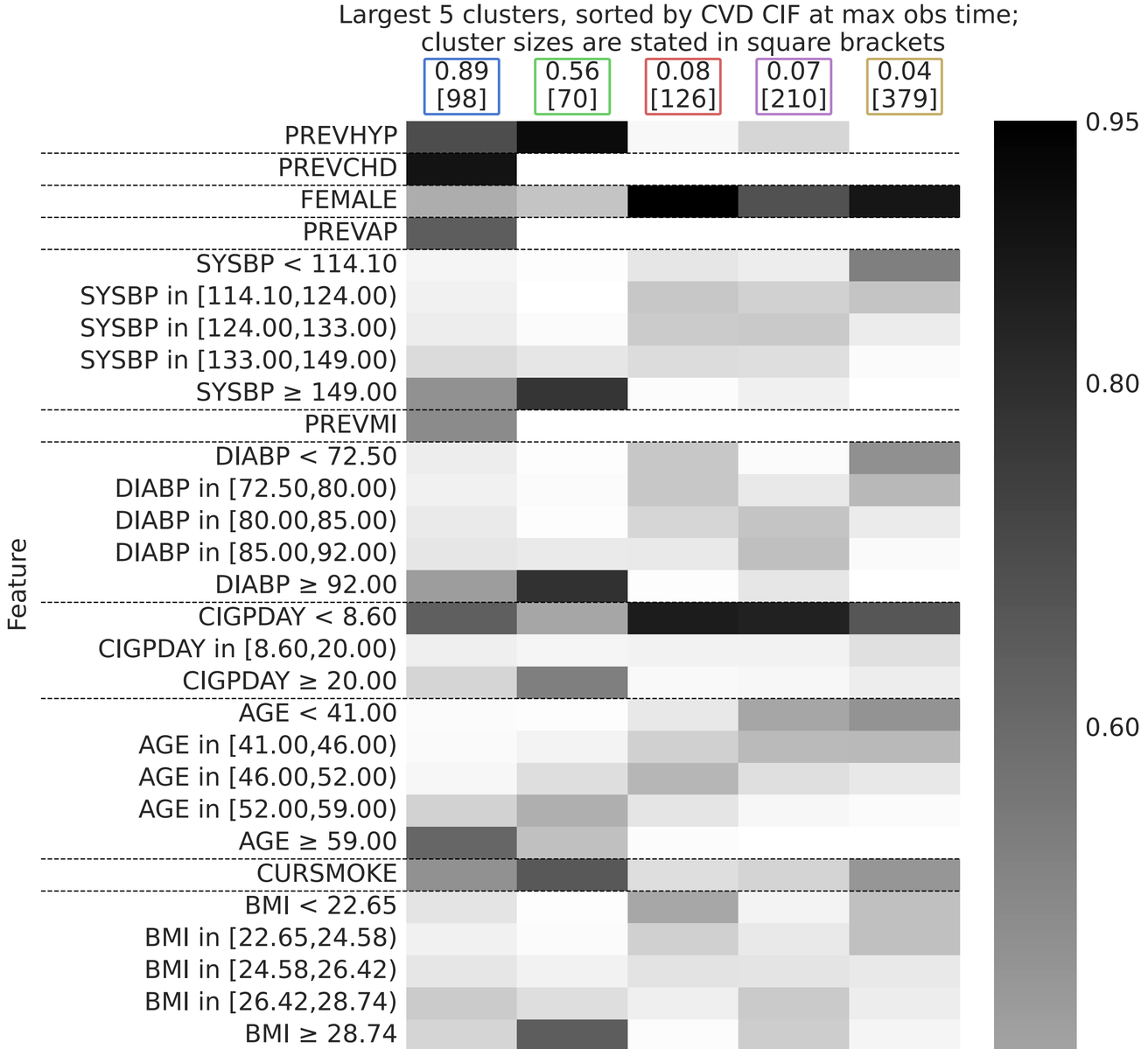

Each column: a different cluster
columns sorted by "**risk of CVD death**"

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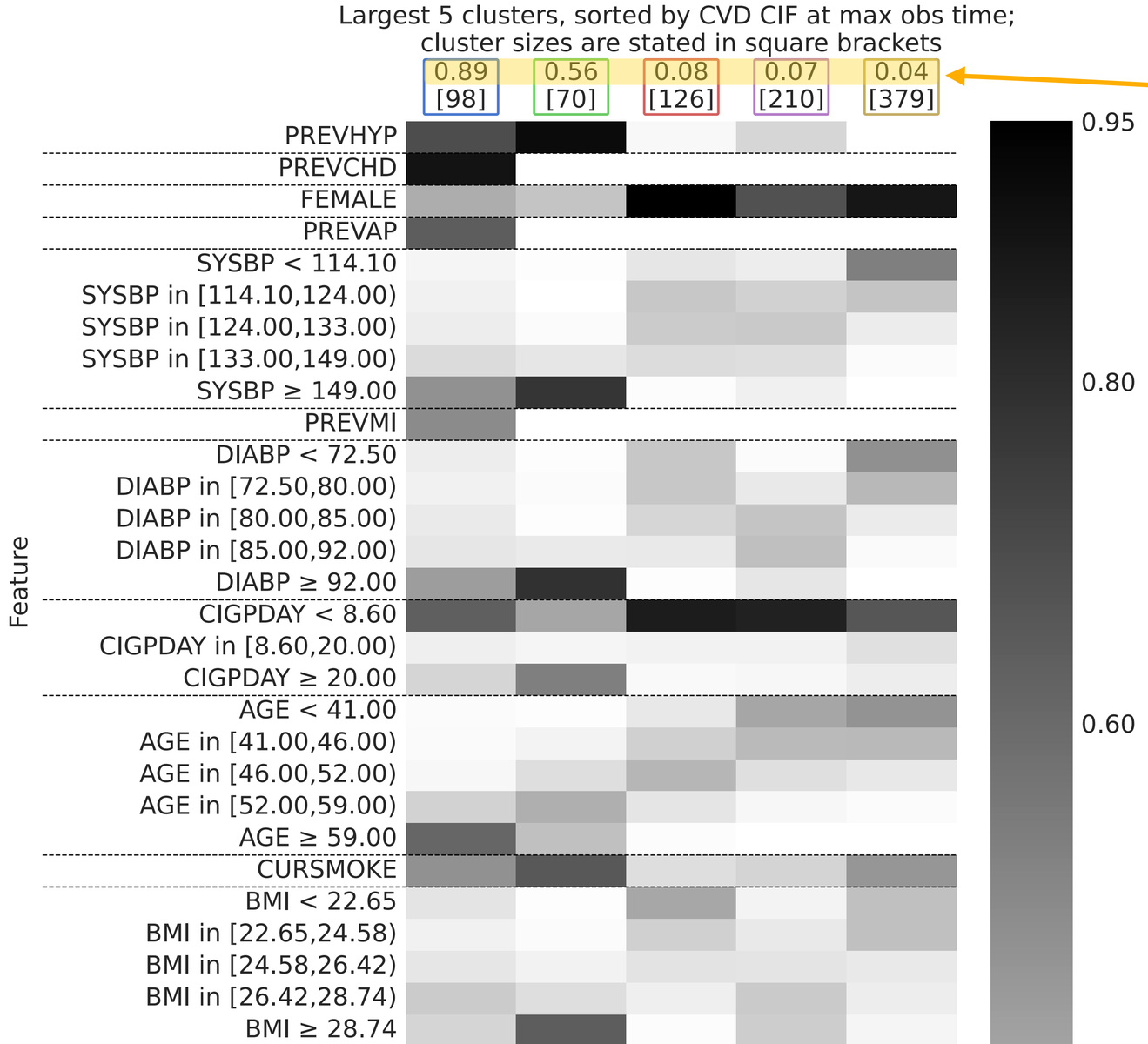

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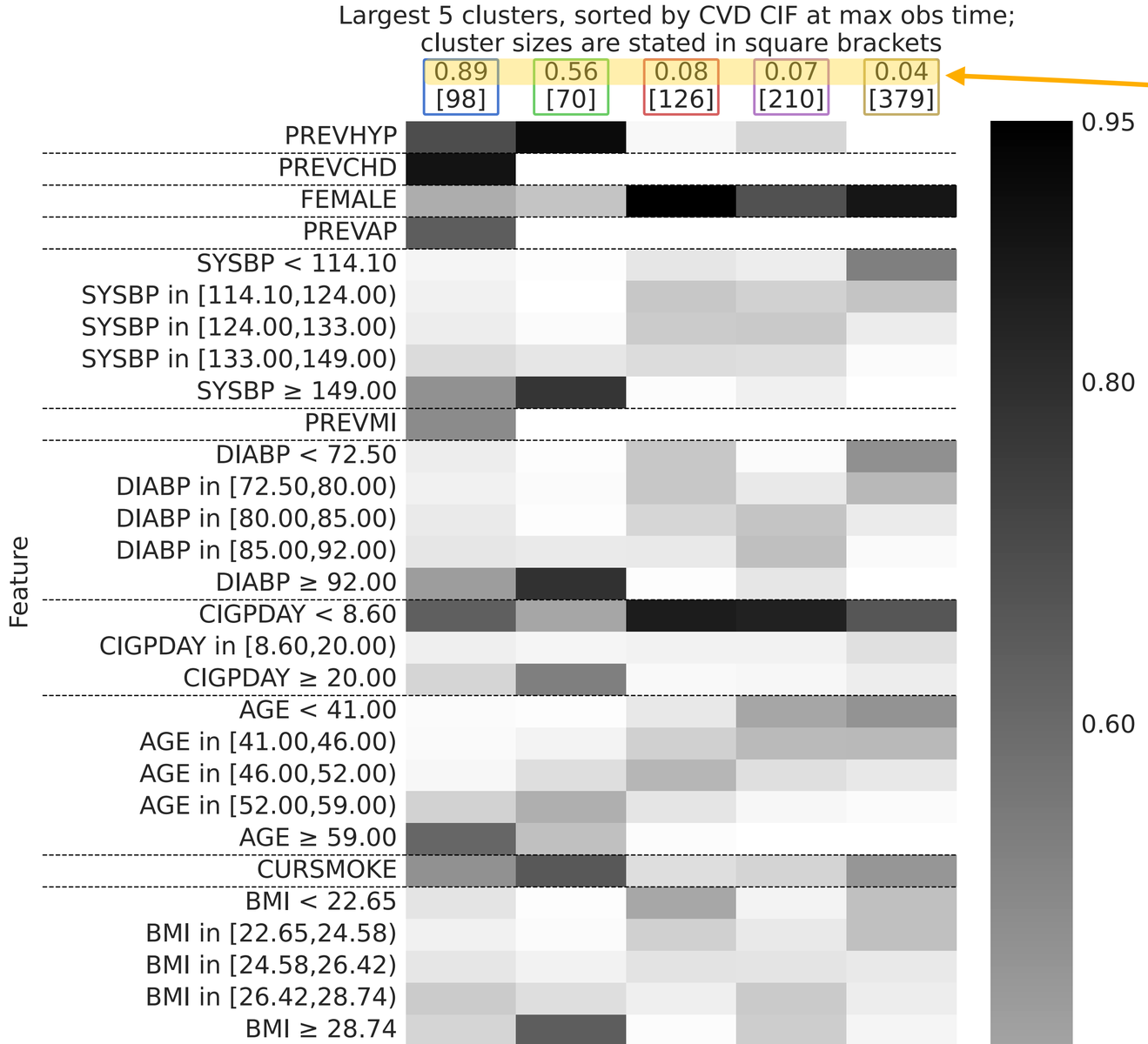

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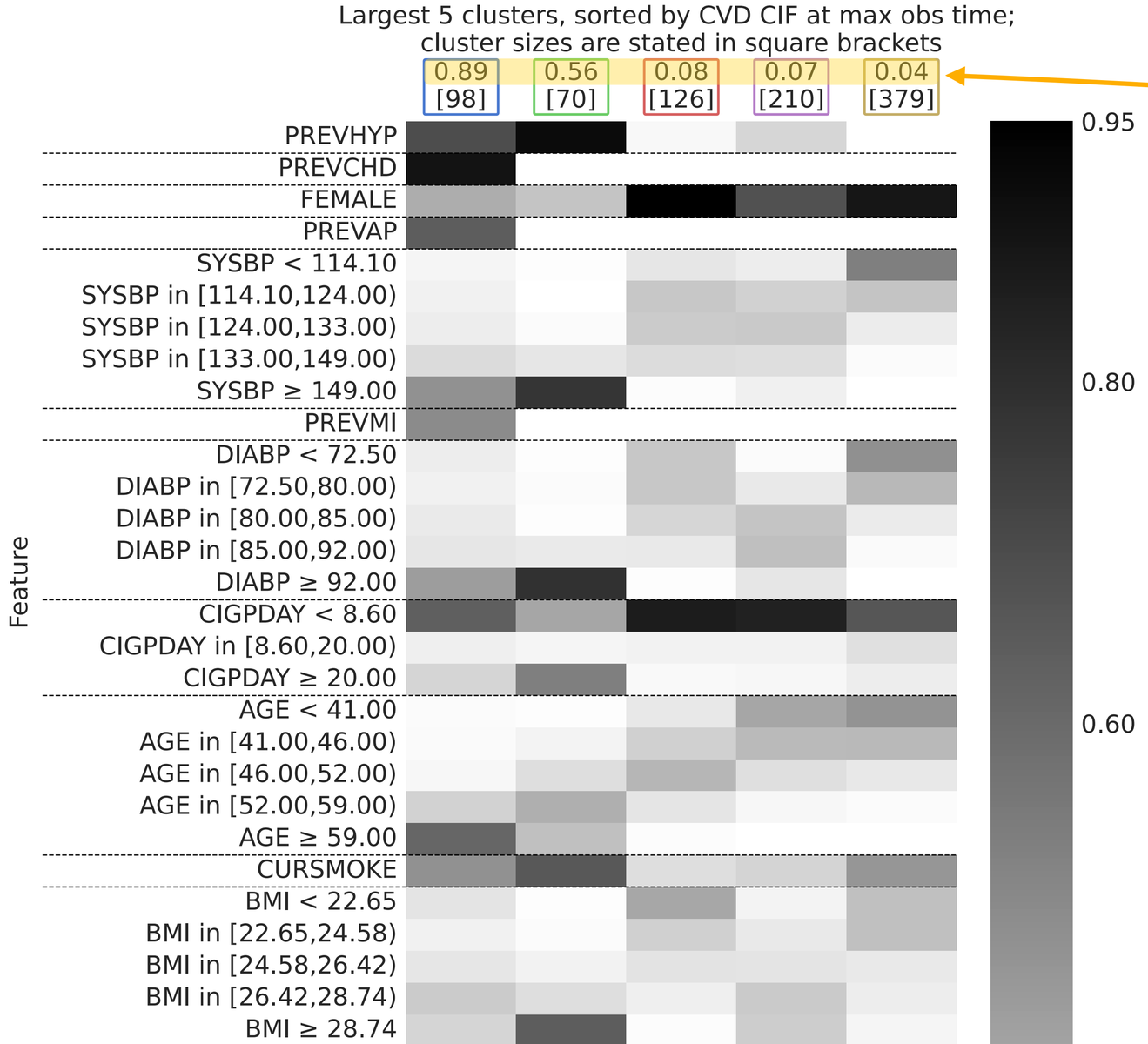

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Intensity value: fraction of people in a cluster with a particular raw feature

hypertension, coronary heart disease

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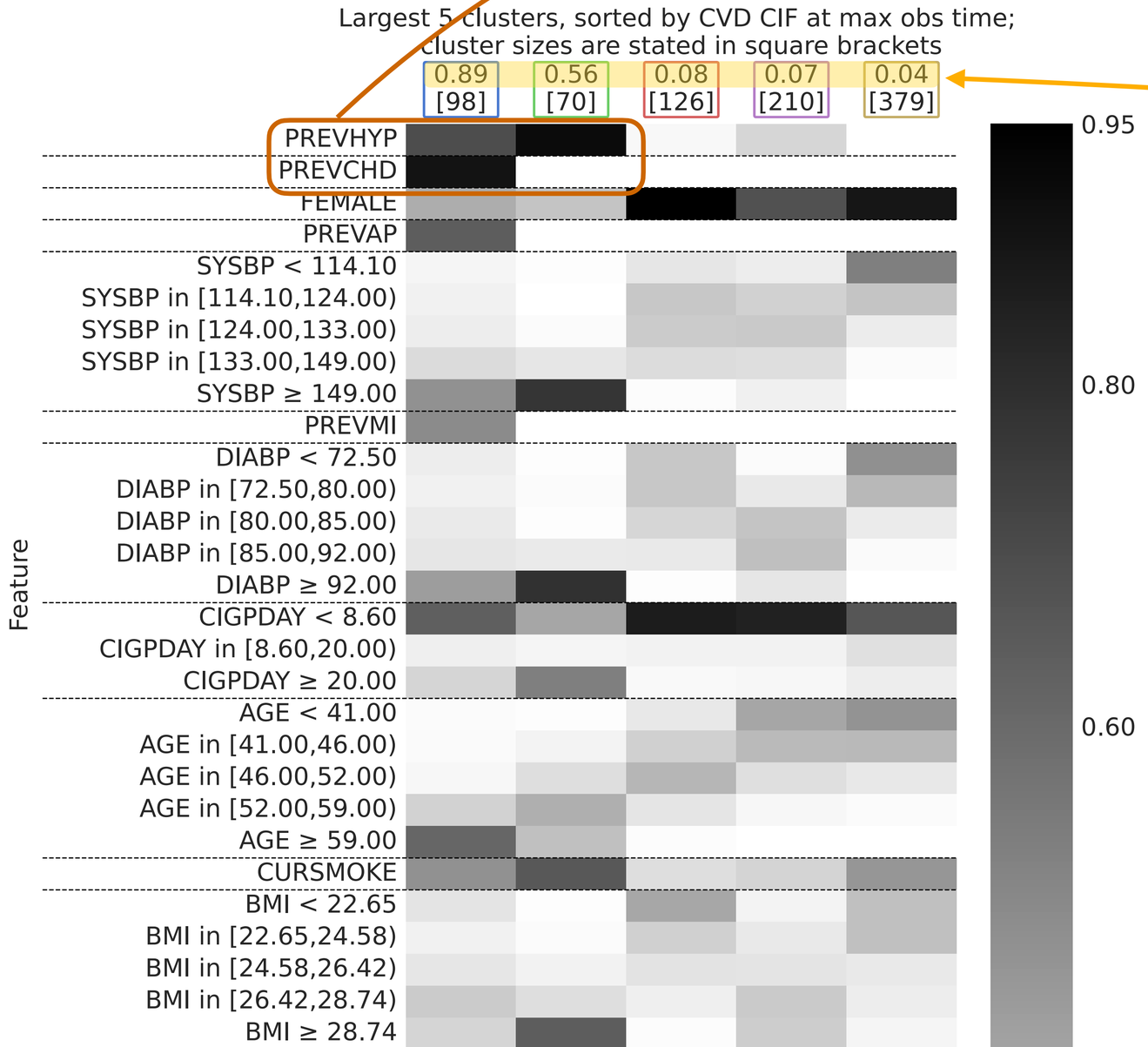

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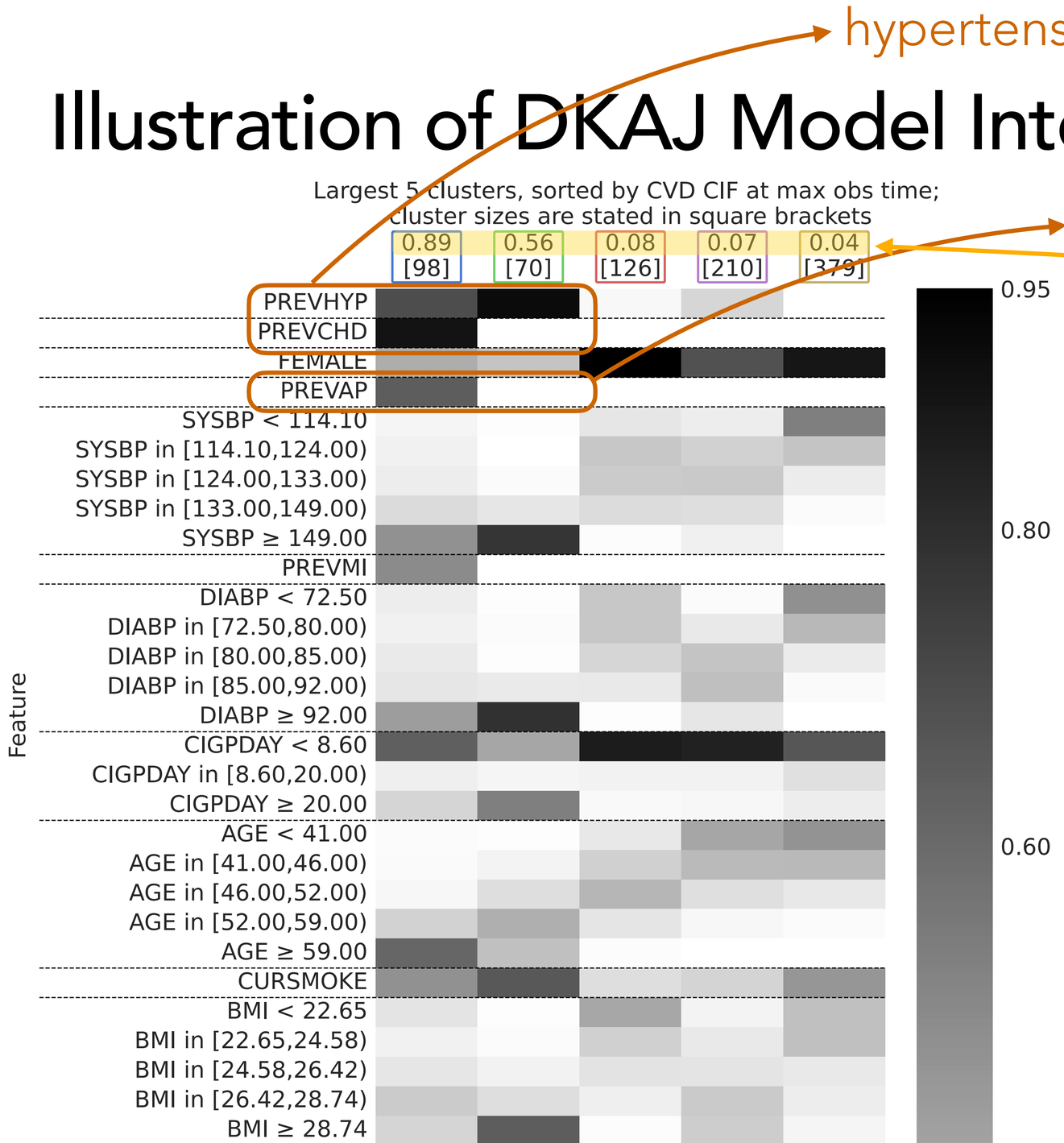

