An Analytic Approach to Strong Correlations

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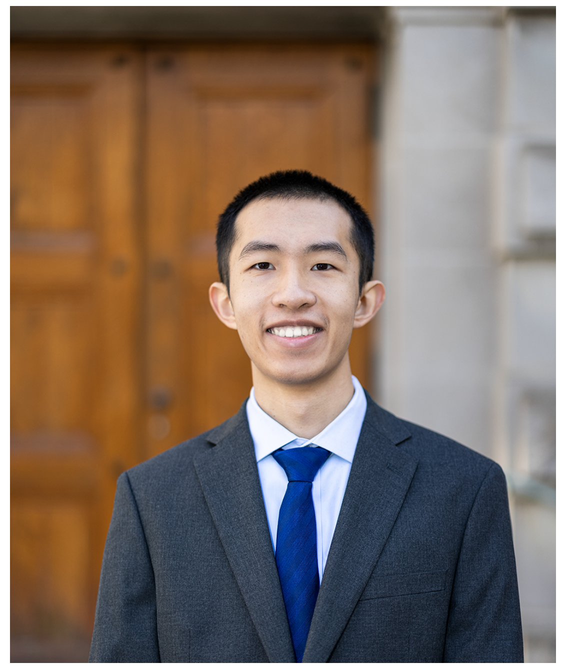
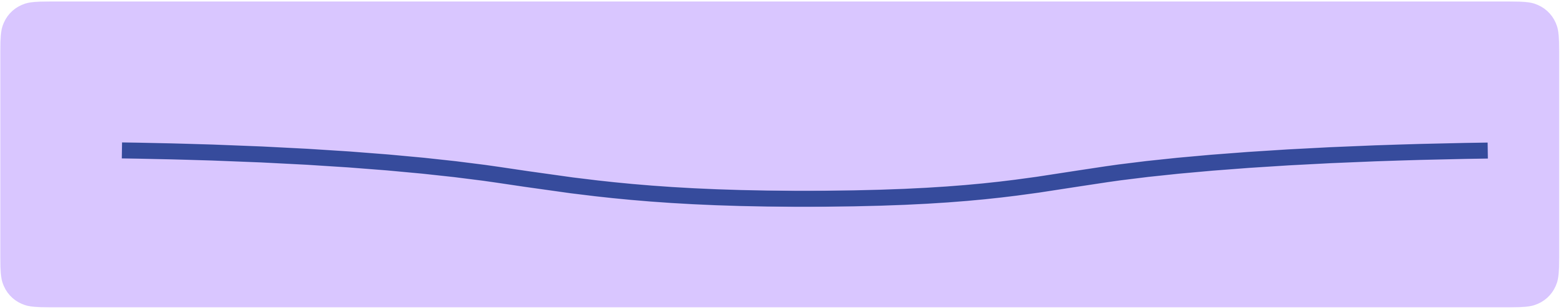


Junkai Dong

Pavel

Phys. Rev. X **15**, 021087 arXiv:2505.08779 arXiv:2605.20171

Diversity of Strongly Correlated Systems



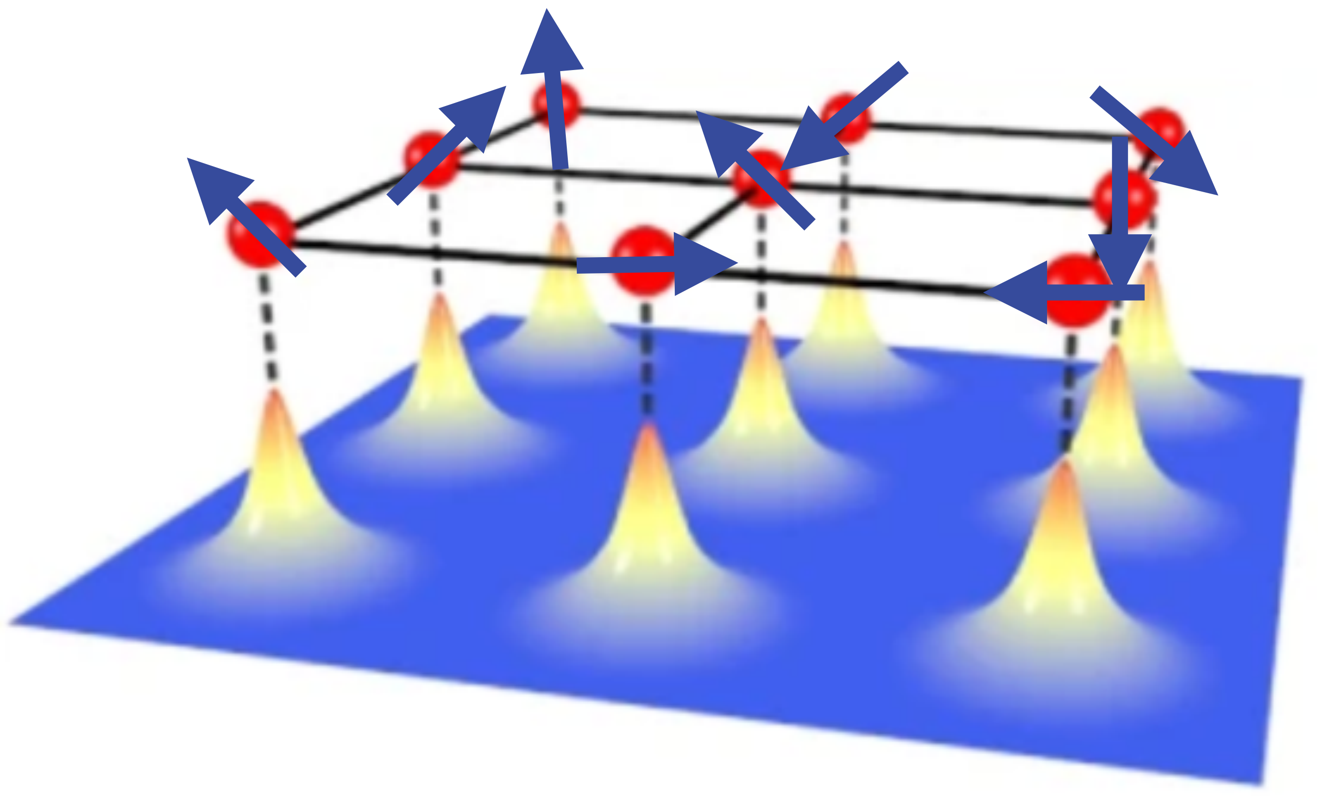
U

$$H_{\text{int}} = \sum_{i<j} P_i P_j V(\boldsymbol{r_i} - \boldsymbol{r_j}) P_i P_j \qquad P = \sum_{\boldsymbol{k}} |\psi_{\boldsymbol{k}}\rangle\langle\psi_{\boldsymbol{k}}|$$

Single particle wavefunctions of band determine effective Coulomb interactions

Localized wavefunctions $\Longrightarrow H_{\text{int}} = \sum_{\boldsymbol{R},\boldsymbol{R}'} U(\boldsymbol{R} - \boldsymbol{R}')\hat{n}_{\boldsymbol{R}}\hat{n}_{\boldsymbol{R}'}.$

Classical interactions $\Longrightarrow$“Mott