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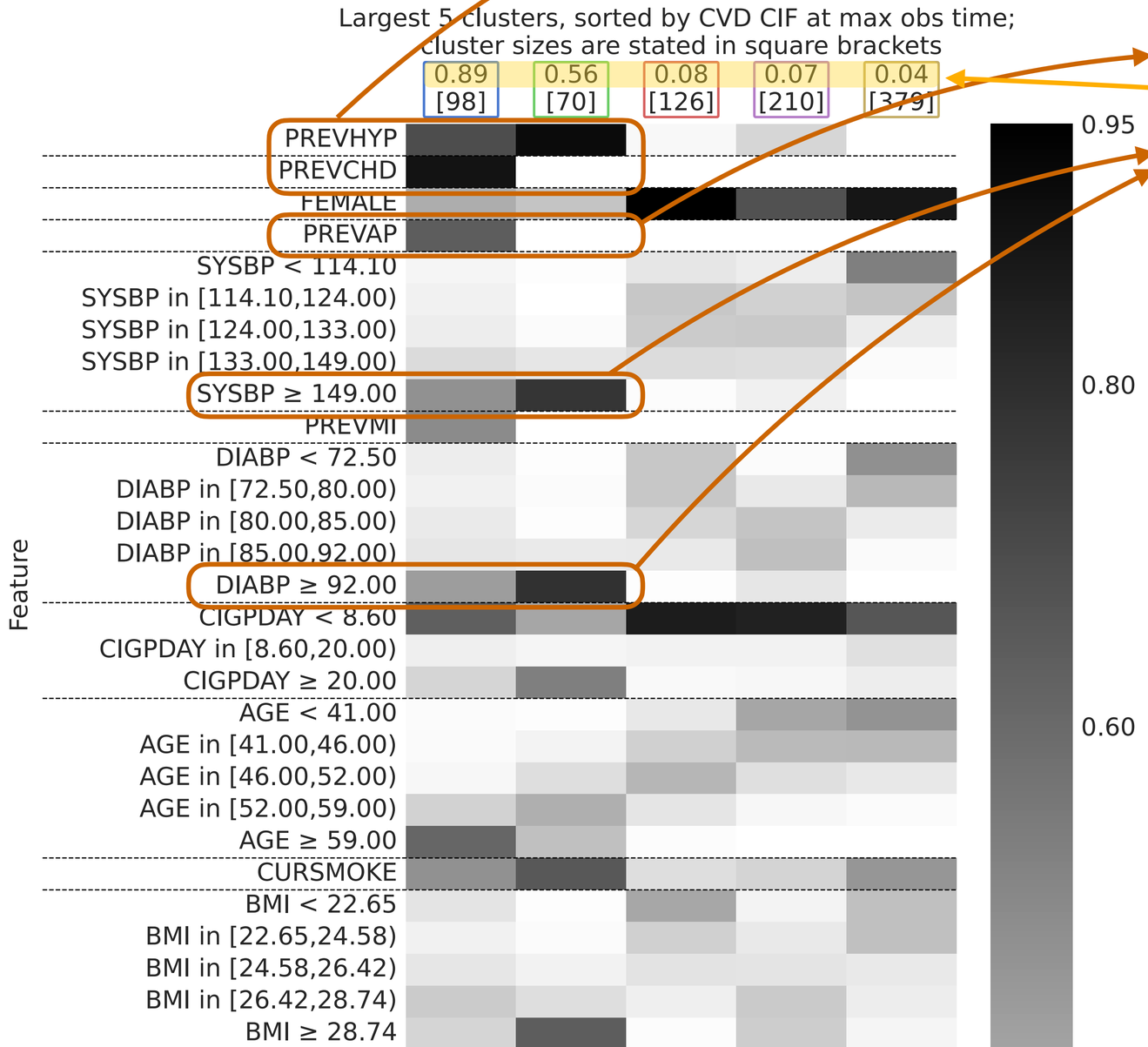

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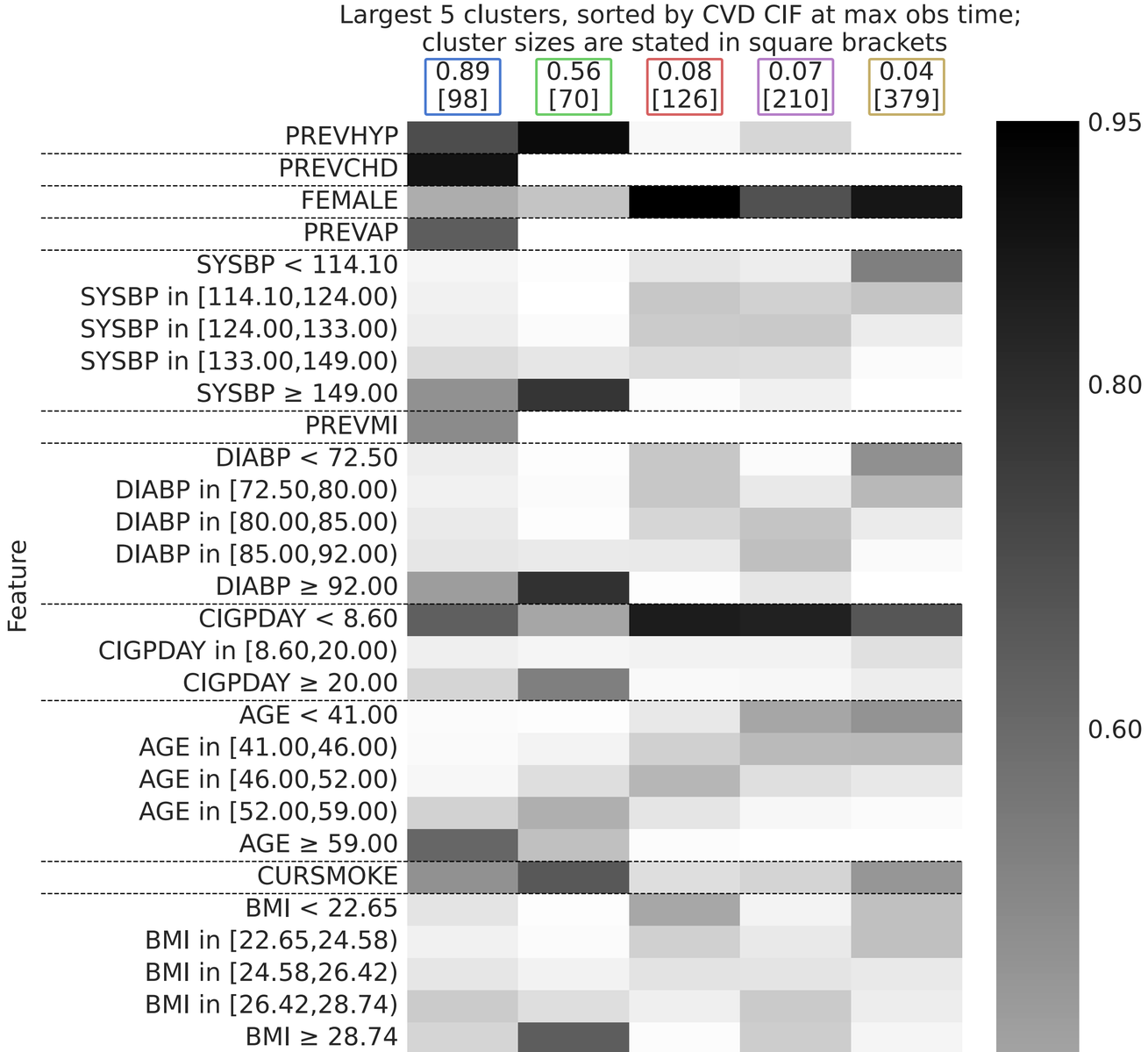

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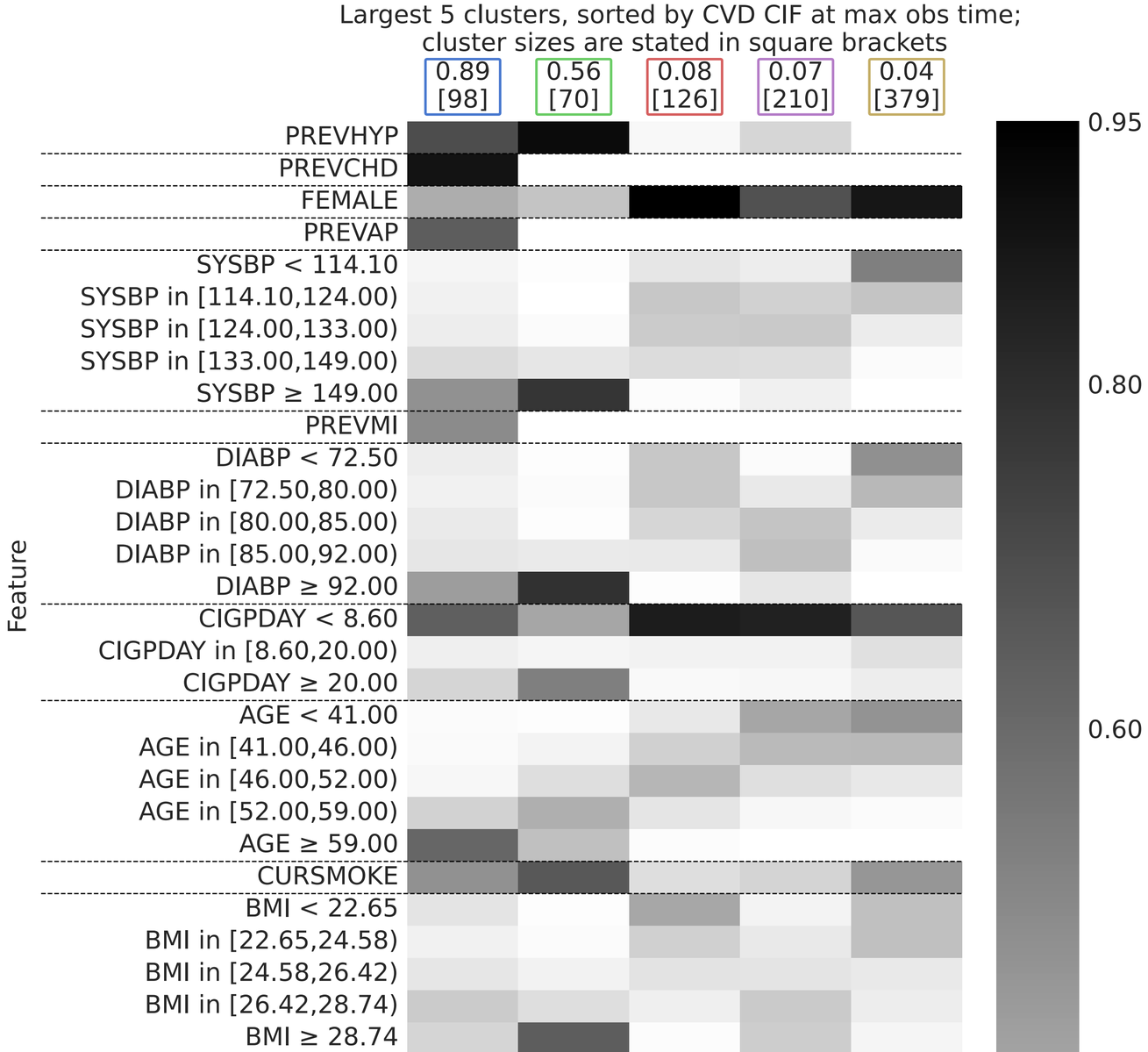

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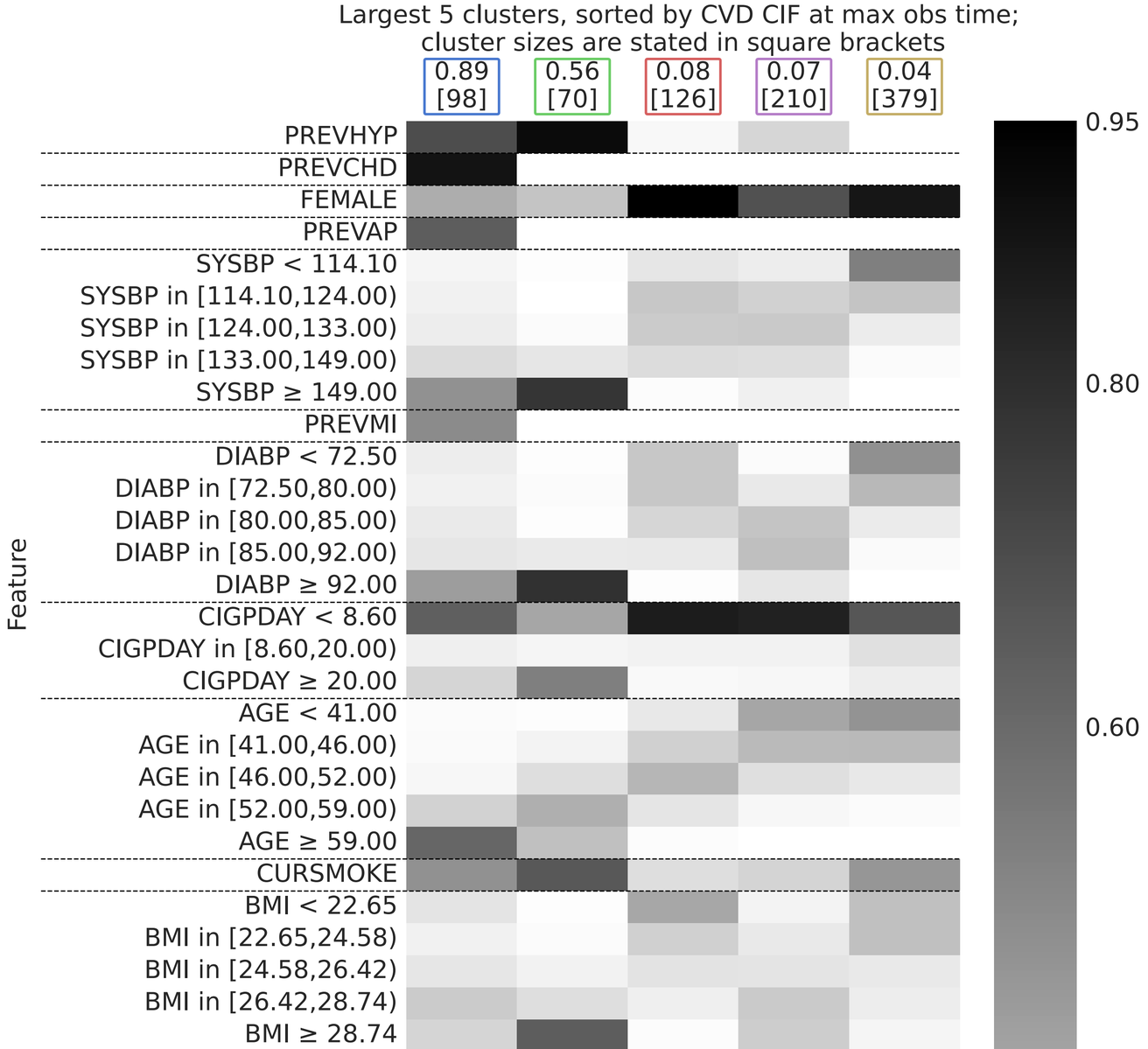

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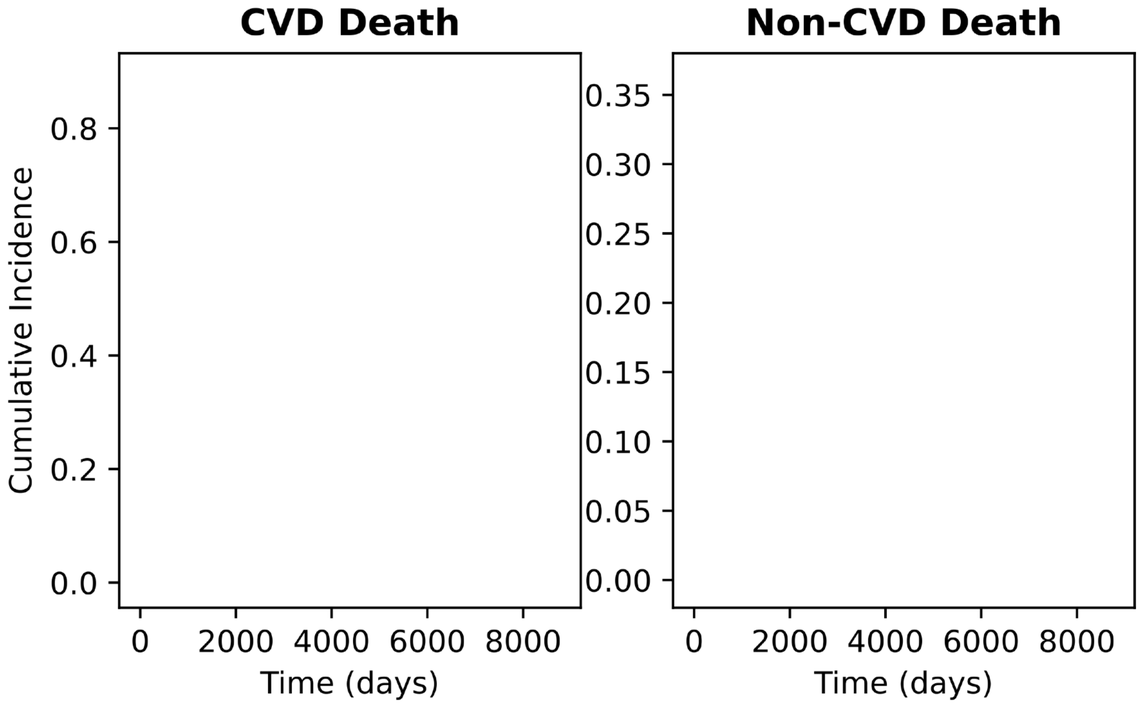

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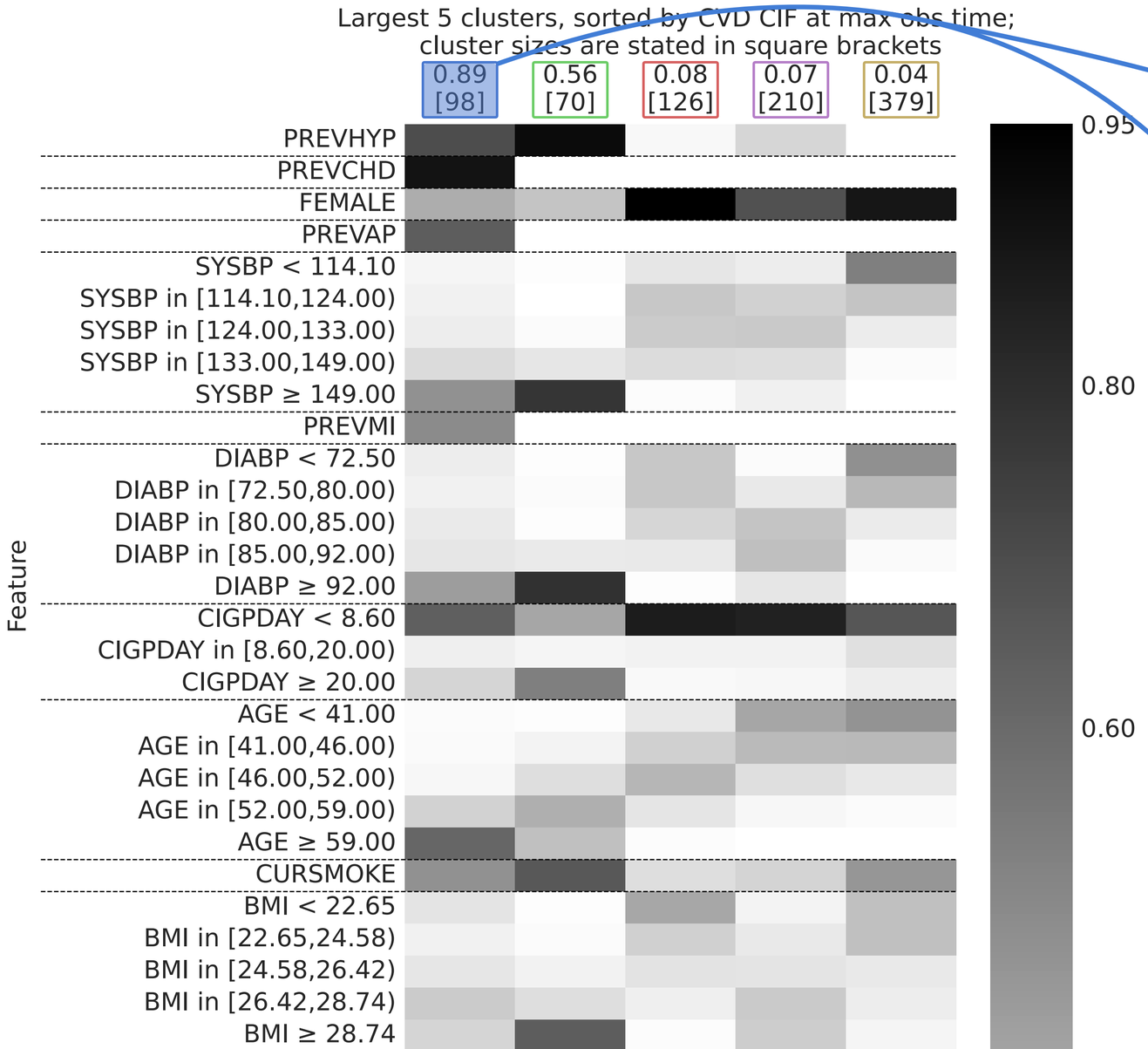

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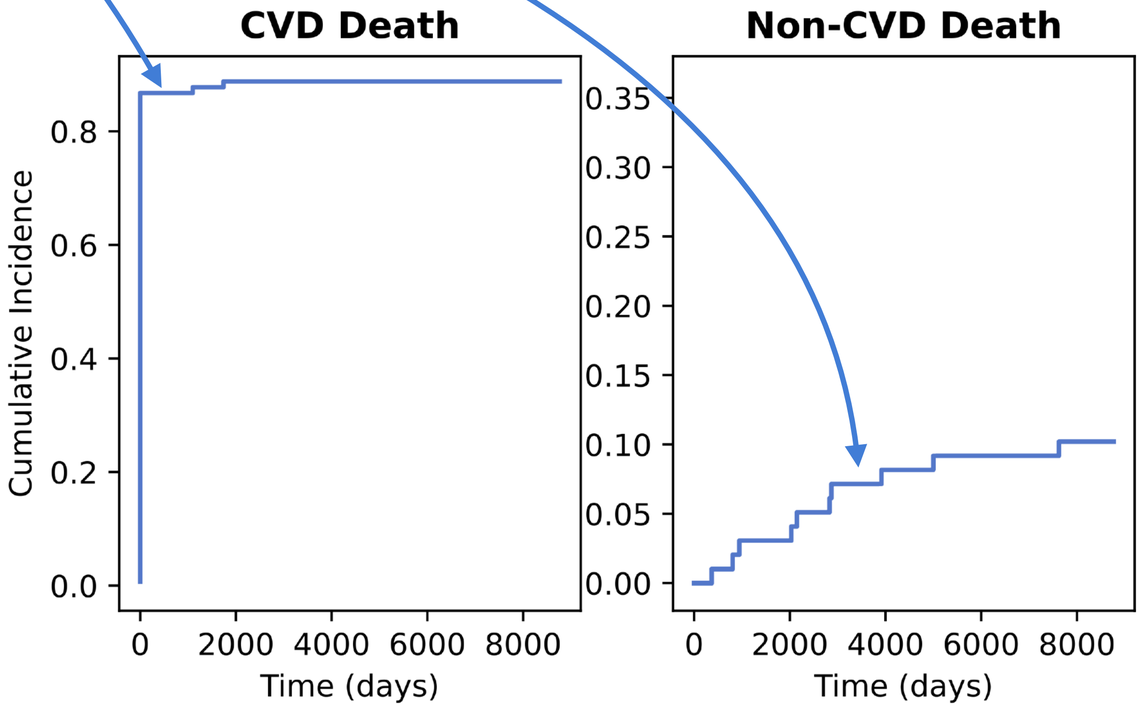

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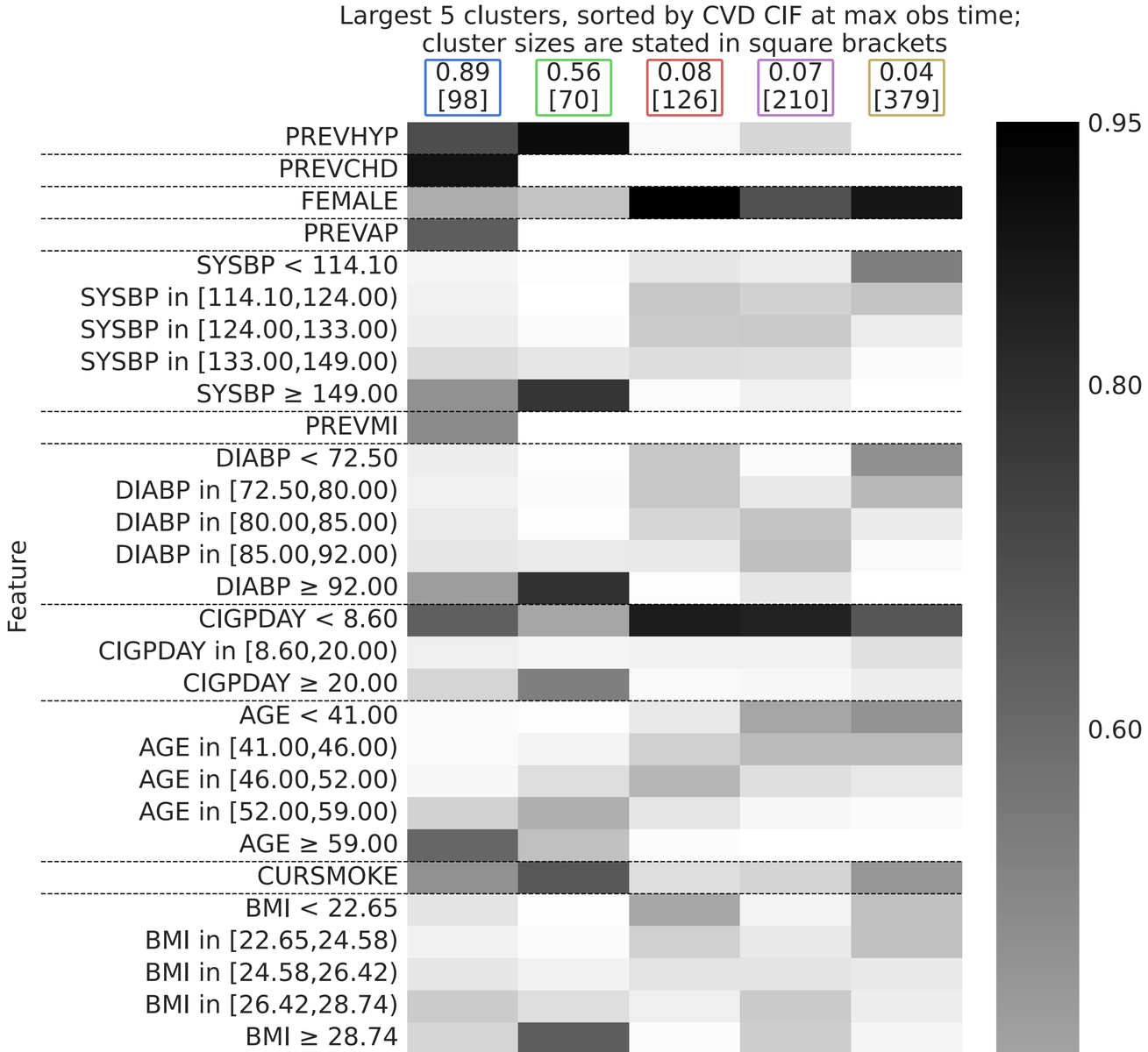

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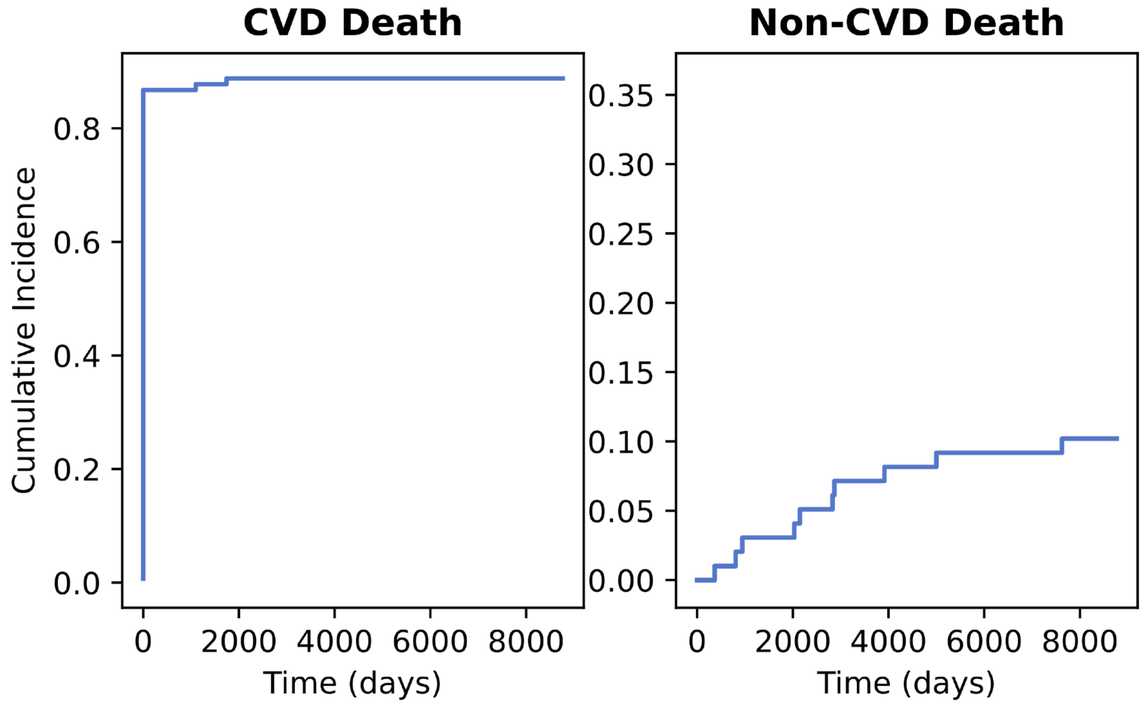

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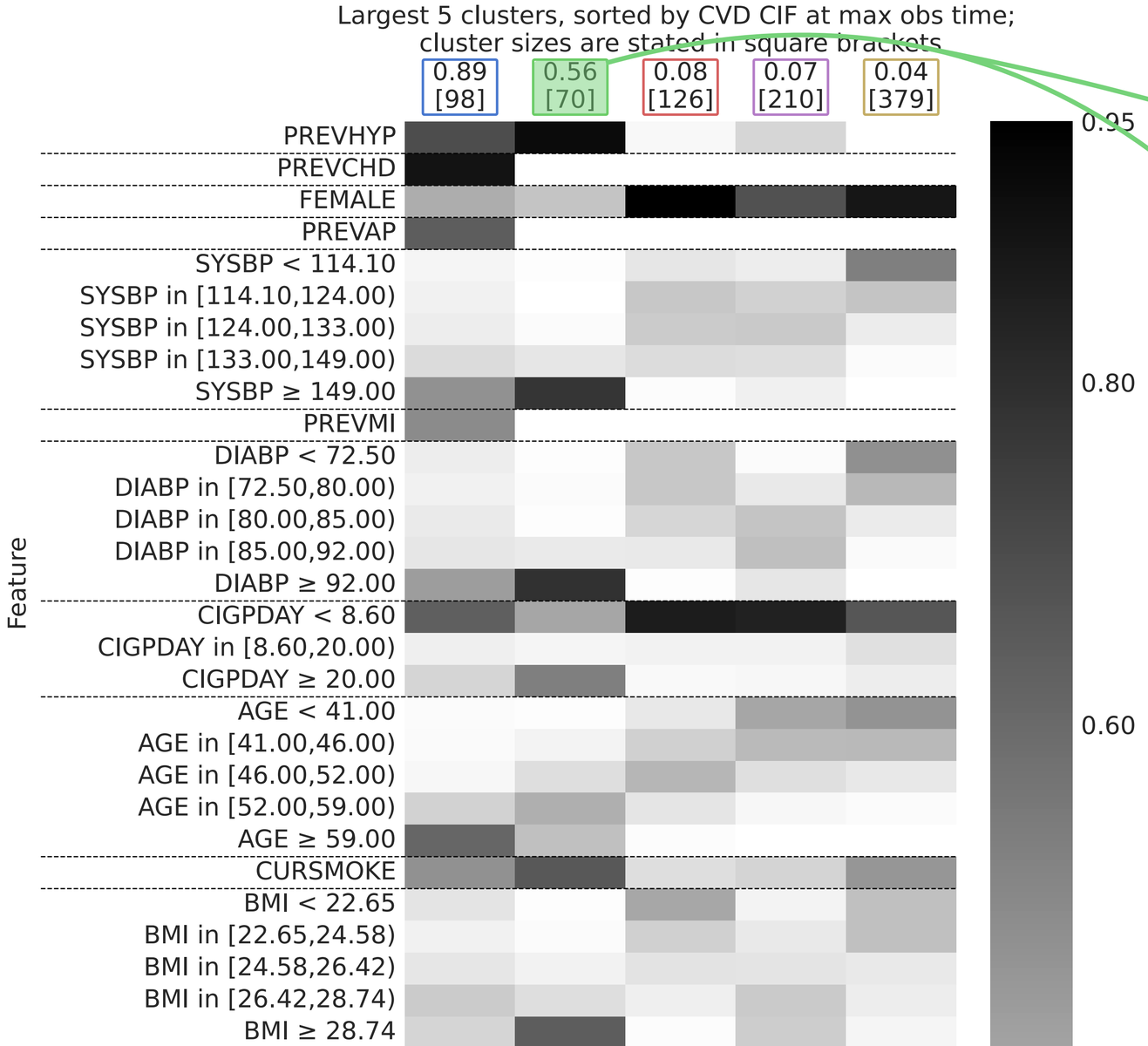

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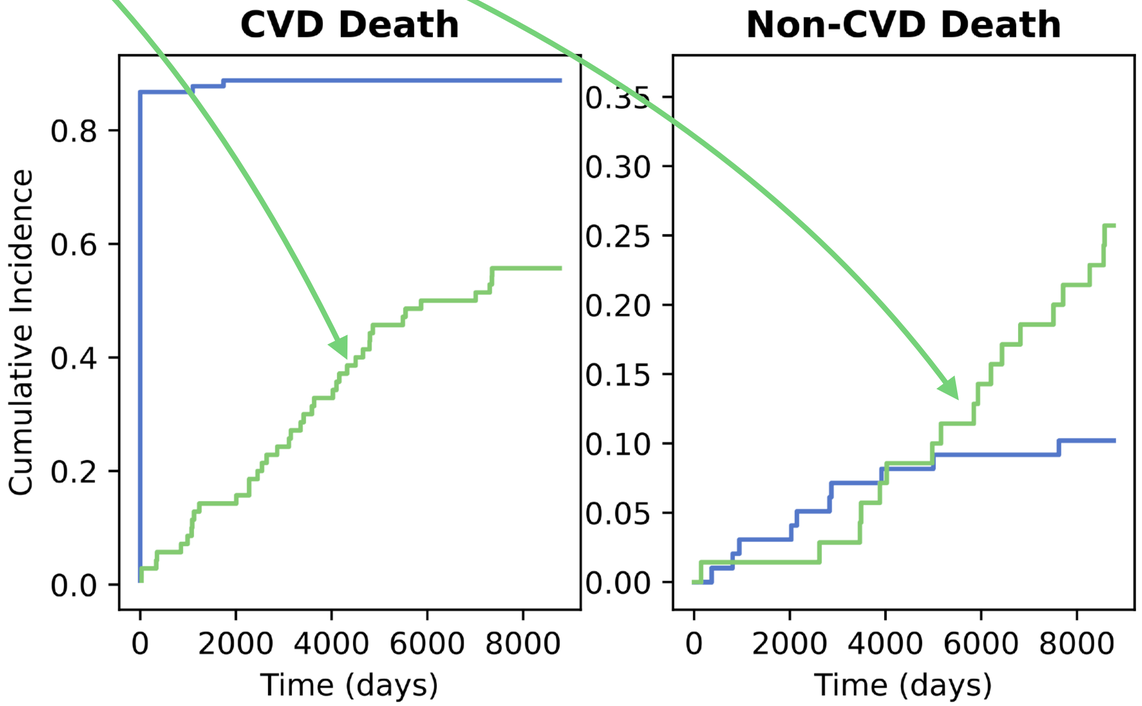

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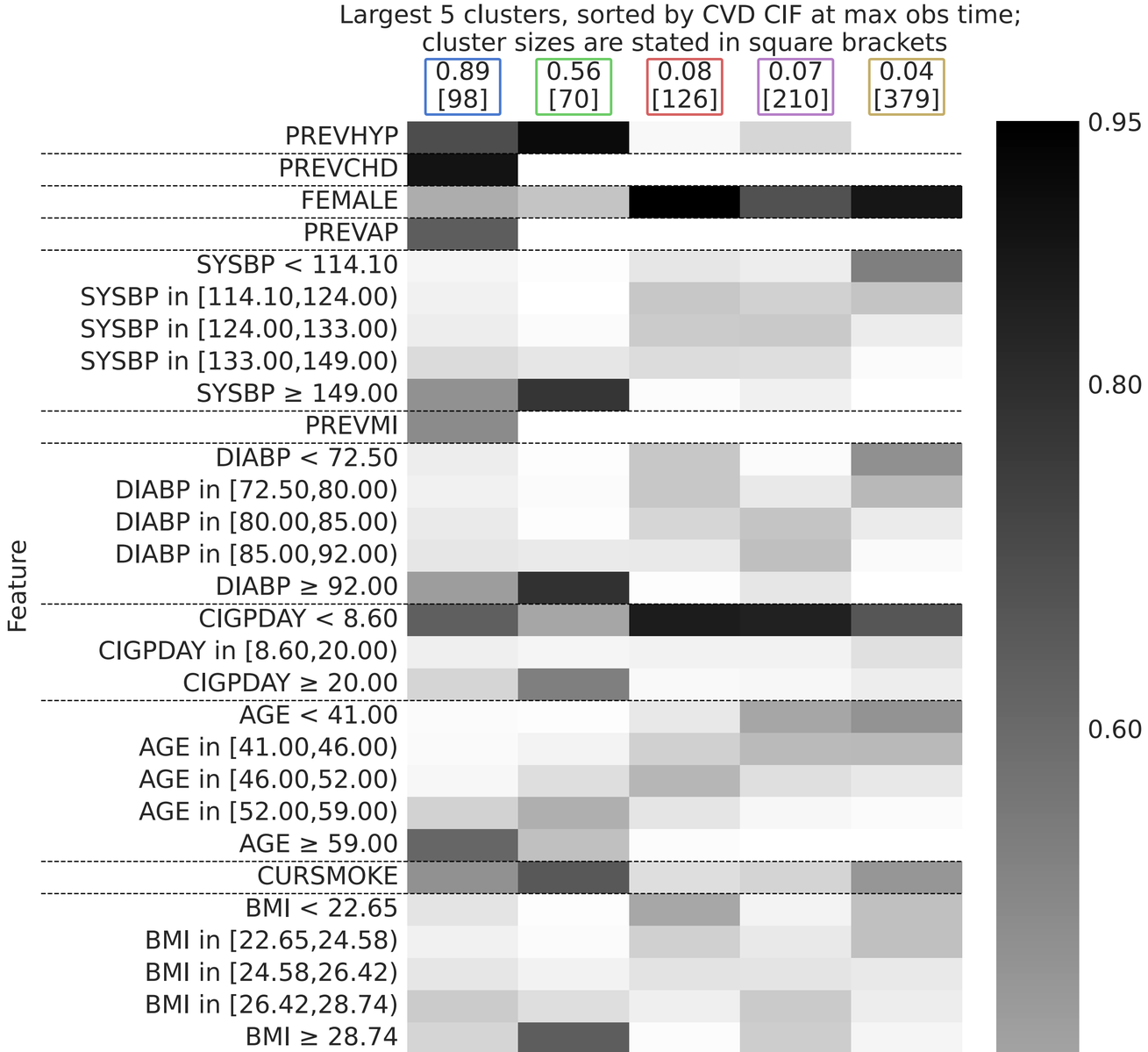

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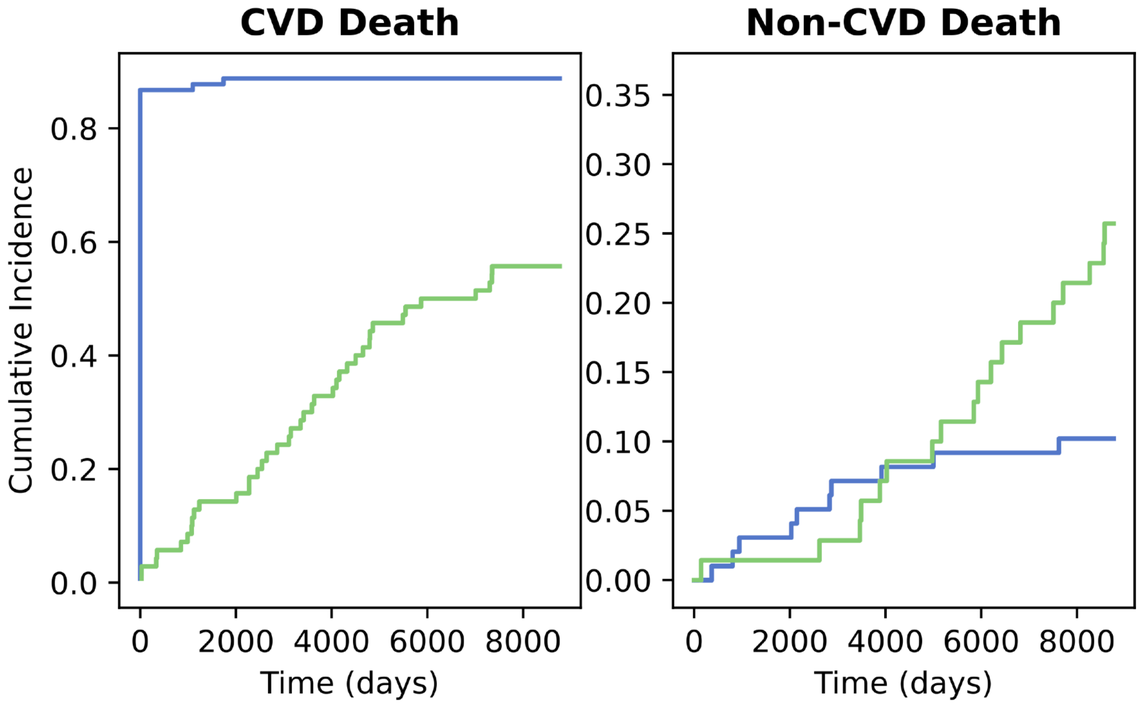

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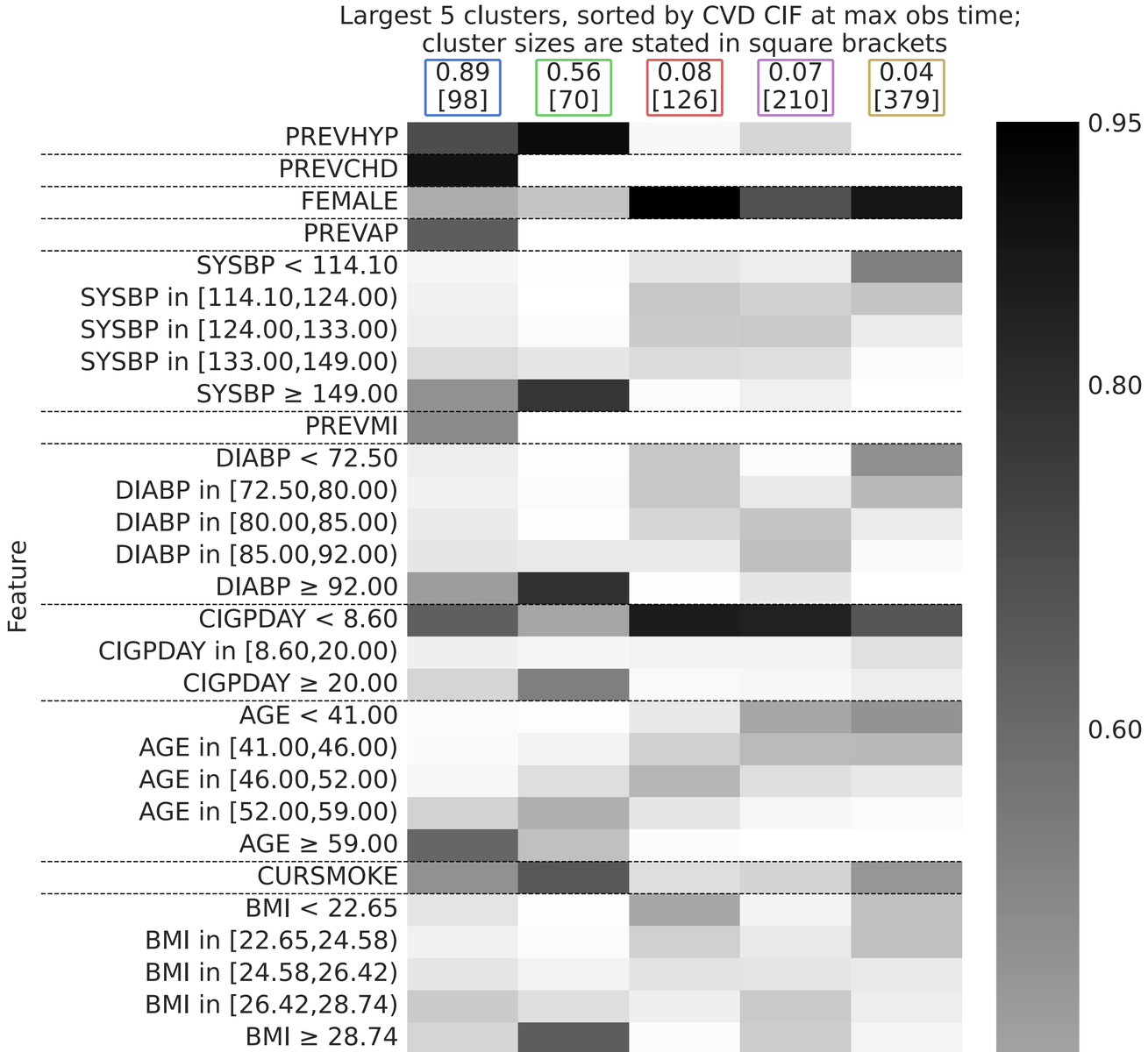

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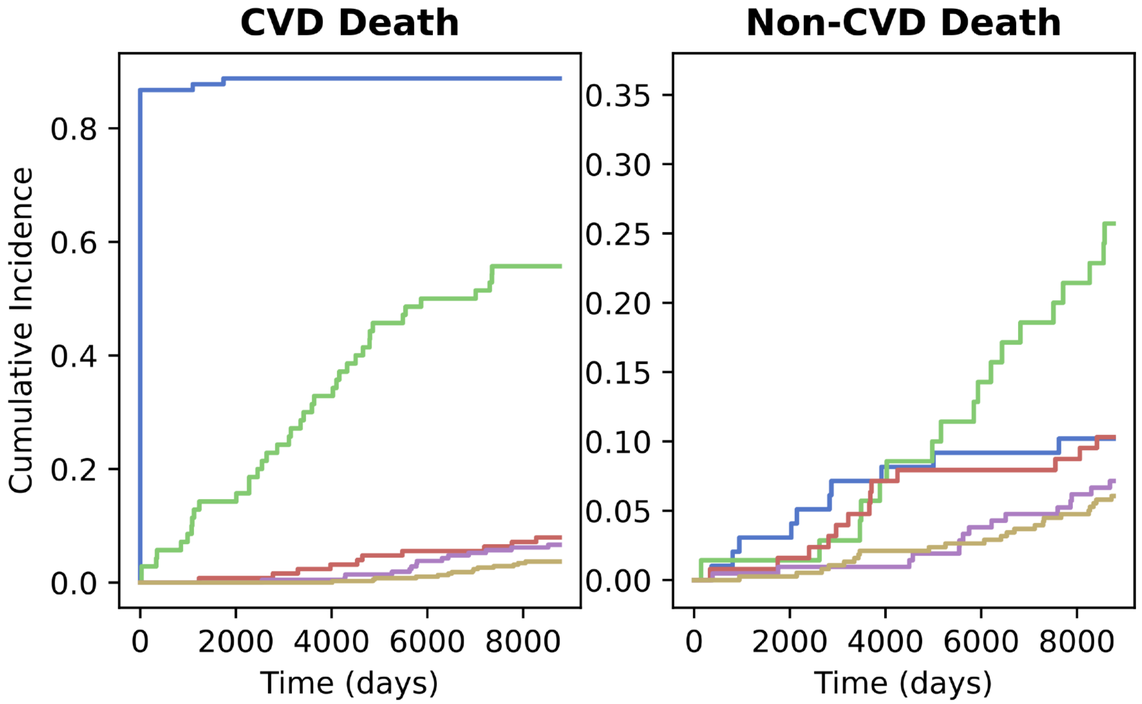

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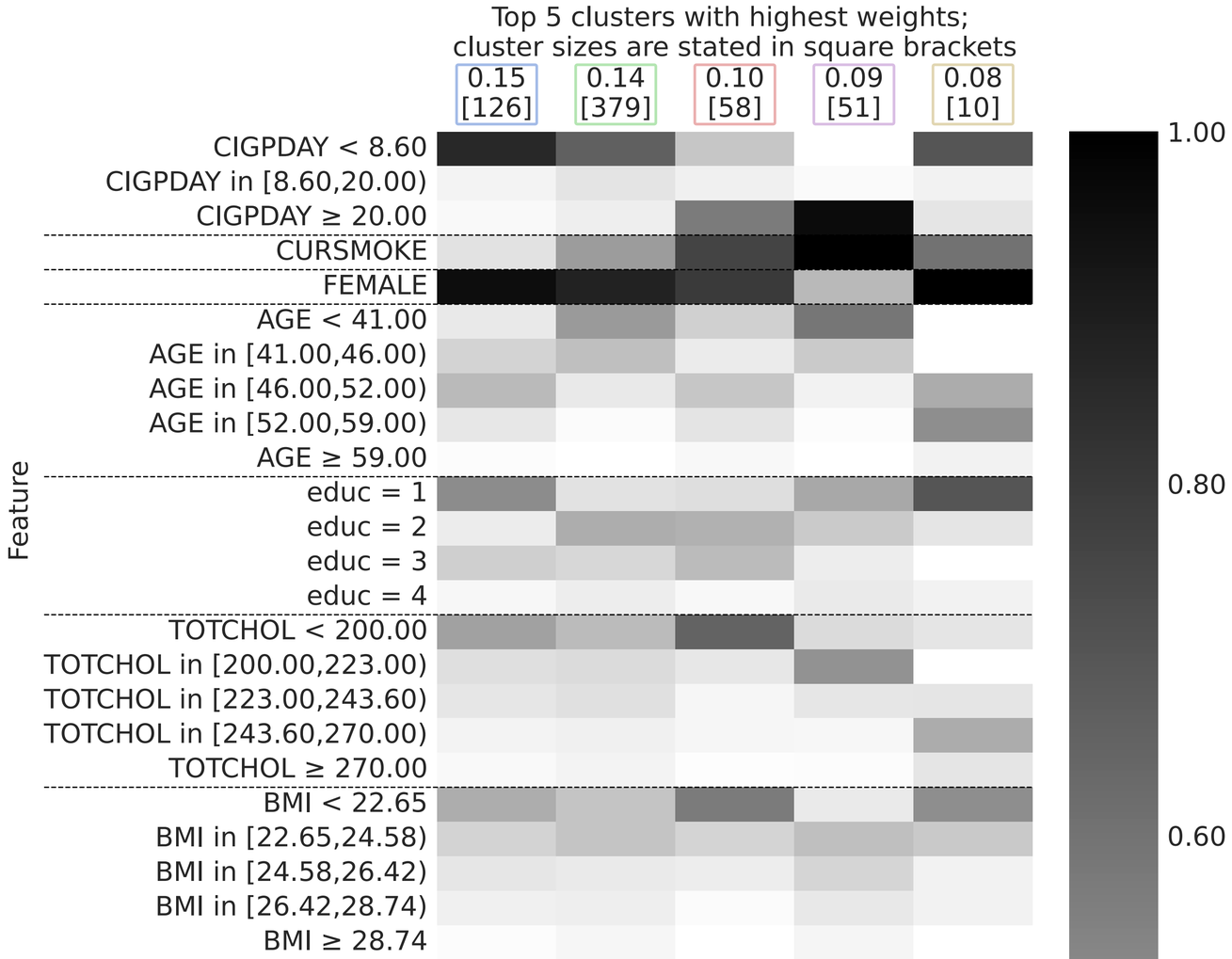

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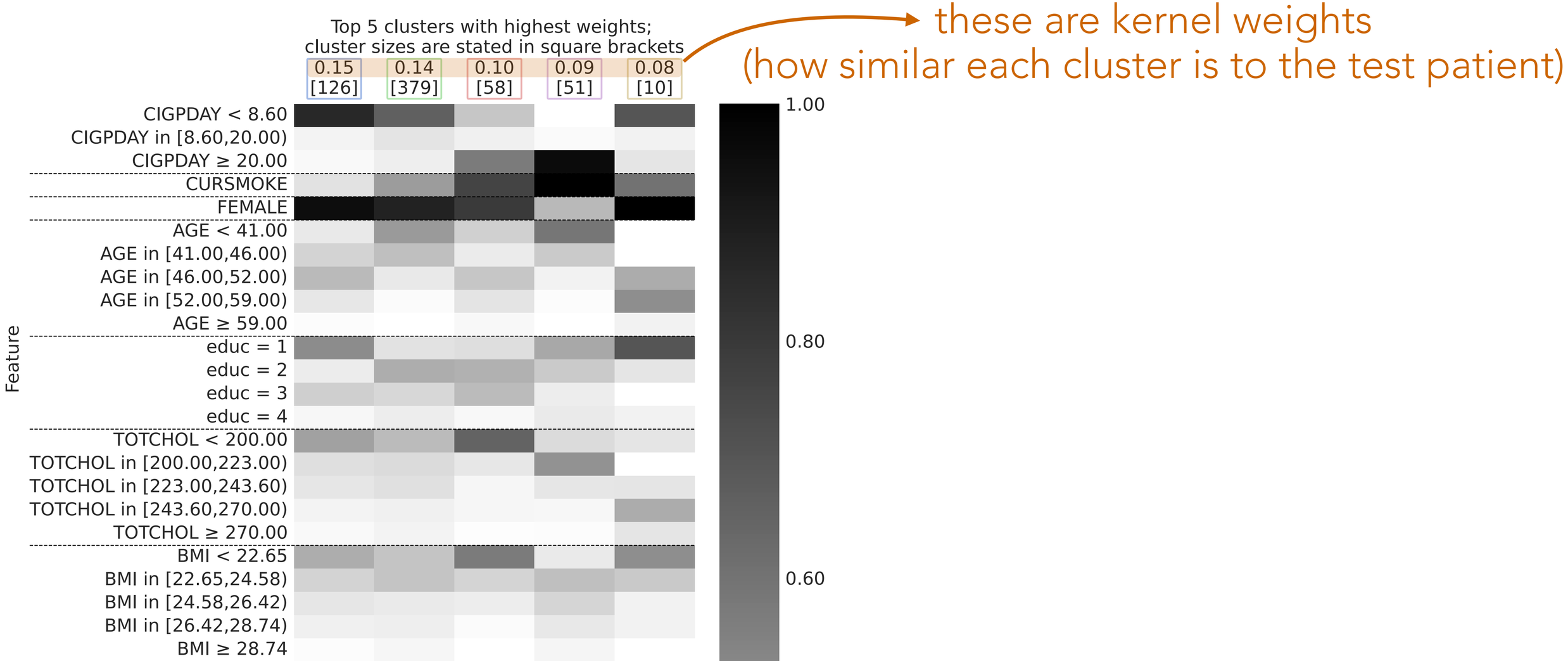

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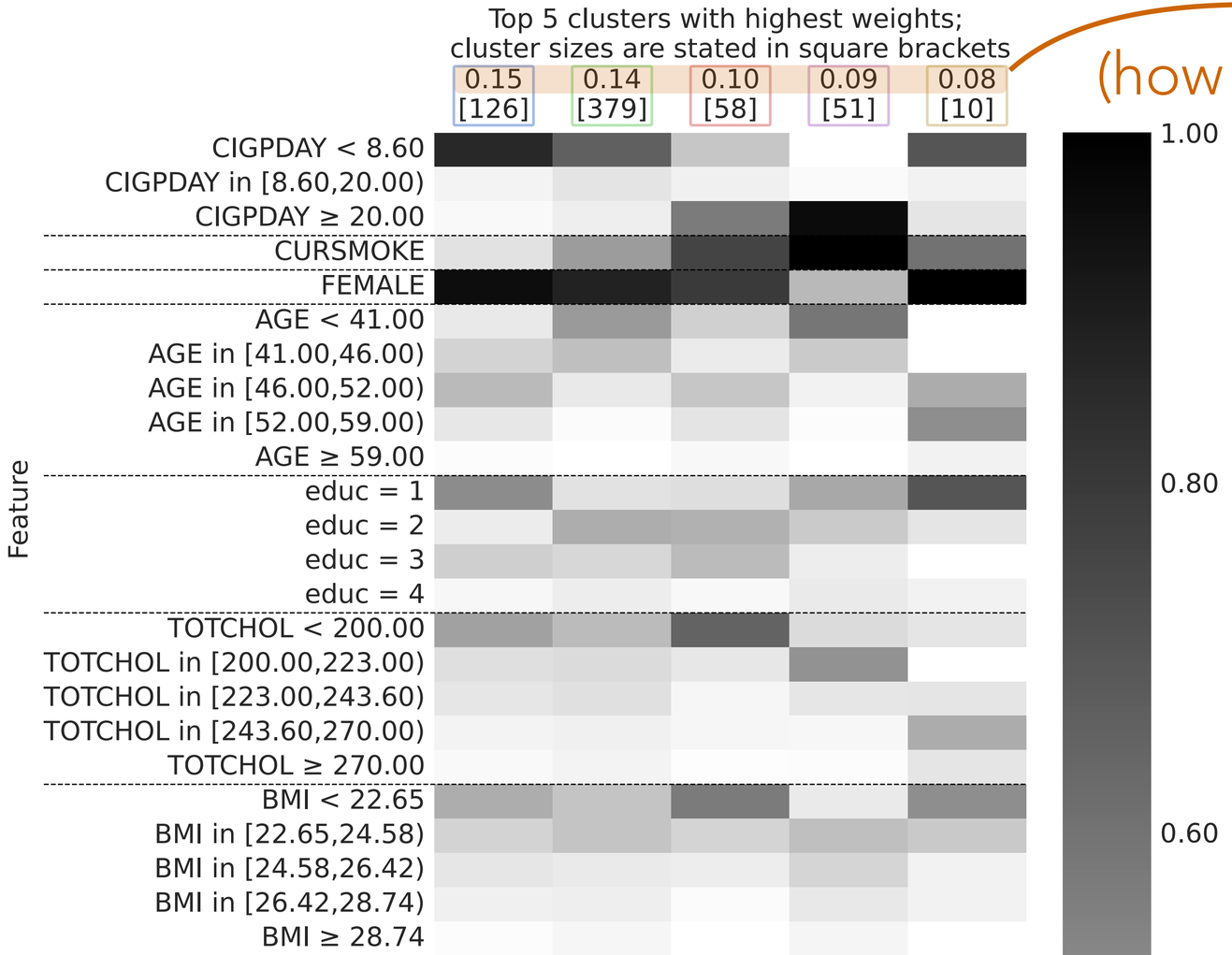
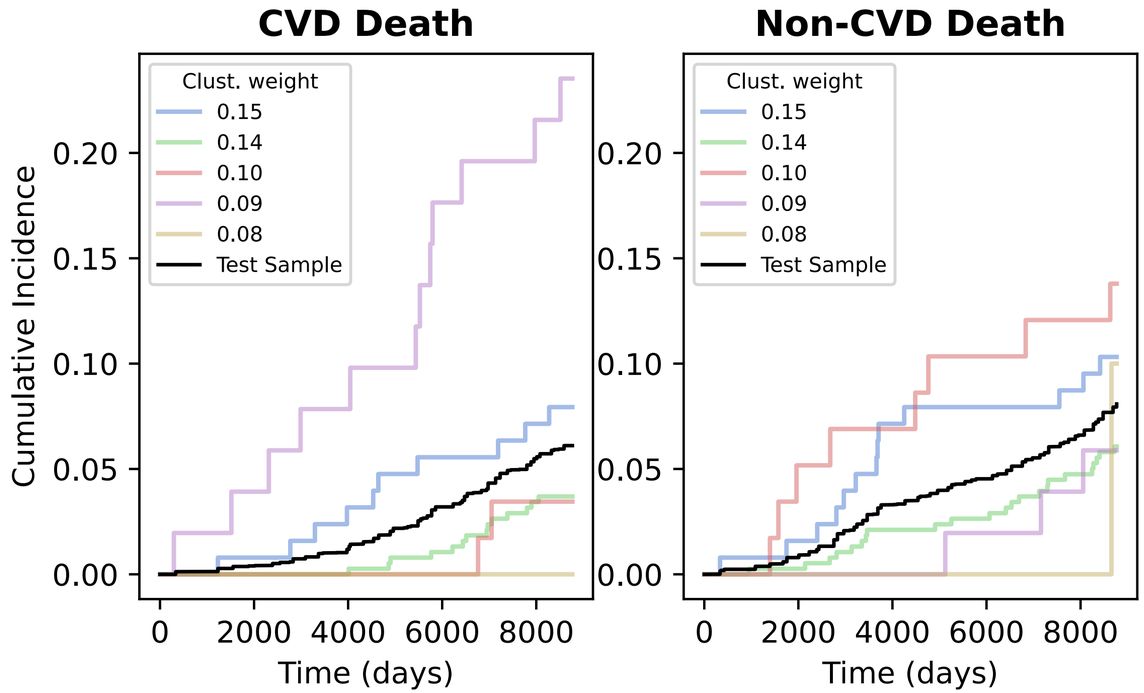
For randomly chosen test patient, can look at which clusters contribute the most

these are kernel weights
(how similar each cluster is to the test patient)





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Code: https://github.com/xiaobin-xs/dkaj

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Funded by NSF CAREER award #2047981

You have reached the end of the main talk slides

Backup Slides

Also Important: The Survival Function

The survival function also shows up in later equations:

$$S(t|x) \triangleq \mathbb{P}(T > t \mid X = x)$$

This is the probability that a patient with feature vector x
experiences their earliest critical event (any of the m events) after time t

Knowing CIFs enables us to recover the survival function: $S(t|x) = 1 - \sum_{\delta=1}^{m} F_\delta(t|x)$

The AJ estimator will depend on the population-level survival function:

$$S^{\text{pop}}(t) \triangleq \mathbb{P}(T > t)$$

This can be estimated by the
Kaplan-Meier estimator [1958]

Loss Function: Negative Log Likelihood

Recall that the CIF for event δ is $F_\delta(t|x) = \mathbb{P}(\Delta^* = \delta, T \leq t \mid X = x)$

A transformed version of the CIF yields the *cause-specific hazard function*:

$$\lambda_\delta(t|x) = \frac{1}{S(t|x)} \frac{\mathrm{d}F_\delta(t|x)}{\mathrm{d}t}$$

$$\text{where } S(t|x) = 1 - \sum_{\widetilde{\delta}=1}^{m} F_{\widetilde{\delta}}(t|x)$$

Standard competing risks likelihood (under uninformative censoring):

$$\mathcal{L} = \prod_{i=1}^{n} \left(\lambda_{\Delta_i}(Y_i|X_i)\right)^{\mathbb{1}\{\Delta_i \neq 0\}} S(Y_i|X_i) \quad \Rightarrow \text{can use loss } -\log \mathcal{L}$$

If we exclude feature vectors, the classical AJ estimator maximizes this likelihood (with some pre- and post-processing steps)

measures how similar x and X_j are

$$d_{\delta,\ell}(x) \triangleq \sum_{j=1}^{n} \mathbb{1}\{\Delta_j = \delta, Y_j = t_\ell\} K(x, X_j)$$

times event δ occurs at time t_ℓ among those who look like x

$$n_\ell(x) \triangleq \sum_{j=1}^{n} \mathbb{1}\{Y_j \geq t_\ell\} K(x, X_j)$$

at risk at time t_ℓ among those who look like x

What loss function do we use to train this neural net?

$$\text{loss} = -\log \mathcal{L} = -\log \left\{ \prod_{i=1}^{n} \left(\lambda_{\Delta_i}(Y_i|X_i)\right)^{\mathbb{1}\{\Delta_i \neq 0\}} S(Y_i|X_i) \right\}$$

$$K(x, x') = \exp(-\|f(x;\theta) - f(x';\theta)\|^2)$$

kernel function parameterized by a neural net $f(\cdot;\theta)$

3. Output: $\widehat{F}_\delta^{\mathrm{KAJ}}(t|x) \triangleq \sum_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \leq t}} \widehat{S}^{\mathrm{KKM}}(t_{\ell-1}|x) \frac{d_{\delta,\ell}(x)}{n_\ell(x)}$

$$\widehat{S}^{\mathrm{KKM}}(t|x) \triangleq \prod_{\substack{\ell \in \{1,\ldots,L\} \\ \text{s.t. } t_\ell \leq t}} \left(1 - \frac{\sum_{\delta=1}^{m} d_{\delta,\ell}(x)}{n_\ell(x)}\right)$$

By just coding these equations up in standard neural net software (e.g., PyTorch), we can use minibatch gradient descent to learn neural net parameters θ

Training Loss (First Attempt)

Discrete time index corresponding to Y_i

With a bit of algebra, can write the negative log likelihood loss:

$$\text{loss} = \sum_{i=1}^{n} \sum_{\delta=1}^{m} \left[-\mathbb{1}\{\Delta_i = \delta\} \log \psi_{\delta,\kappa(Y_i)}(X_i;\theta) + \sum_{\ell=1}^{\kappa(Y_i)} \psi_{\delta,\ell}(X_i;\theta) \right] + \text{constant}$$

where

$$\psi_{\delta,\ell}(x;\theta) = \frac{\sum_{j=1}^{n} \mathbb{1}\{\Delta_j = \delta, Y_j = t_\ell\} \exp(-\|f(x;\theta) - f(X_j;\theta)\|^2)}{\sum_{j=1}^{n} \mathbb{1}\{Y_j \geq t_\ell\} \exp(-\|f(x;\theta) - f(X_j;\theta)\|^2)}$$

(this is a predicted cause-specific hazard value)

Problem: i-th training point's loss function uses hazard function prediction that has access to the i-th training point's ground truth

Training Loss

Discrete time index corresponding to Y_i

With a bit of algebra, can write the negative log likelihood loss:

$$\text{LOO loss} = \sum_{i=1}^{n}\sum_{\delta=1}^{m}\left[-\mathbb{1}\{\Delta_i = \delta\} \log \psi^{\neg i}_{\delta,\kappa(Y_i)}(X_i;\theta) + \sum_{\ell=1}^{\kappa(Y_i)} \psi^{\neg i}_{\delta,\ell}(X_i;\theta)\right] + \text{constant}$$

where

$$\psi^{\neg i}_{\delta,\ell}(x;\theta) = \frac{\sum_{j\neq i}\mathbb{1}\{\Delta_j = \delta, Y_j = t_\ell\}\exp(-\|f(x;\theta) - f(X_j;\theta)\|^2)}{\sum_{j\neq i}\mathbb{1}\{Y_j \geq t_\ell\}\exp(-\|f(x;\theta) - f(X_j;\theta)\|^2)}$$

(this is a predicted cause-specific hazard value)

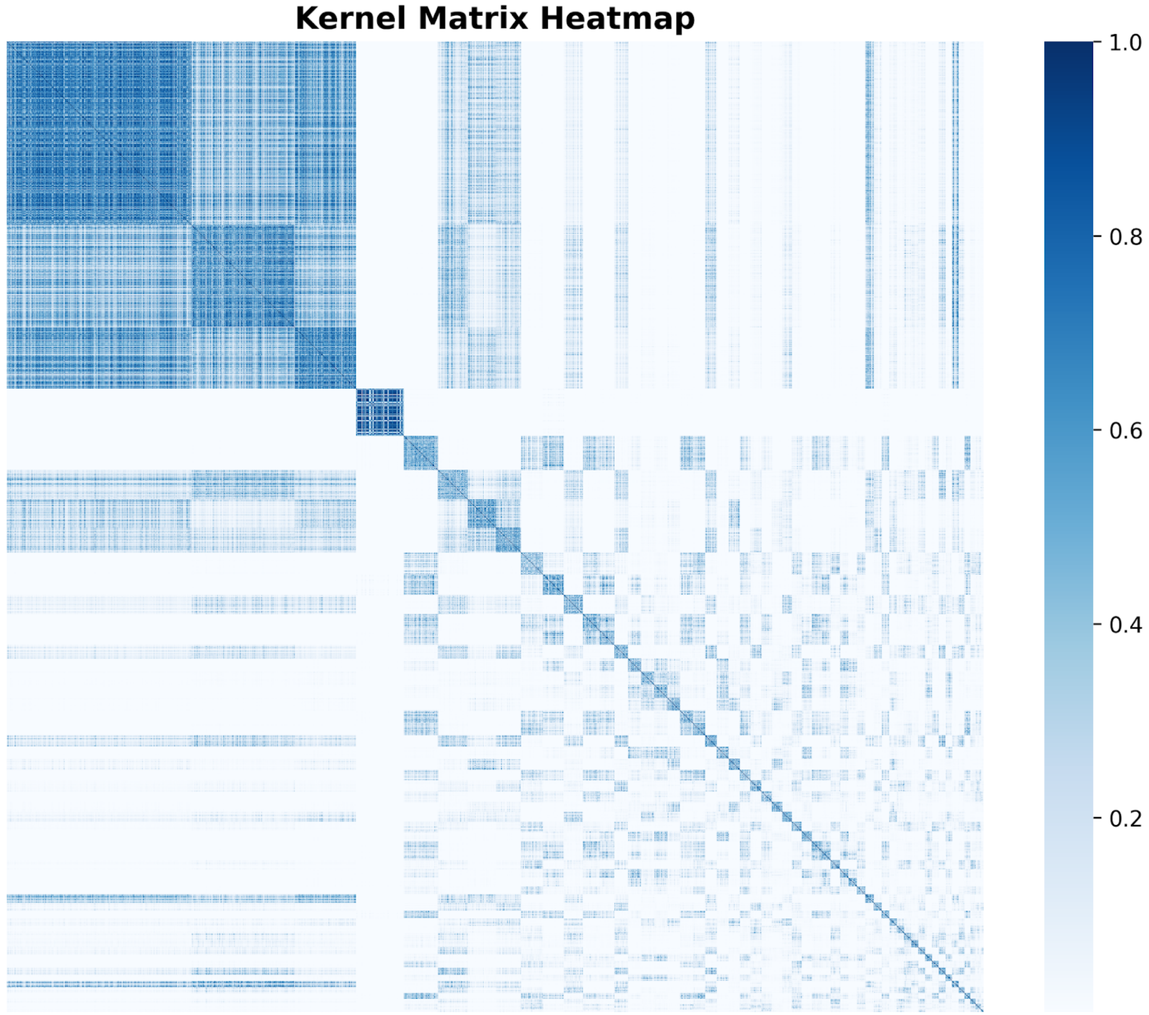
Solution: use a leave-one-out estimate instead

DKAJ Special Cases

- If $\tau = \infty$ (shaded ball has infinite radius):
  - If $\varepsilon = \infty$ for ε-net clustering (all training points are in the same cluster), we get the **classical AJ estimator** [Aalen-Johansen 1978]
  - If $\varepsilon = 0$ (each training point is in its own cluster), we get an approach that is almost the same as that of the **conditional AJ estimator** by Bladt & Furrer [2025]
    - If furthermore the kernel function always outputs 1, we also get the **classical AJ estimator** [Aalen-Johansen 1978]
- If # event types $m = 1$, we get my earlier **survival kernets** model [Chen 2024] (a deep kernel Kaplan-Meier estimator)

DKAJ Visualization of the Learned Kernel Function

- After learning the kernel function, can apply it to all training points to produce a kernel matrix
- Sort rows & columns by cluster index
- Finding: very clear block structure
- Also possible (although we haven't experimented with this yet):
  can understand hierarchical structure via hierarchical clustering algorithms that take a kernel matrix as input



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Model each generic training point (X, Y, Δ) to be sampled as follows:

1. Sample feature vector X from $\mathbb{P}_{\mathsf{X}}$
2. Sample the time until each of the m events happens; we do this in a "joint" fashion by sampling the length-m vector $(T_1, \ldots, T_m)$ from $\mathbb{P}_{\mathsf{T}|\mathsf{X}}(\cdot|X)$

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4. Finally, set $Y \triangleq \min\{T, C\}$, and $\Delta \triangleq \begin{cases} 0 & \text{if } Y = C \\ \Delta^* & \text{otherwise} \end{cases}$

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The AJ estimator estimates the population-level version: $F_\delta^{\text{pop}}(t) \triangleq \mathbb{P}(\Delta^* = \delta, T \leq t)$

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Everything is done in the embedding space (output space of neural net)

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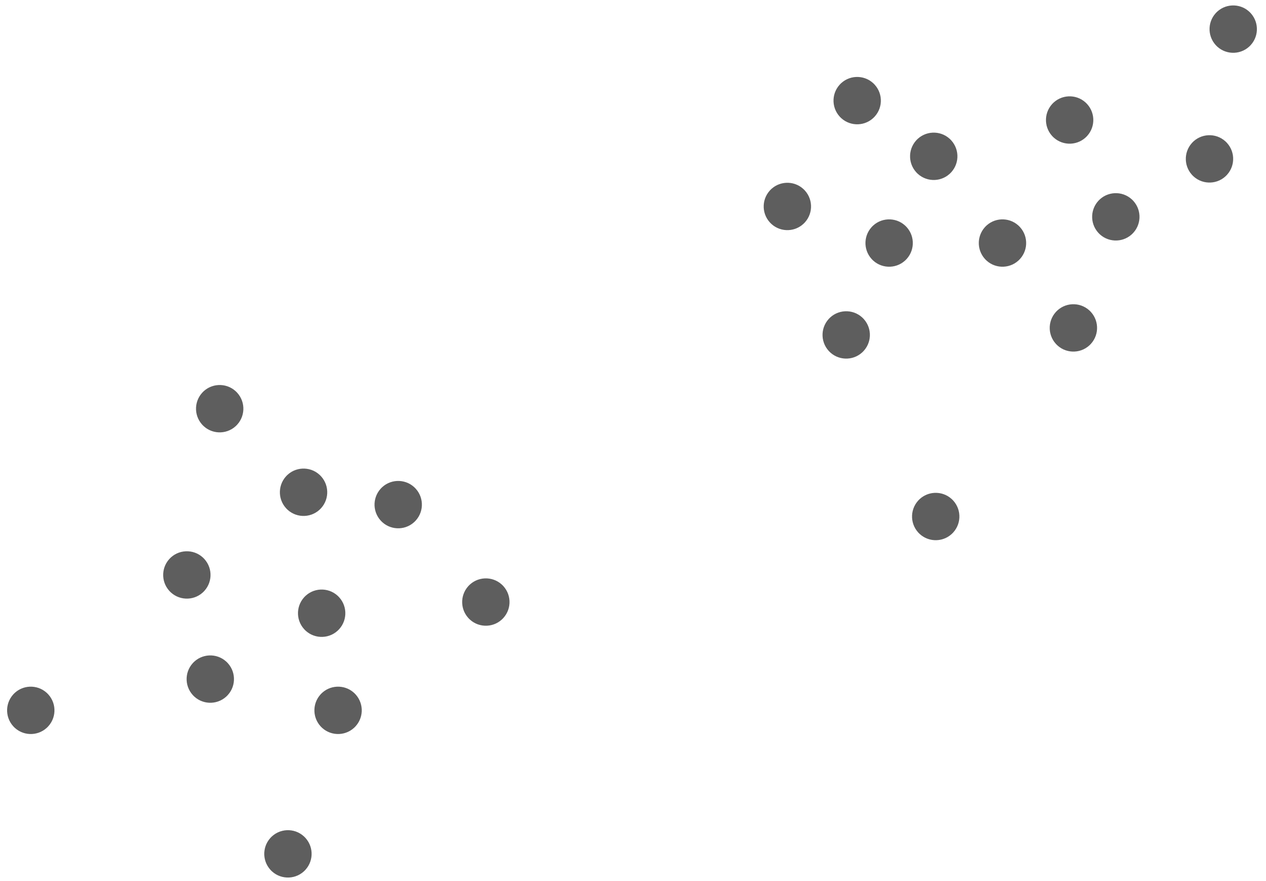

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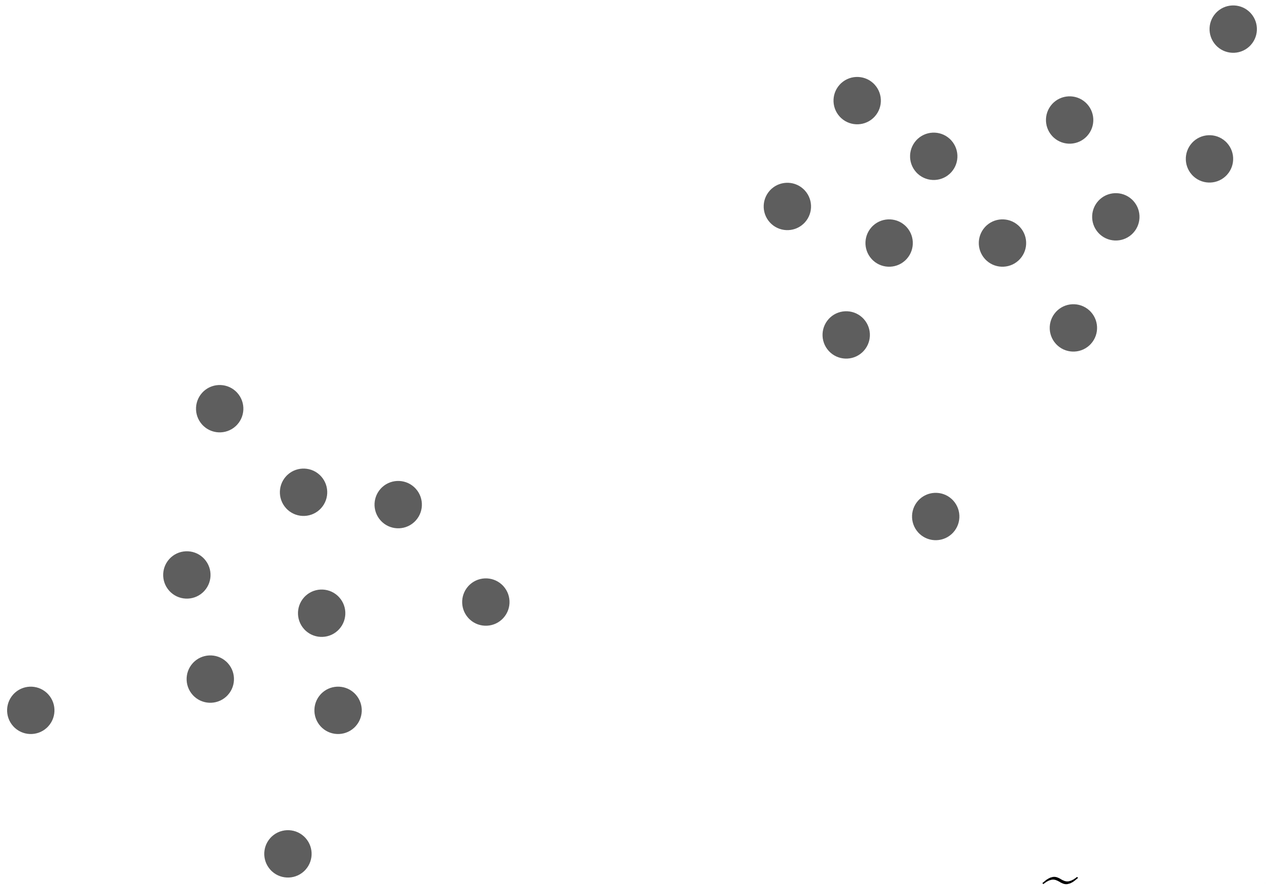

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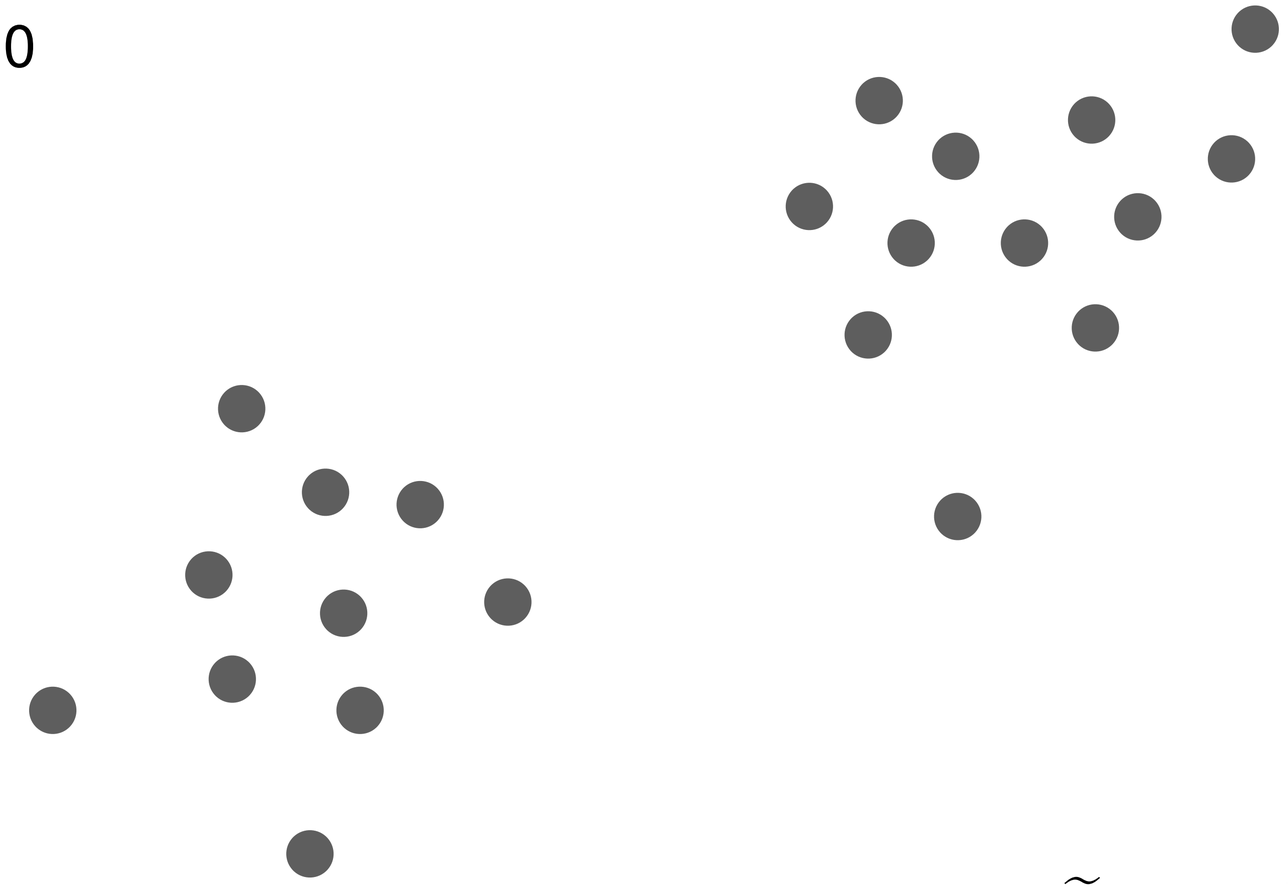

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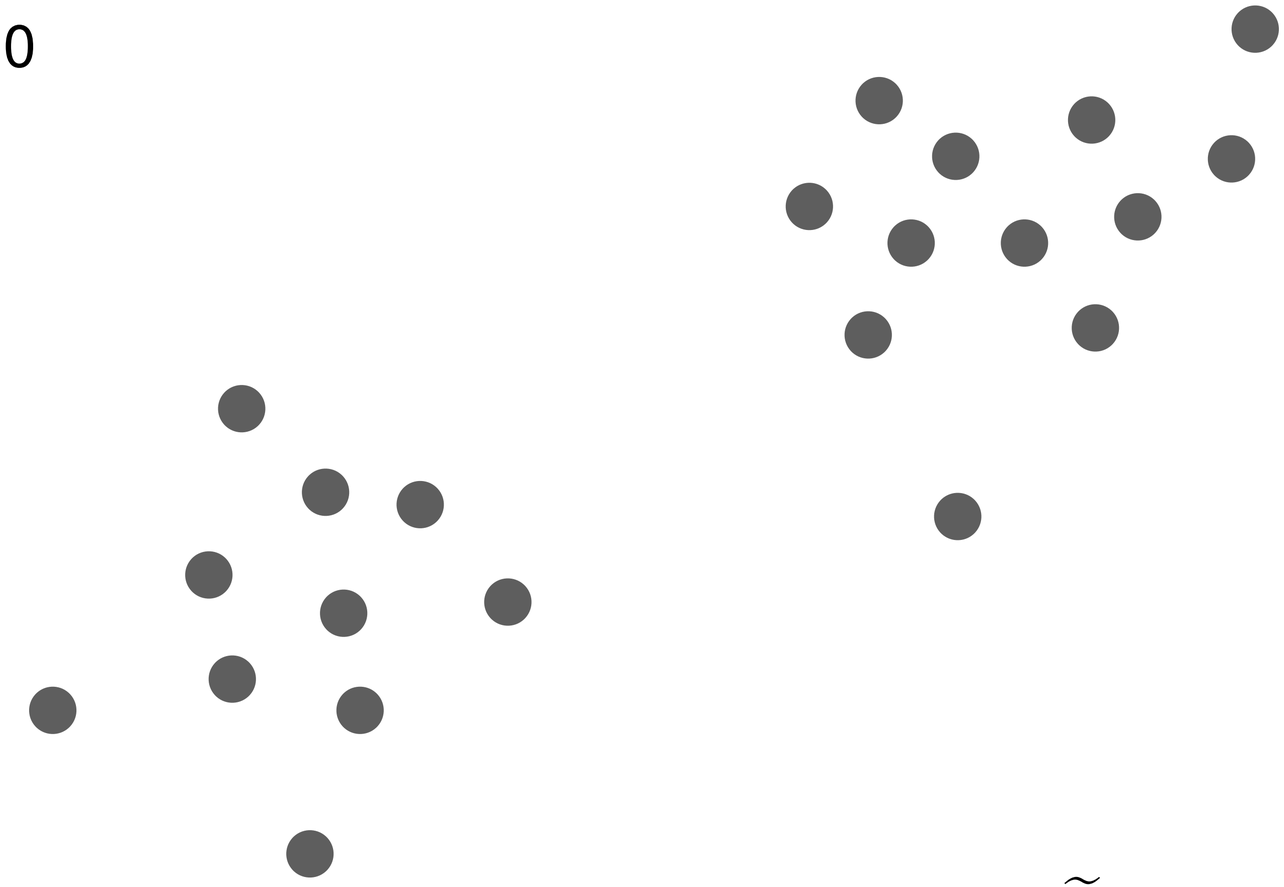

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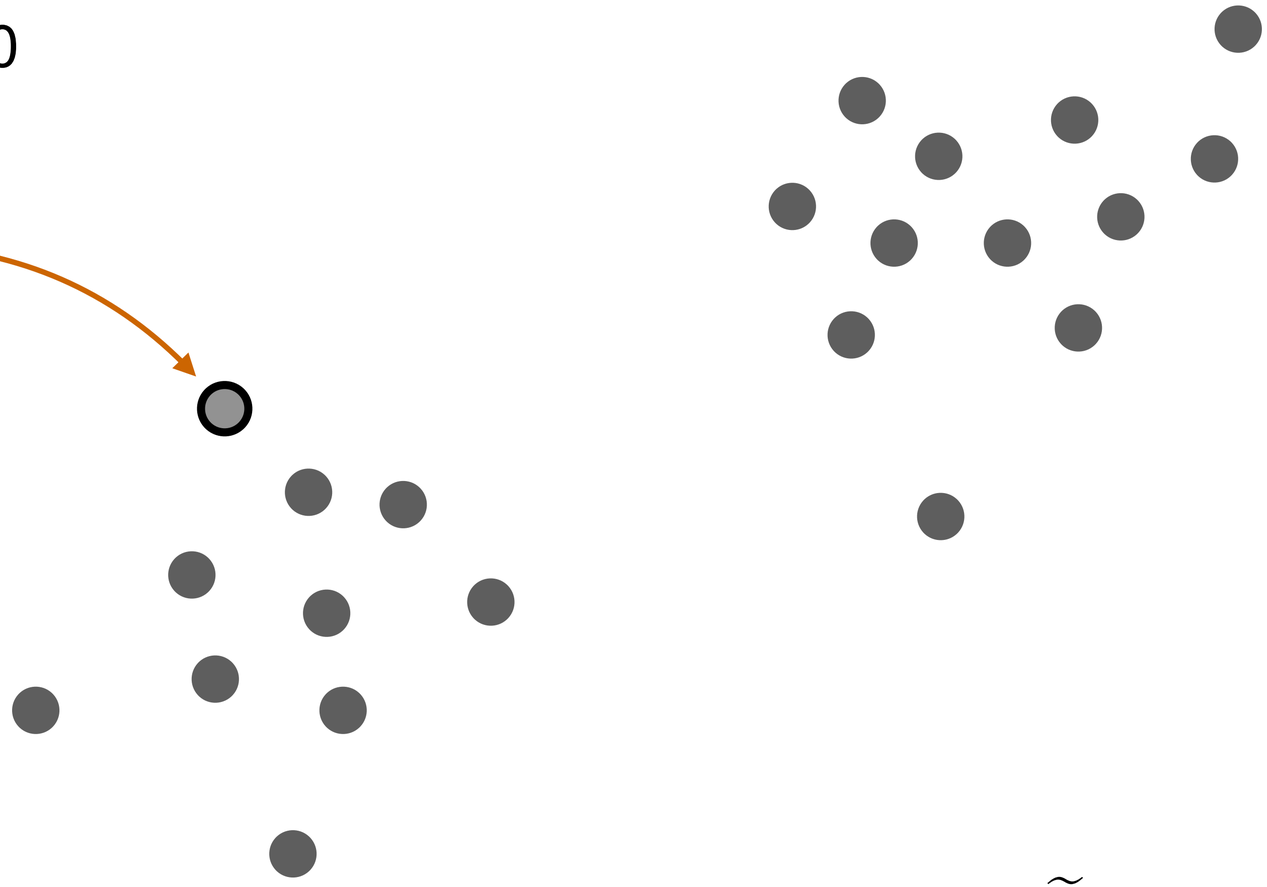

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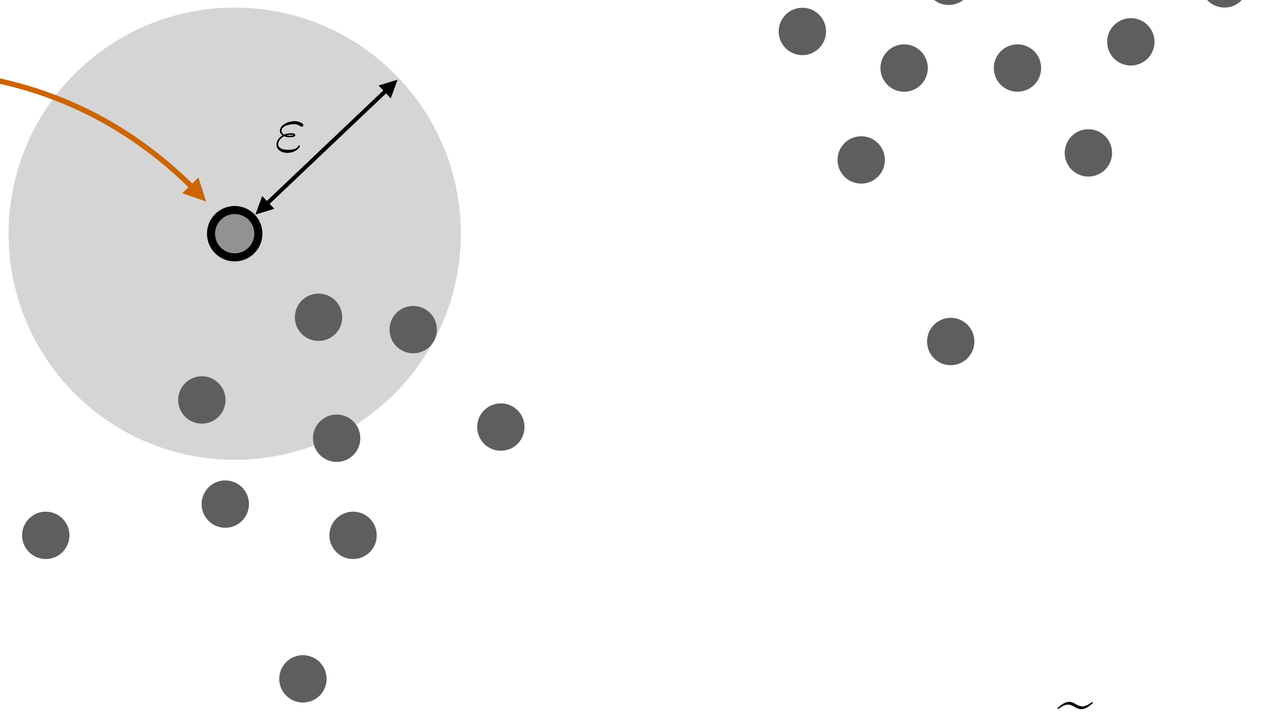

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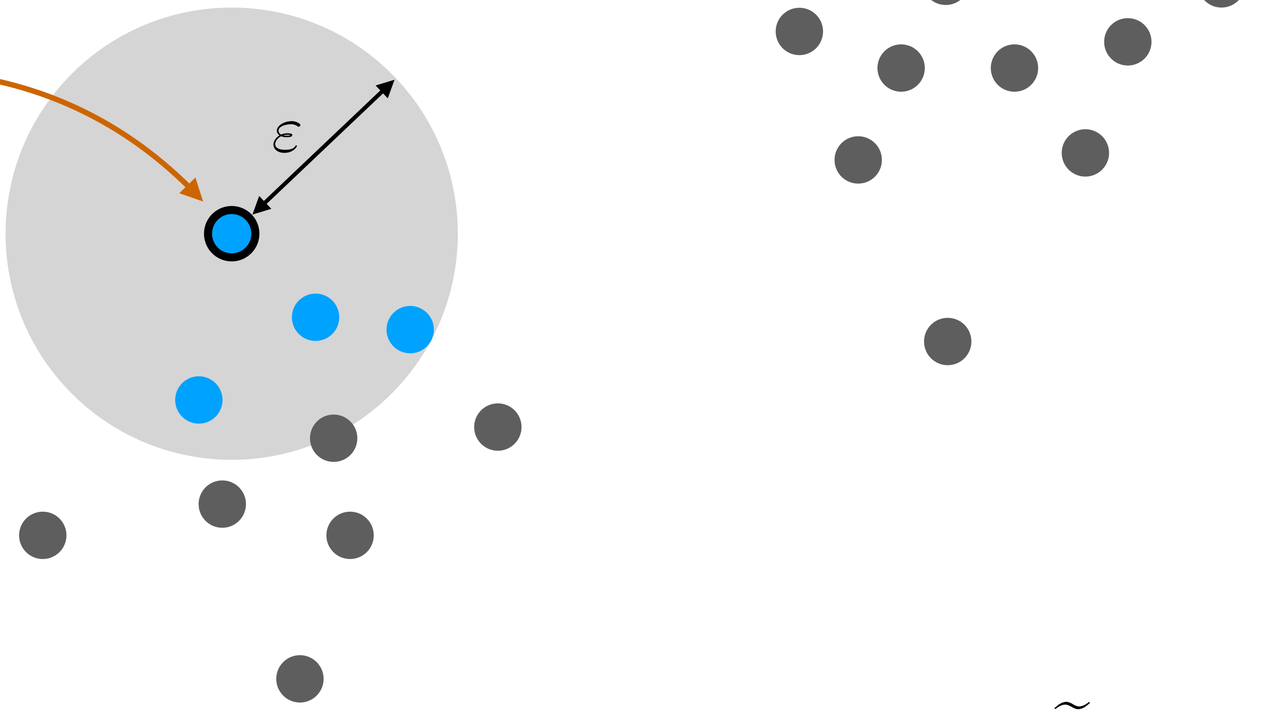

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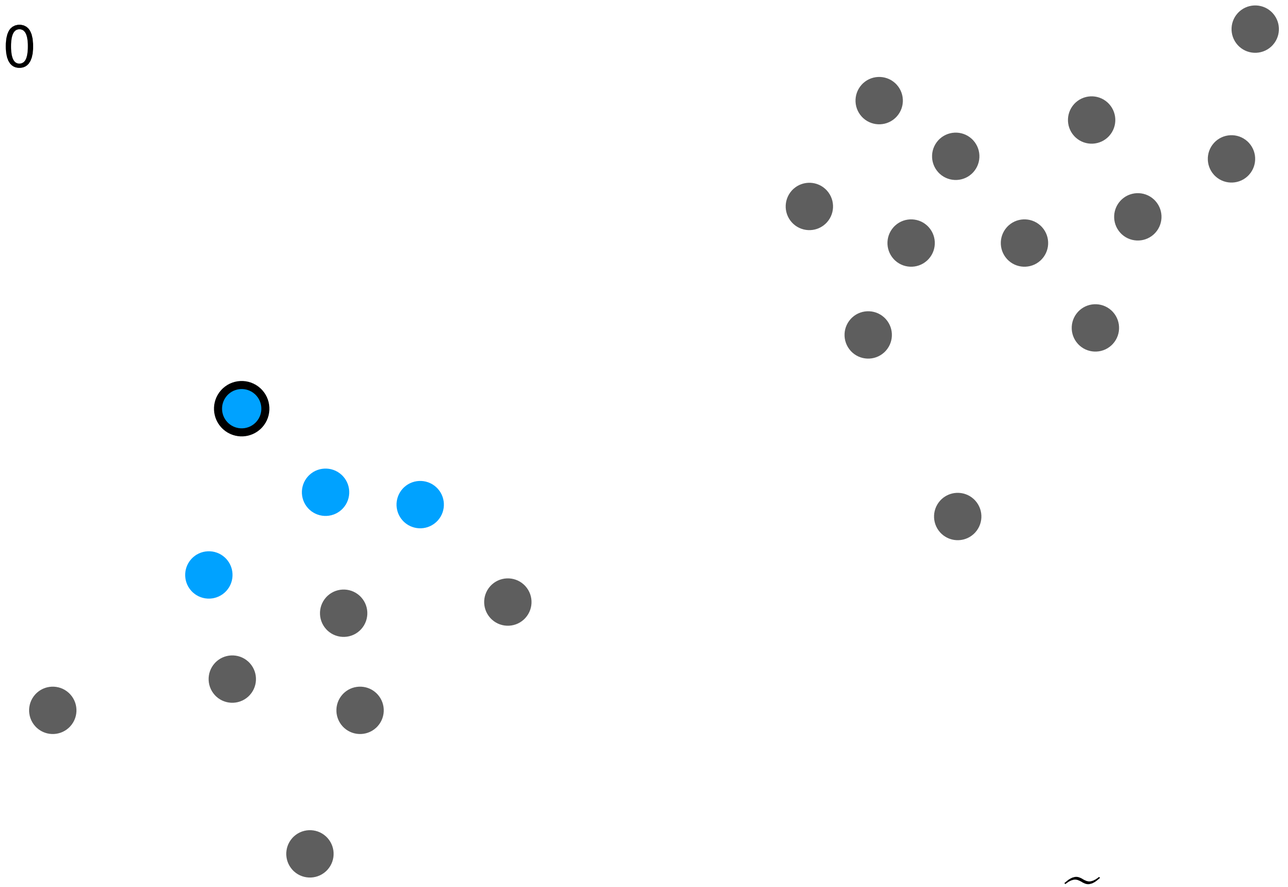

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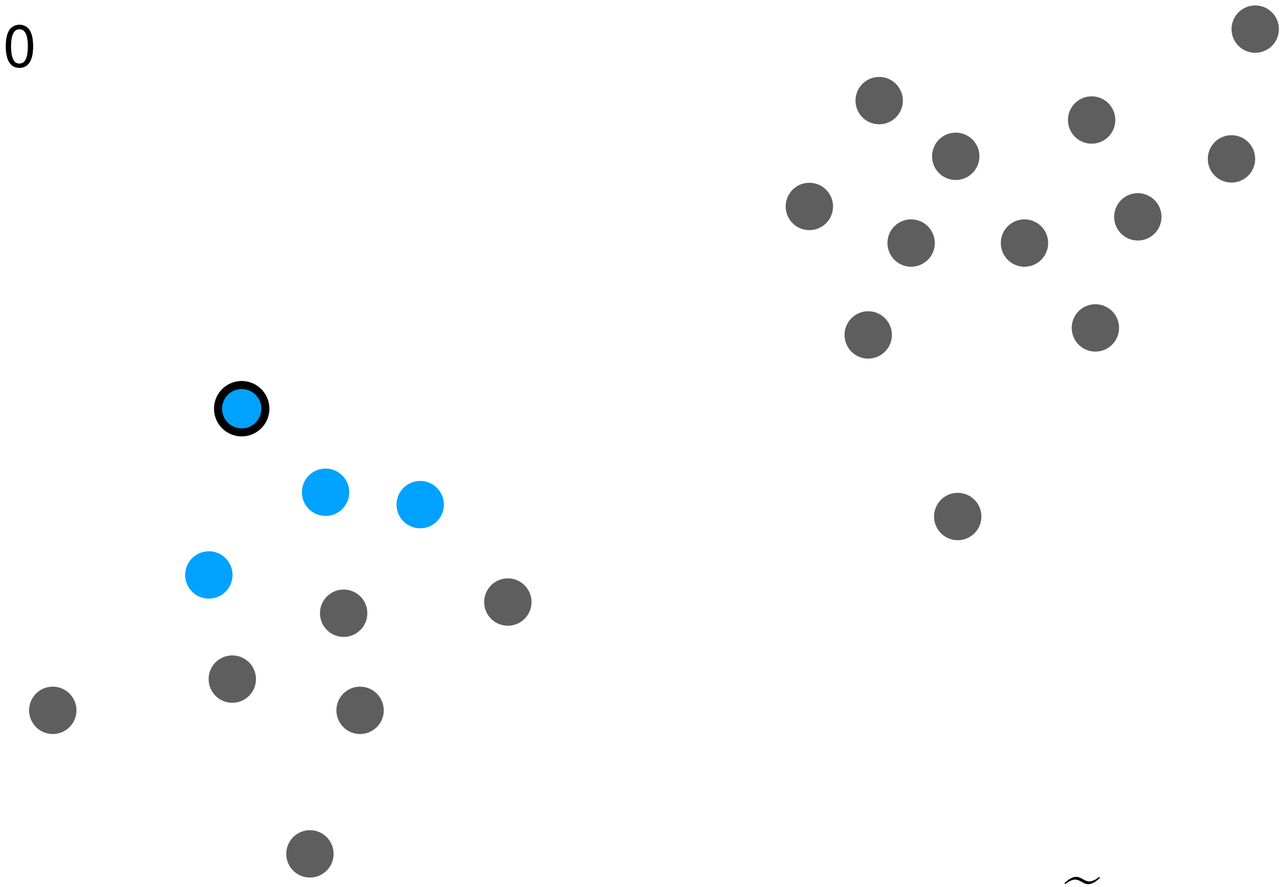

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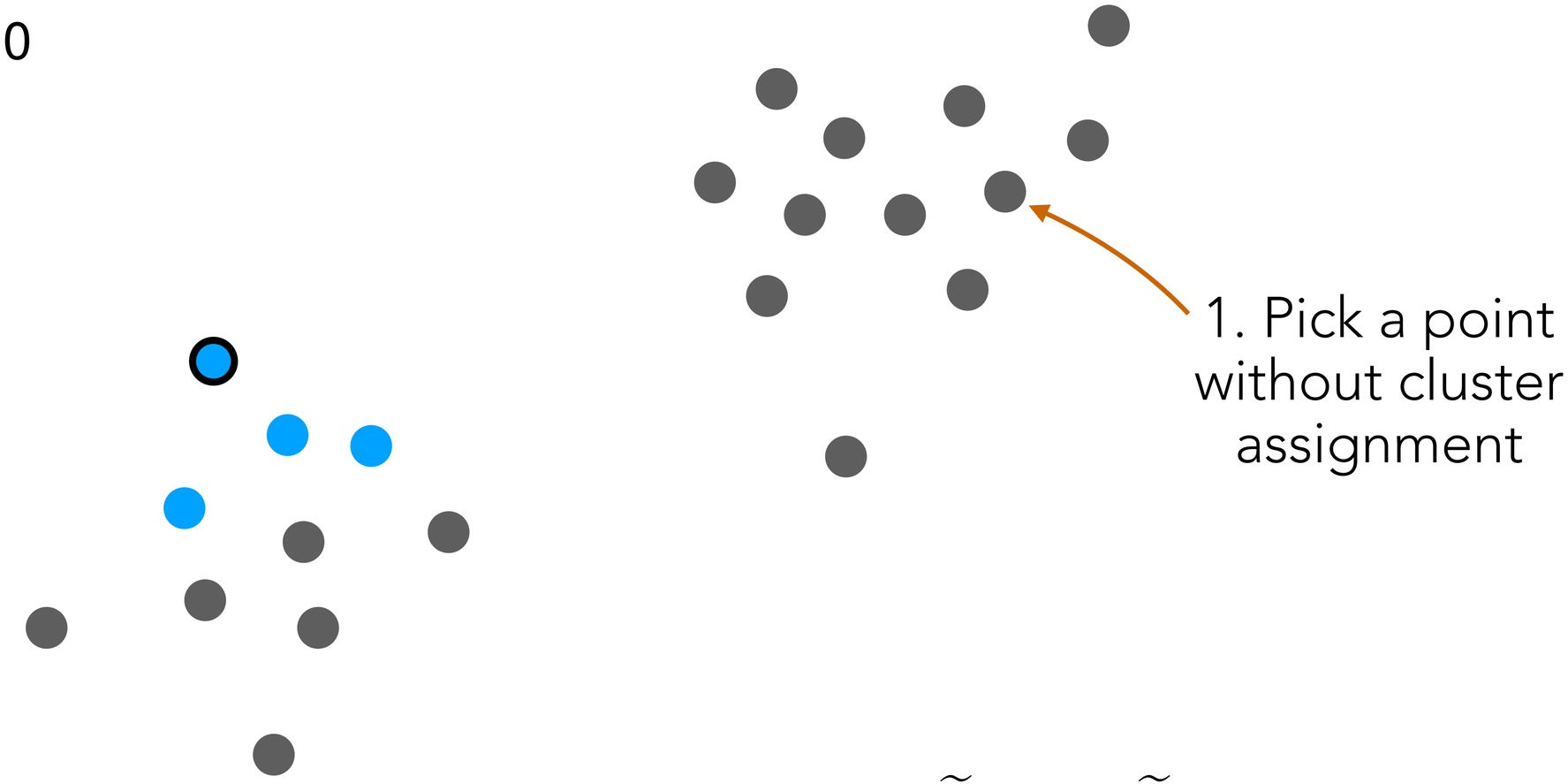

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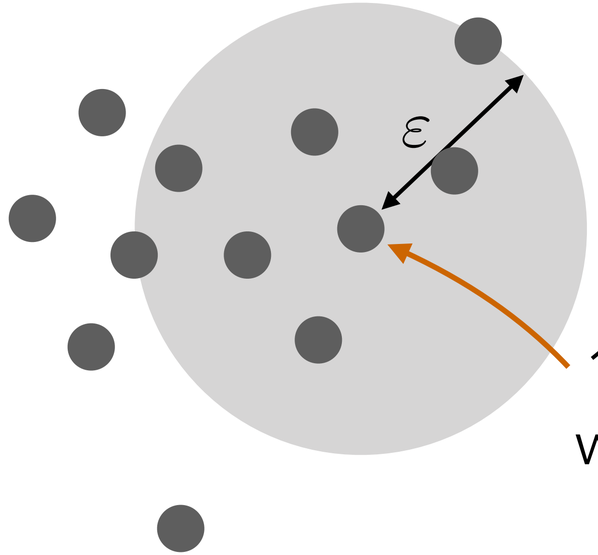

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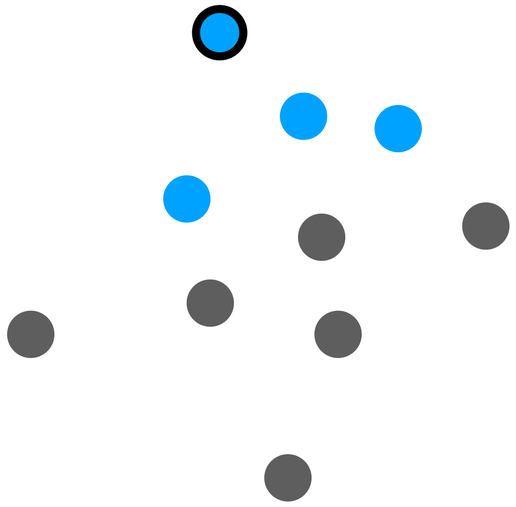

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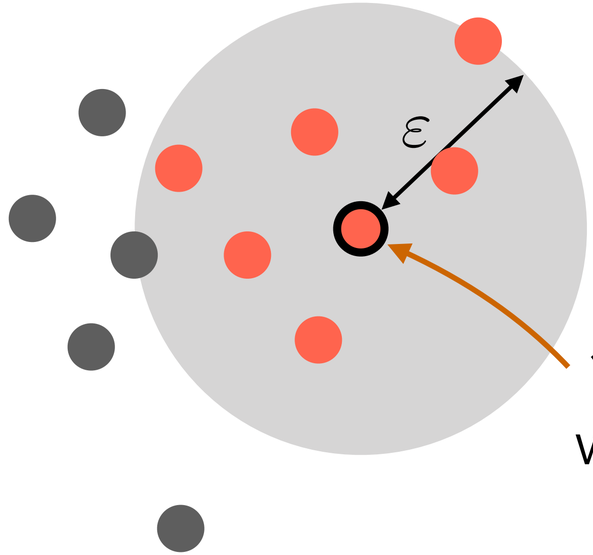

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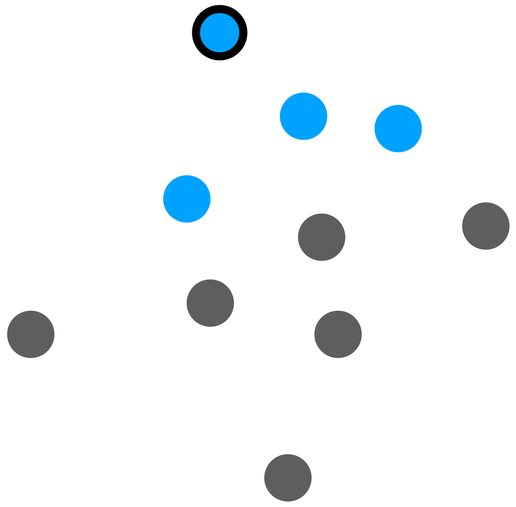

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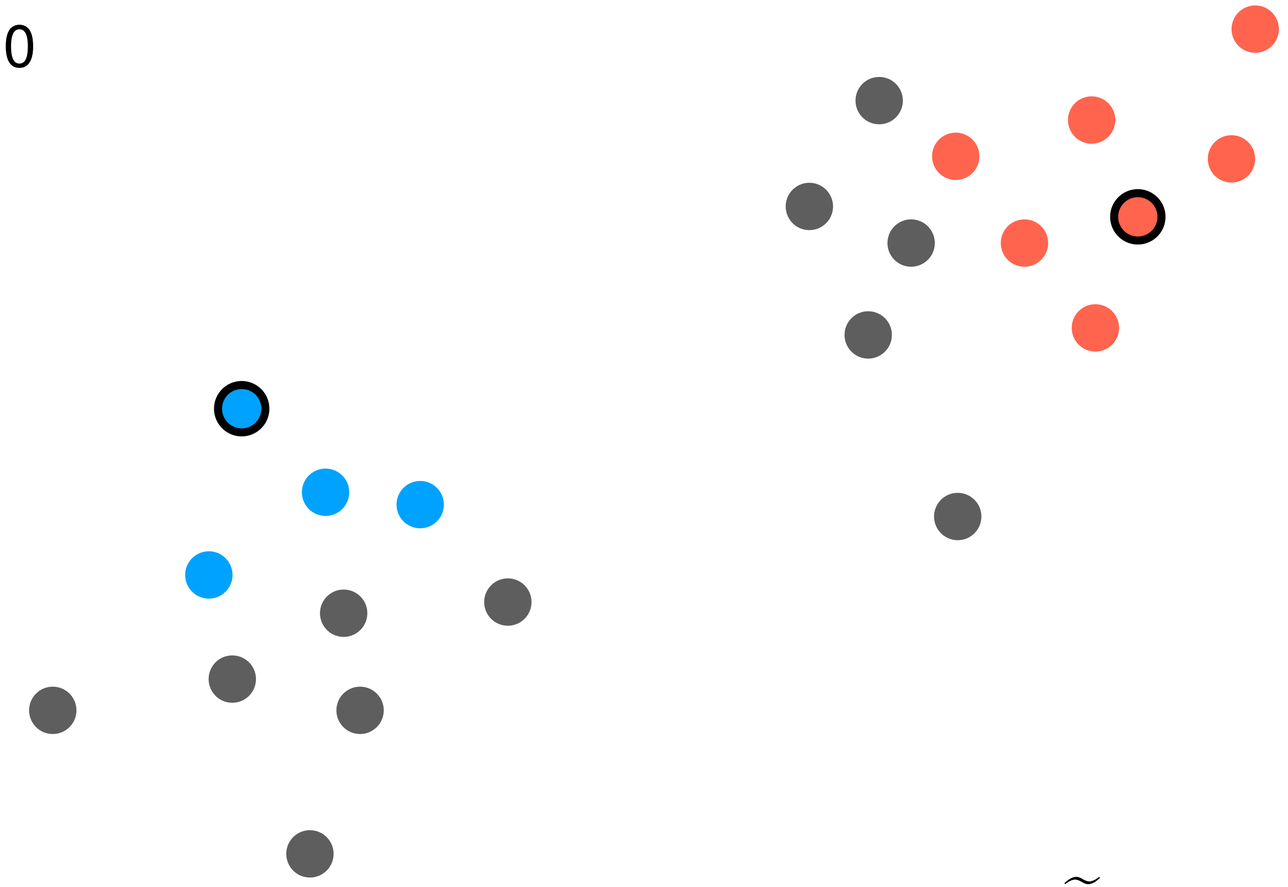

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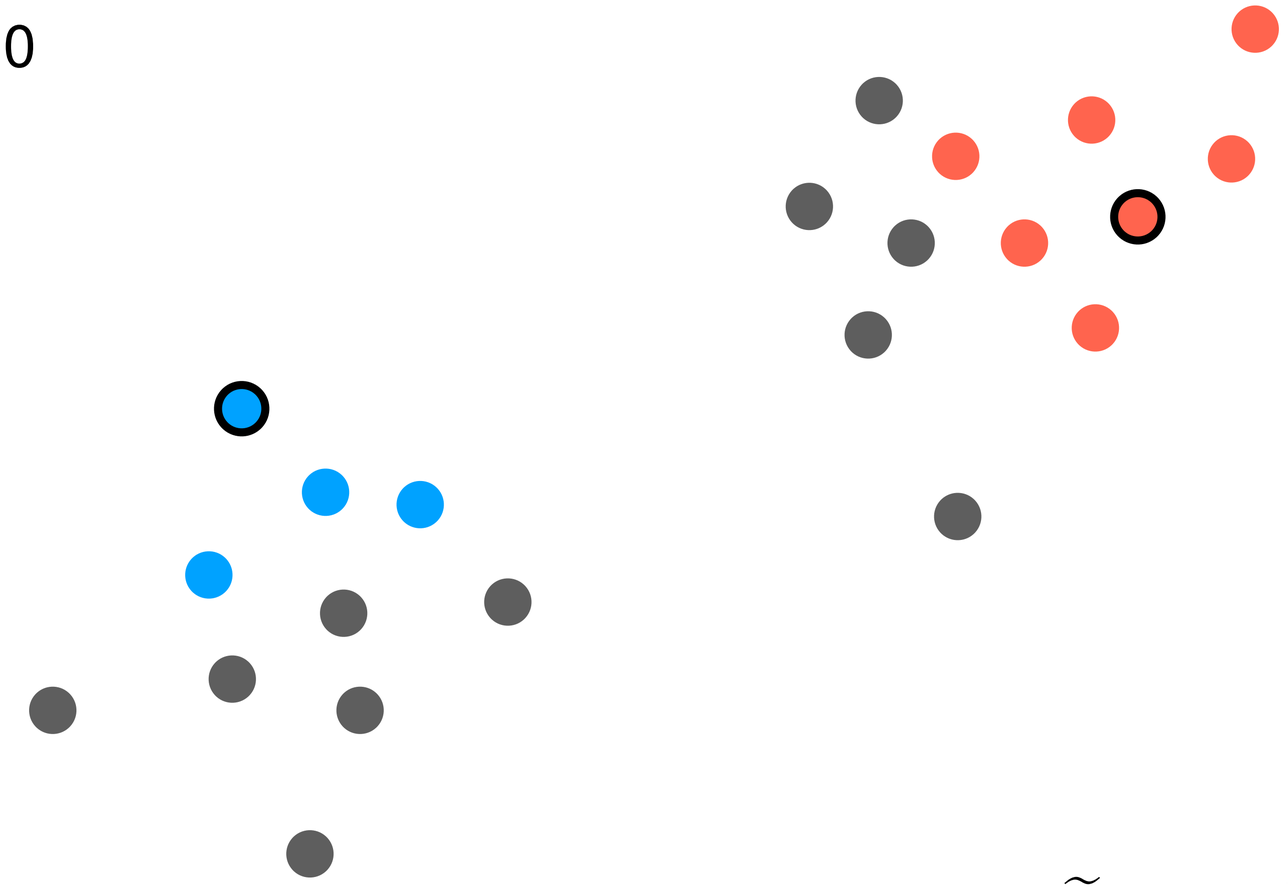

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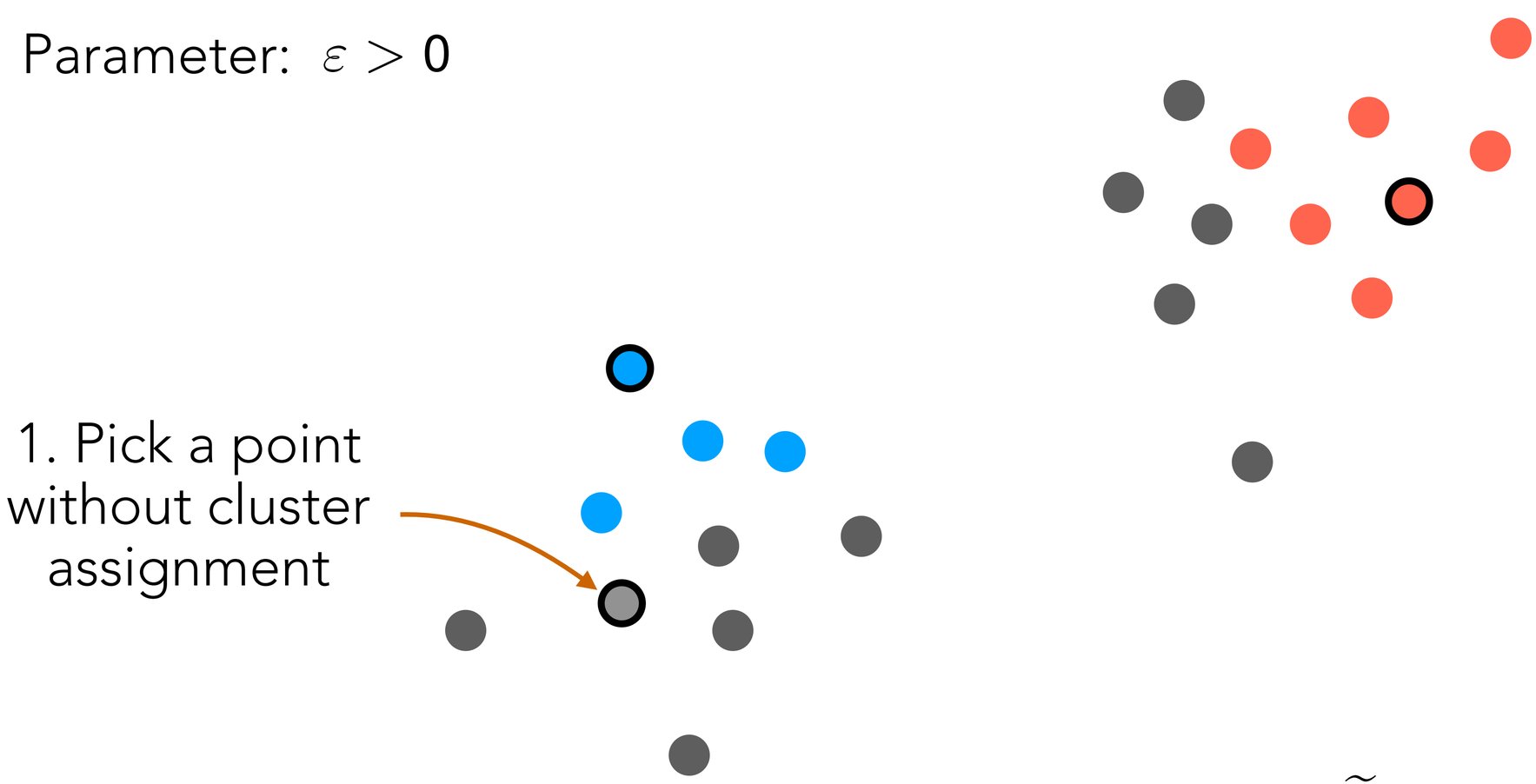

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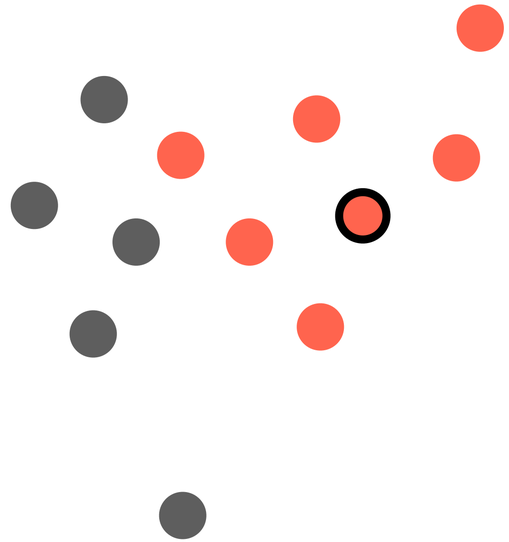

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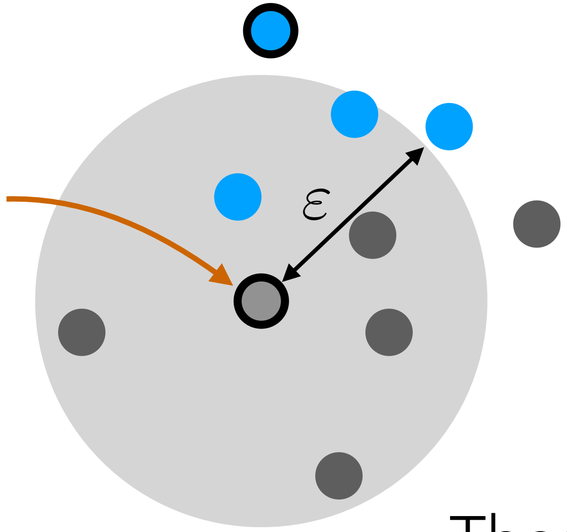

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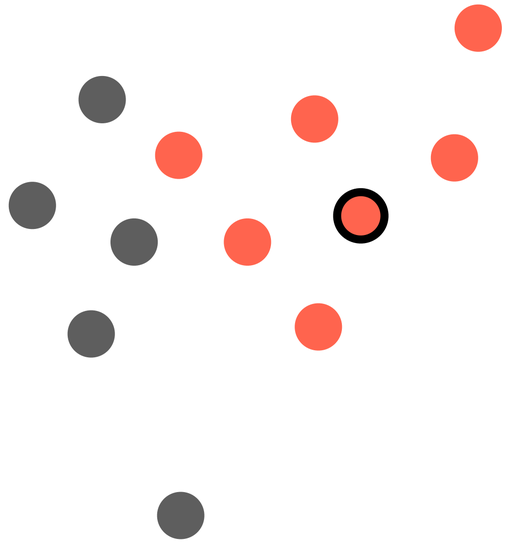

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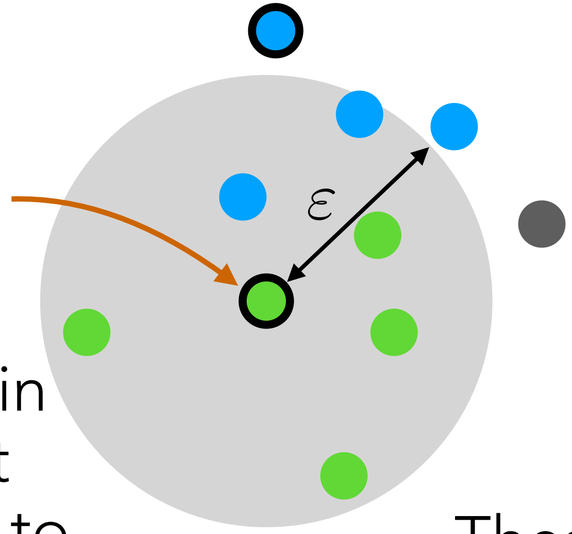

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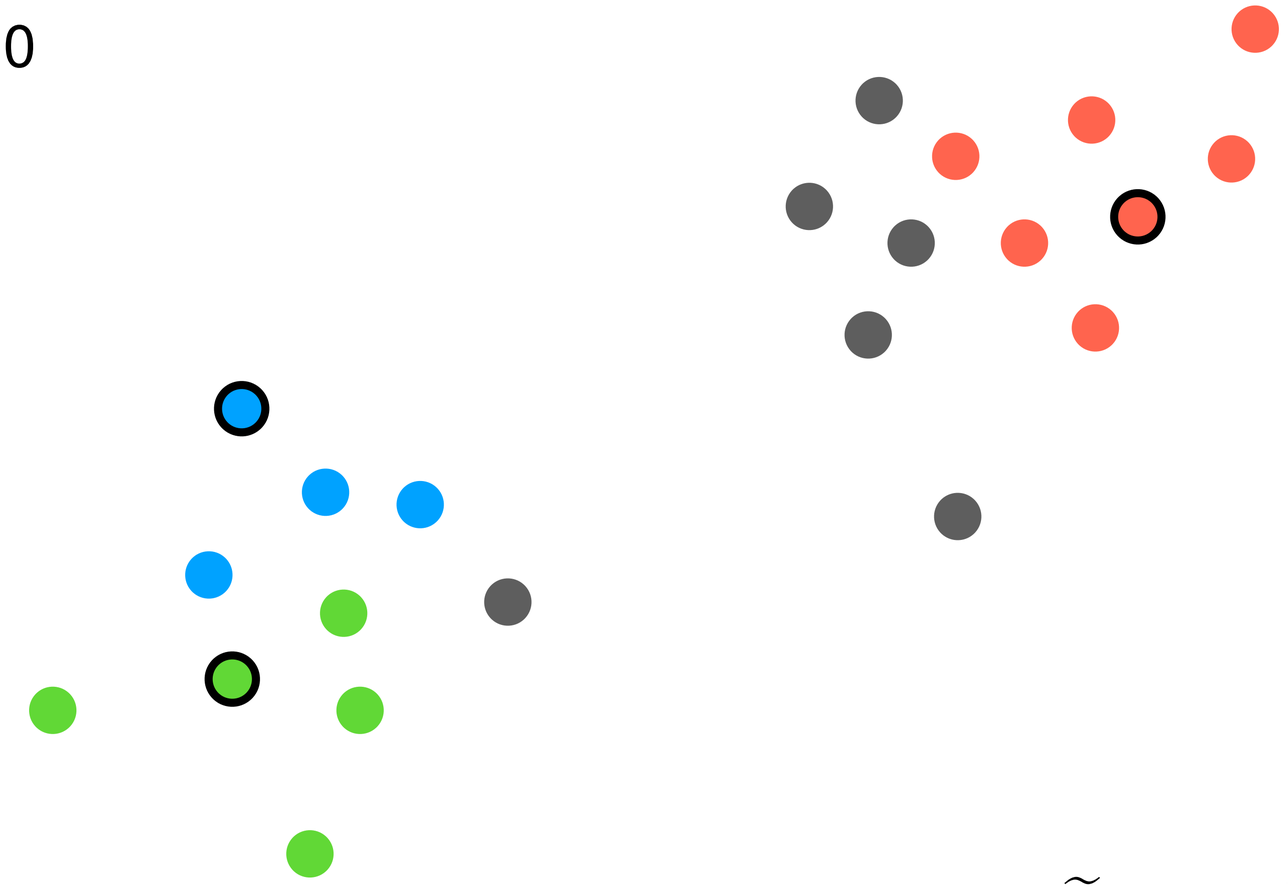

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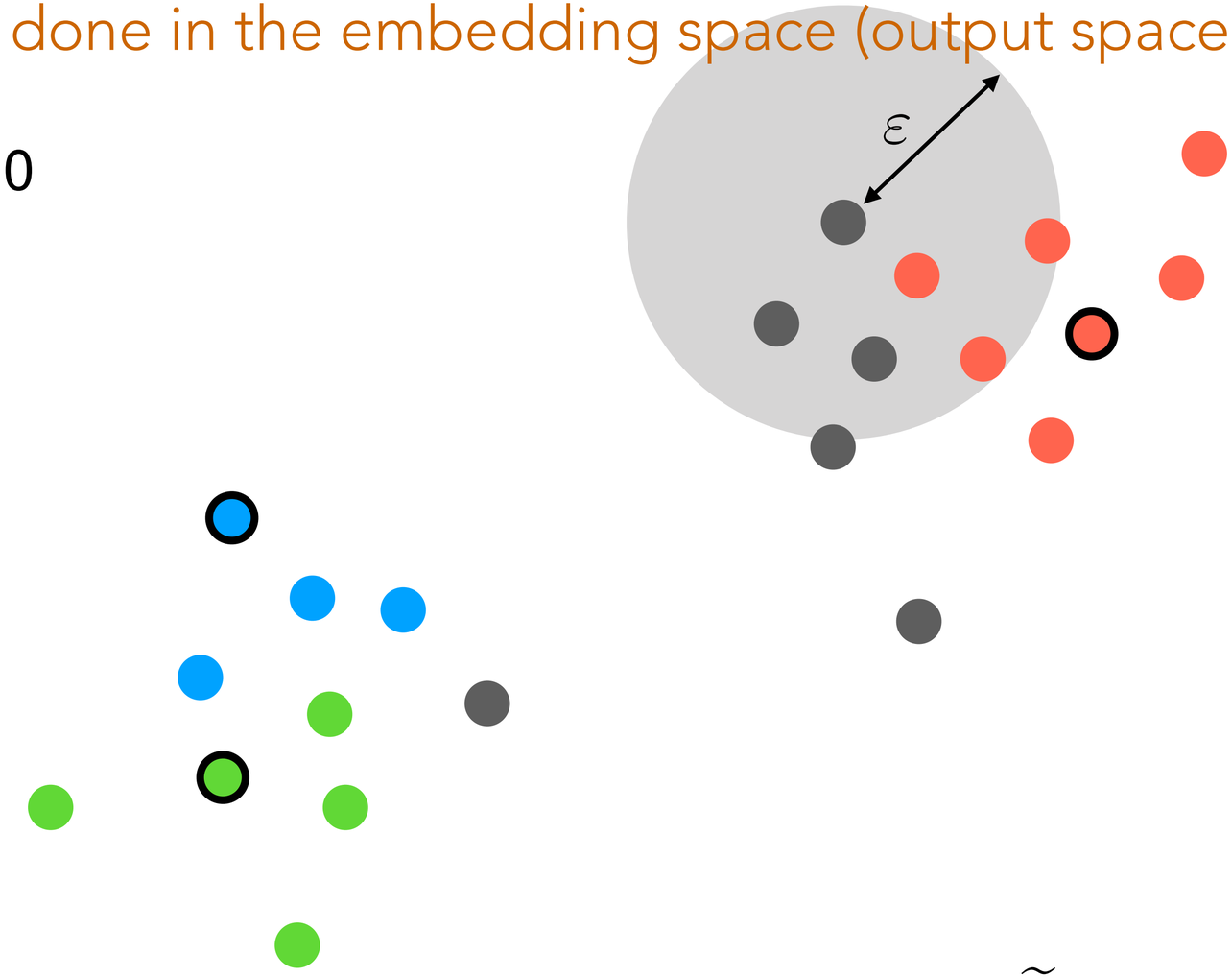

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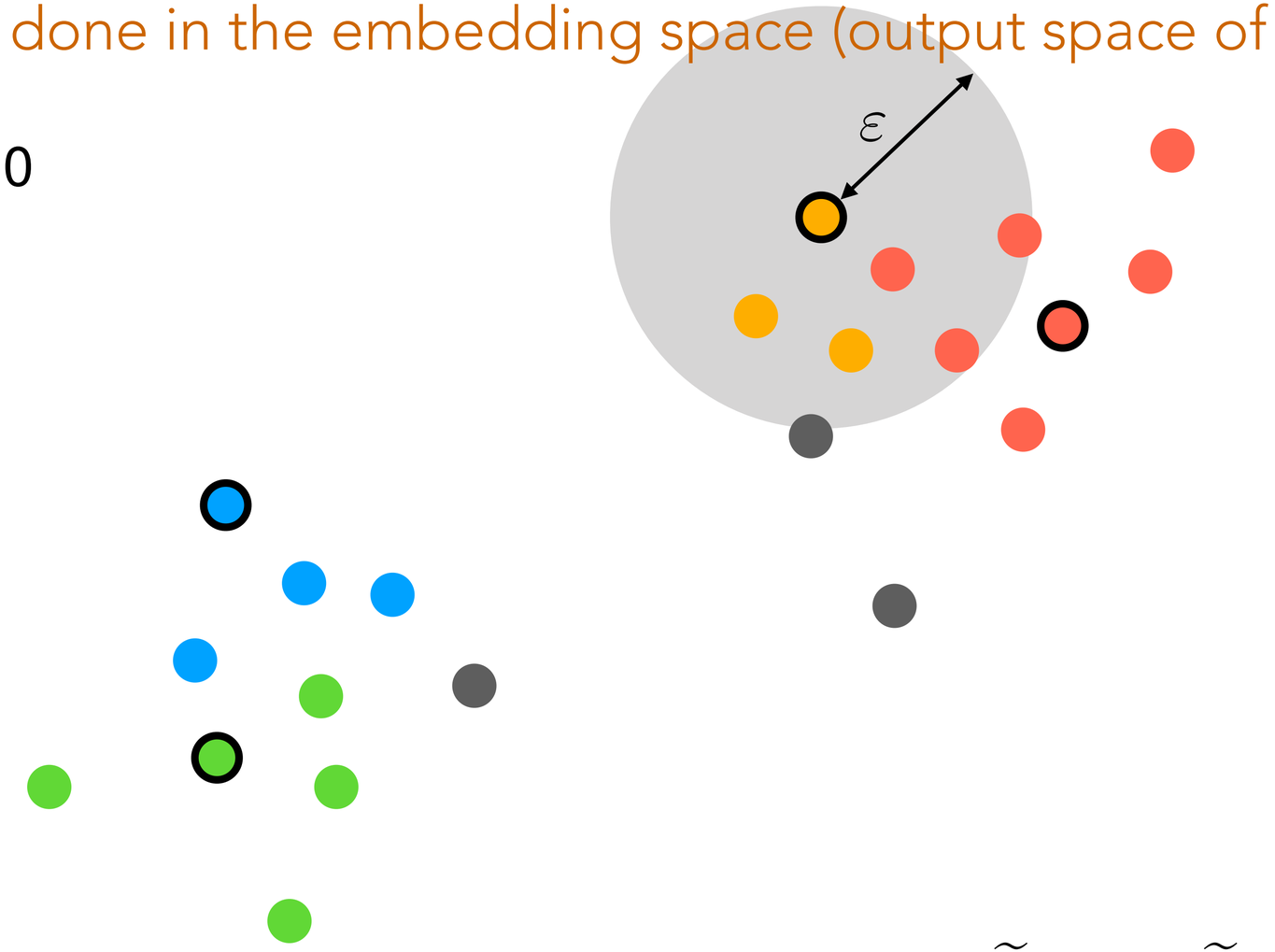

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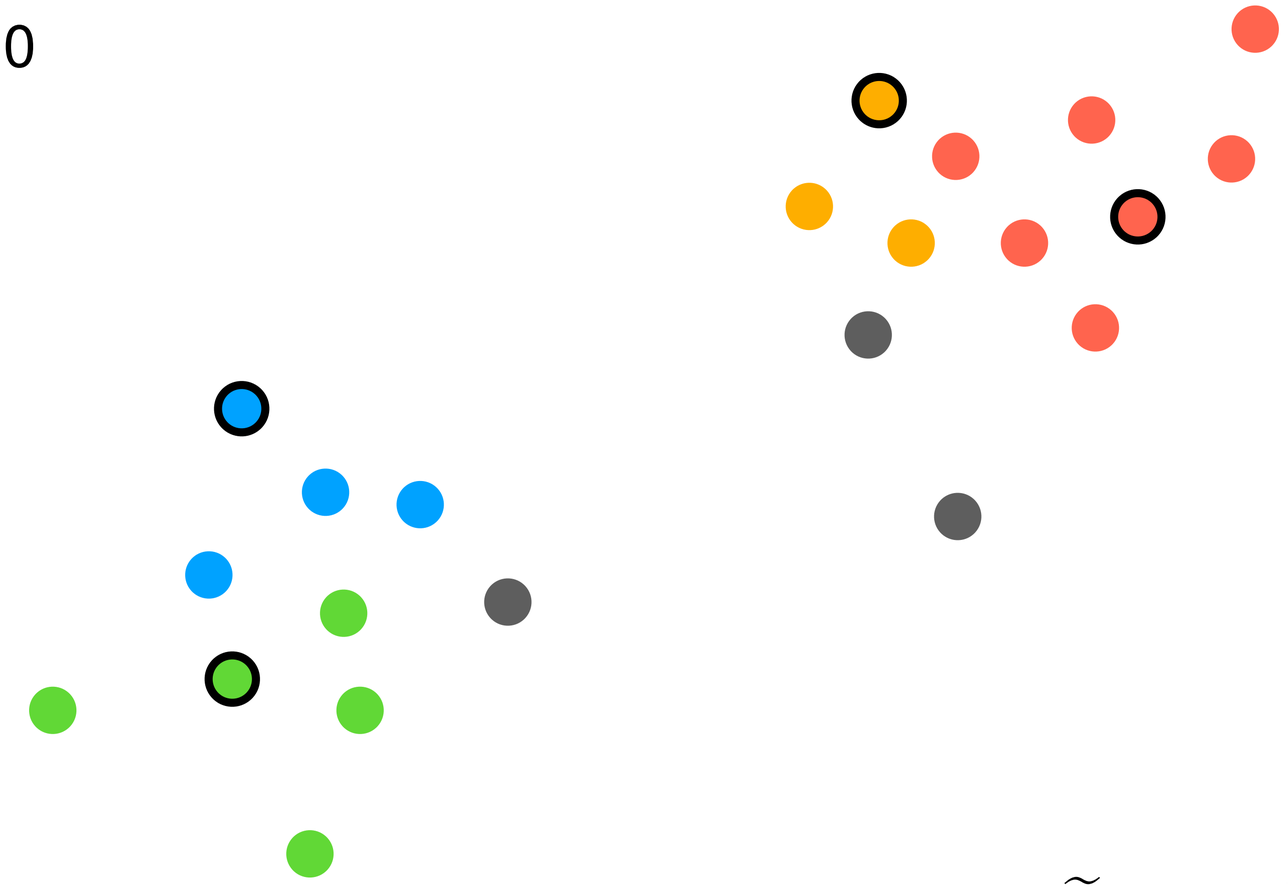

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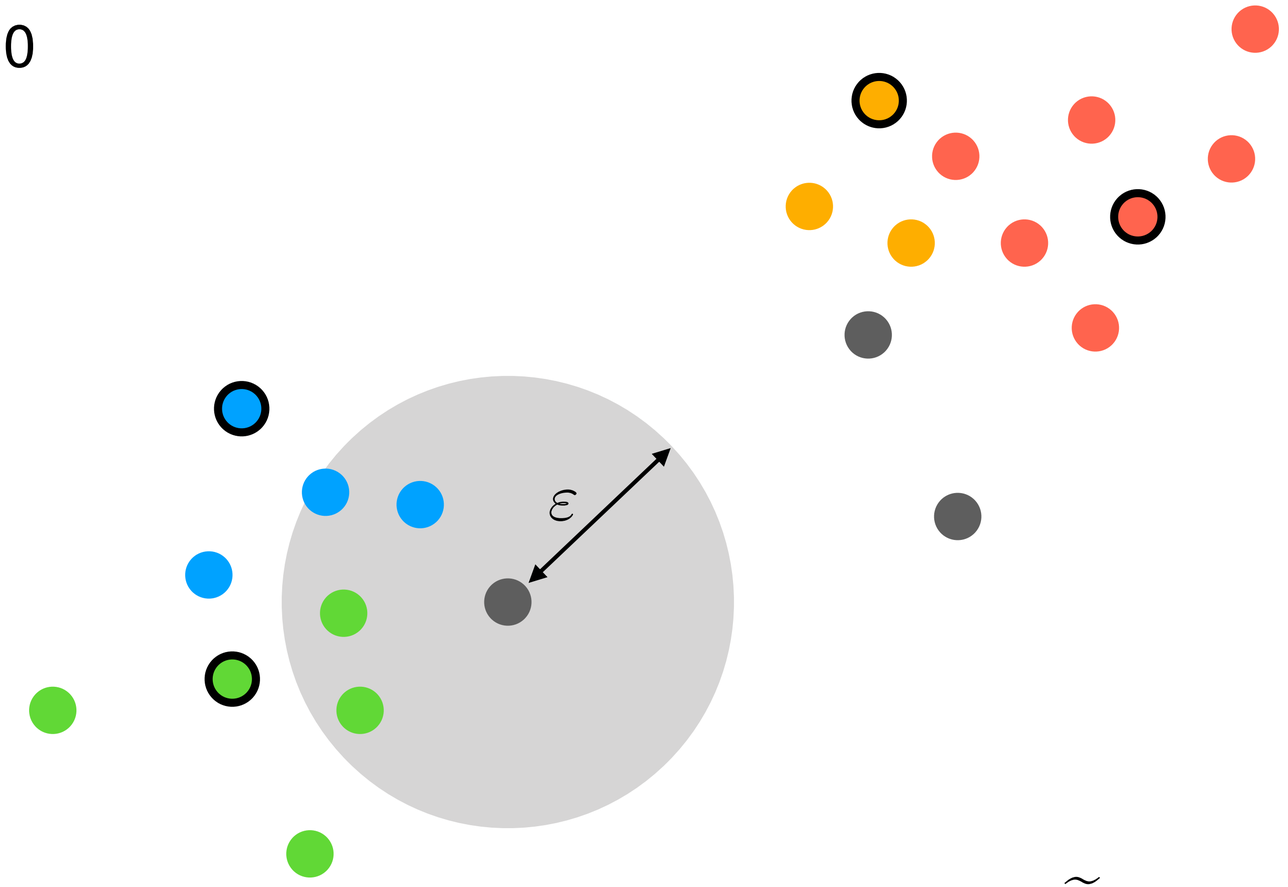

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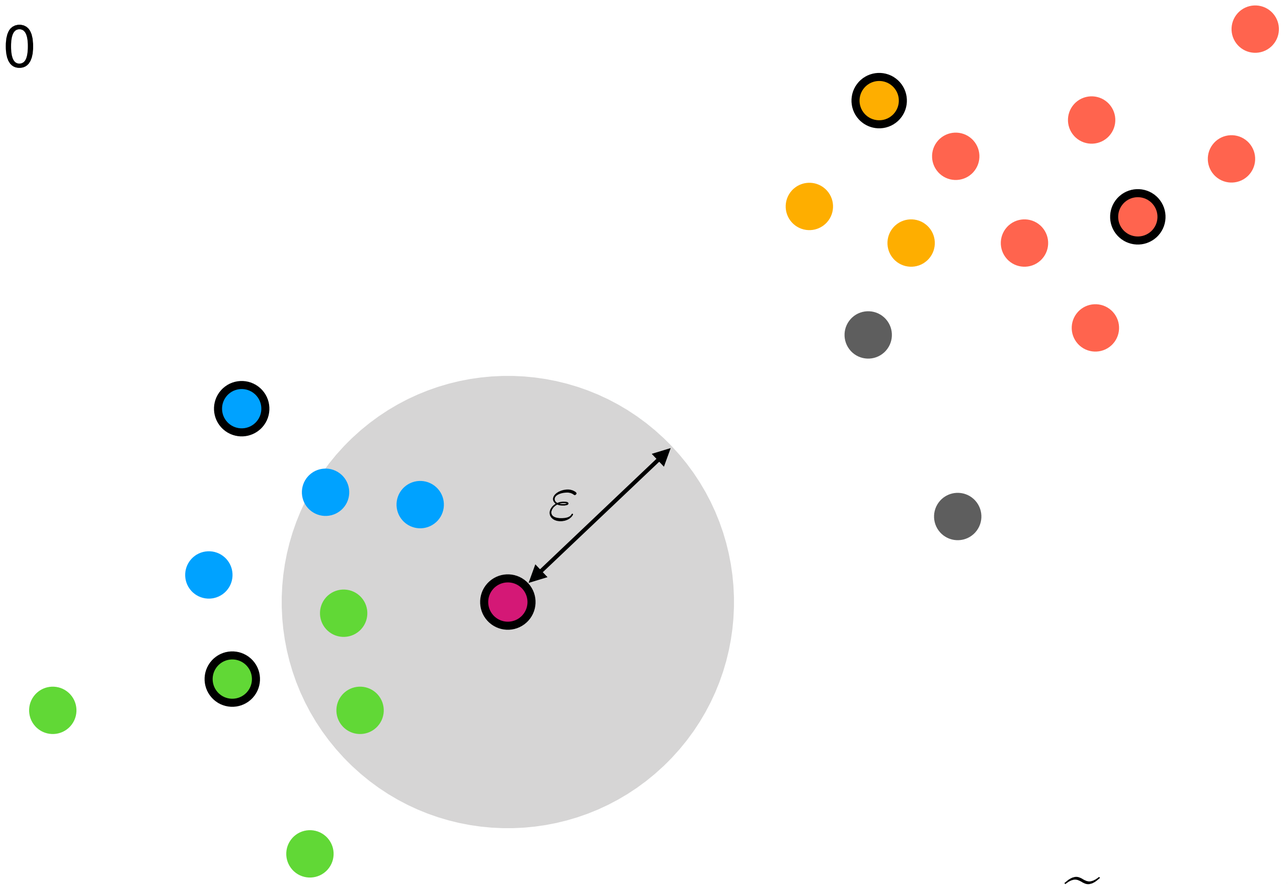

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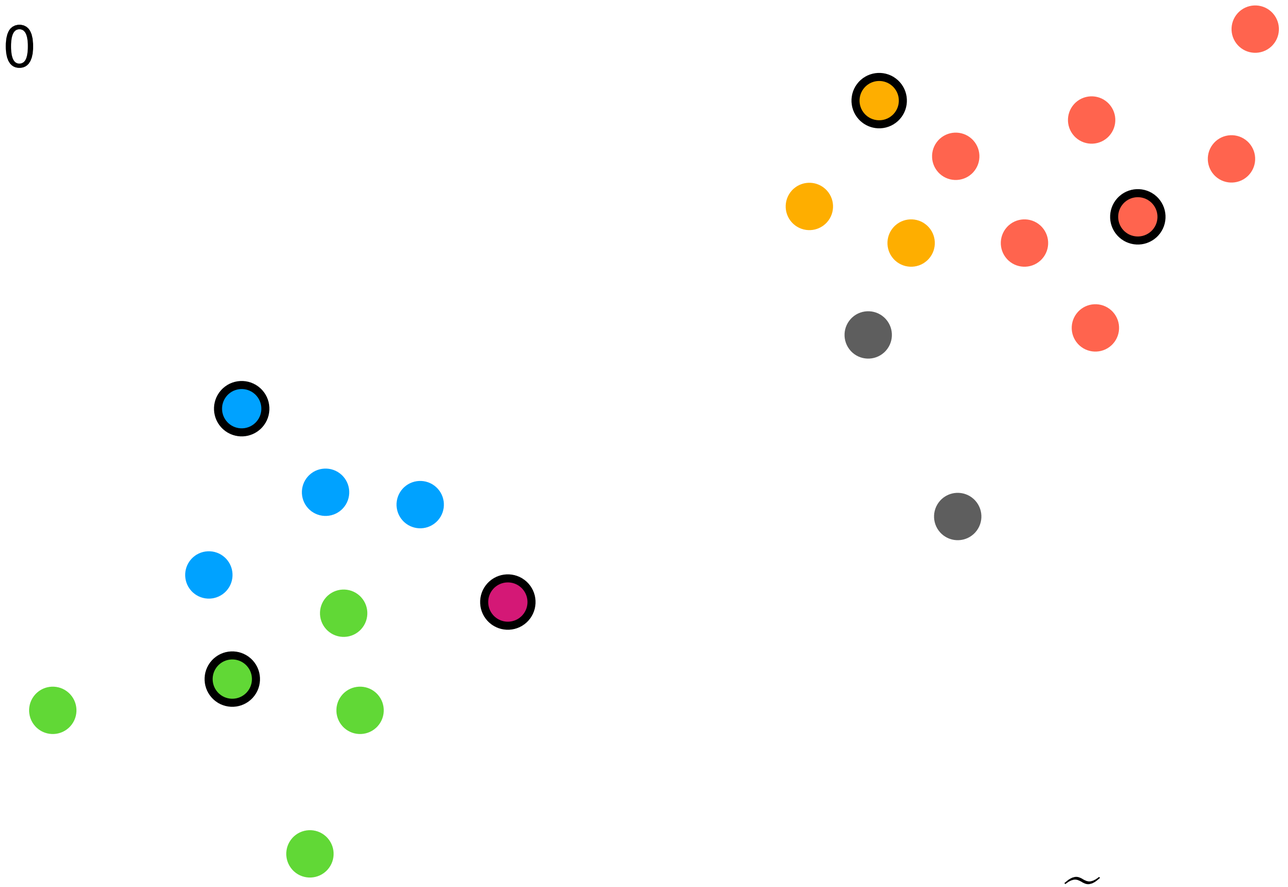

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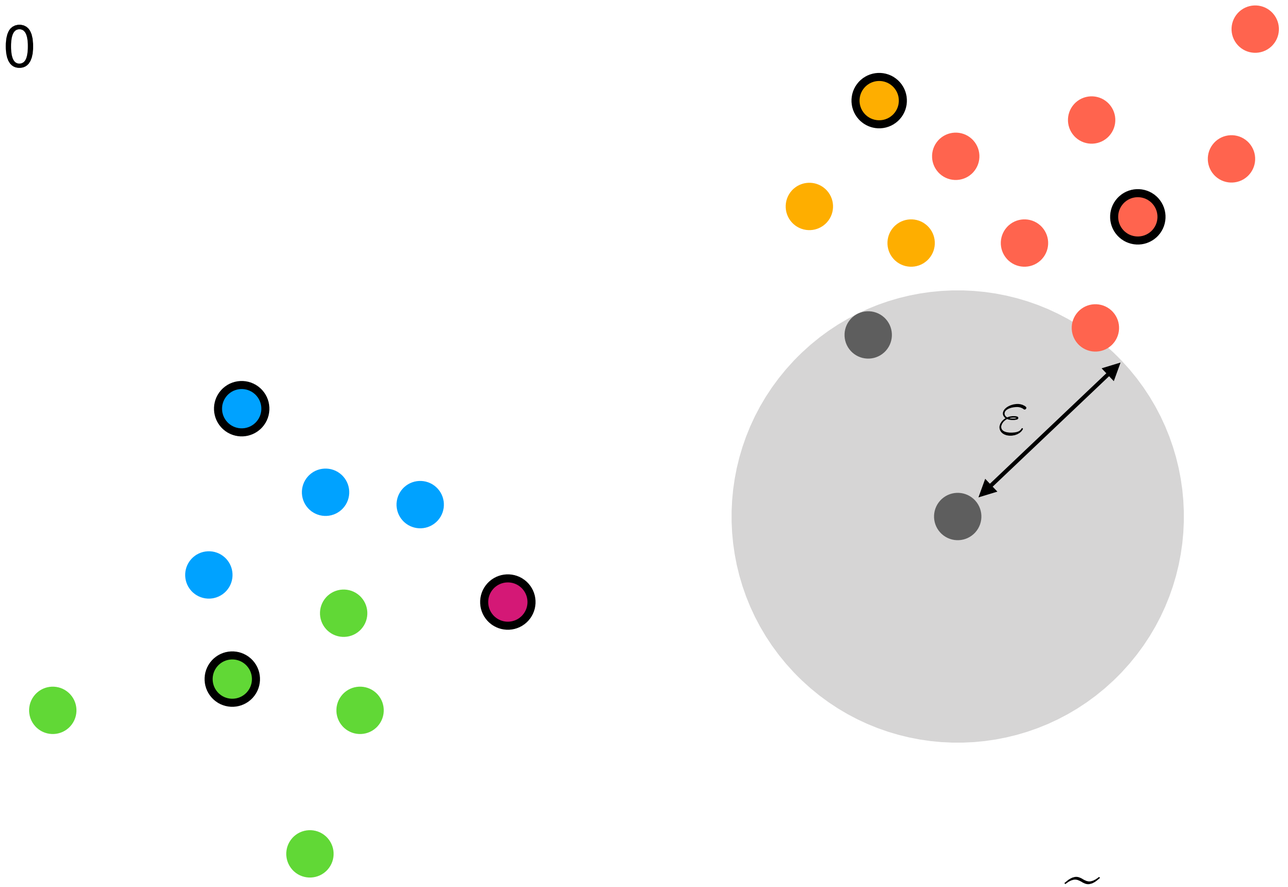

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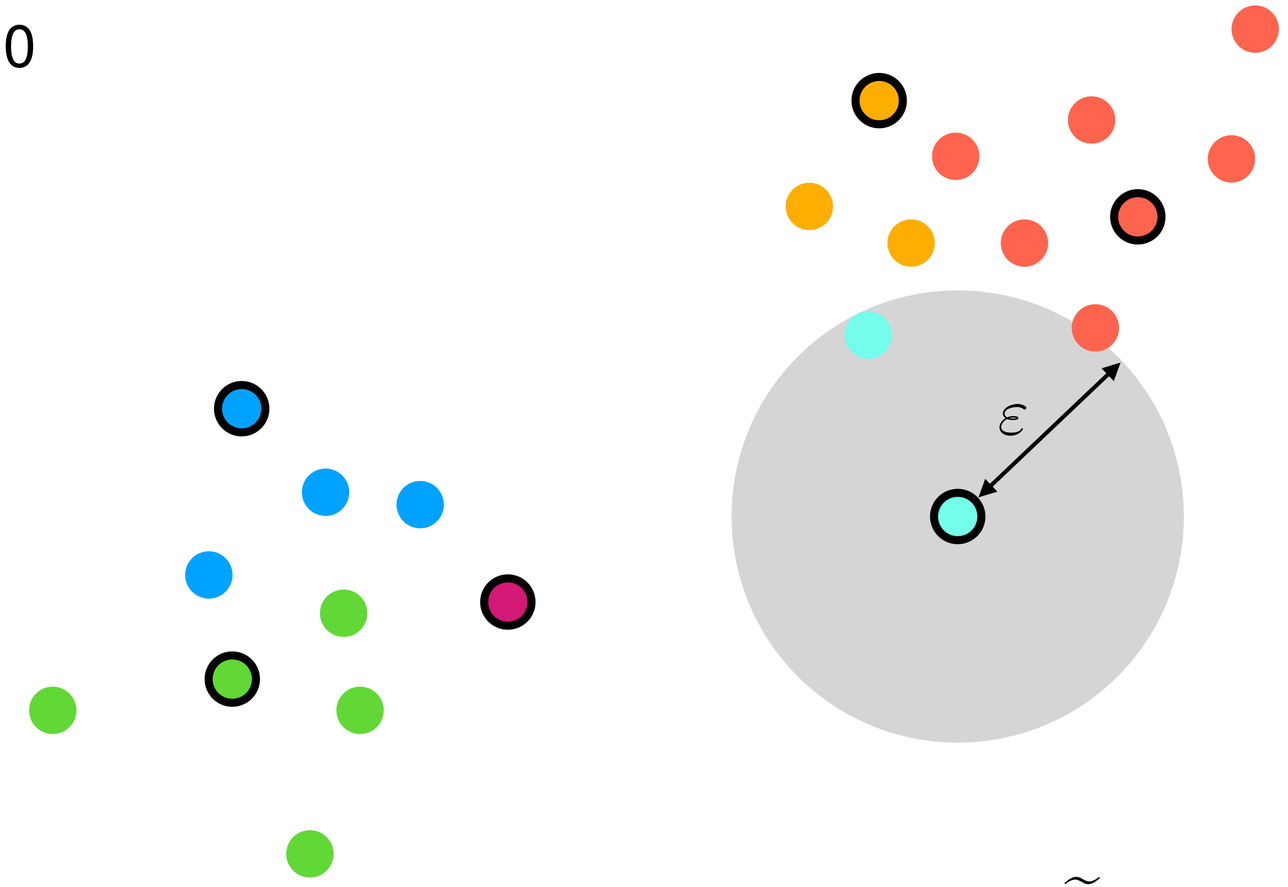

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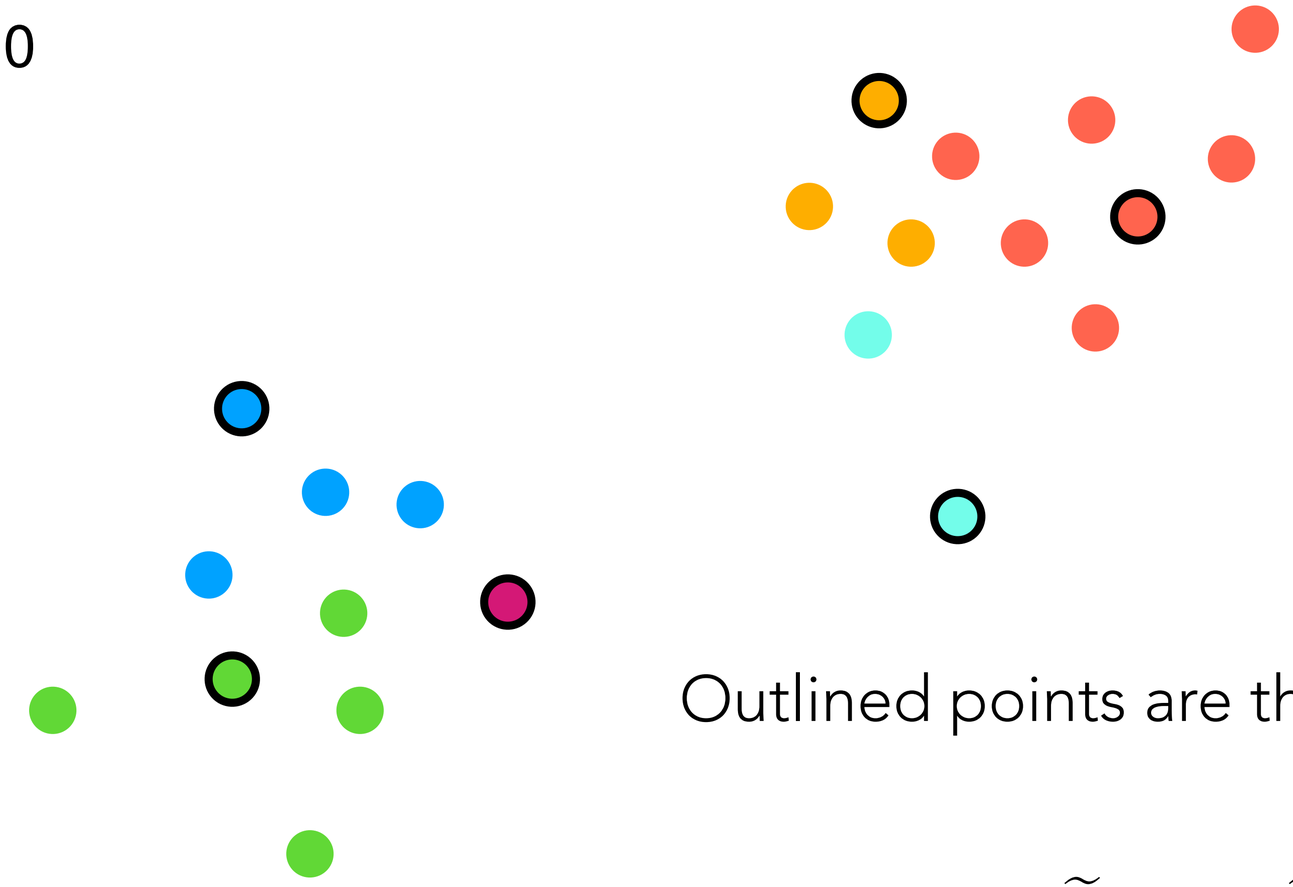
Everything is done in the embedding space (output space of neural net)

Parameter: $\varepsilon > 0$



Outlined points are the *exemplars* $\mathcal{Q}$

These gray points are $\widetilde{X}_1, \ldots, \widetilde{X}_n$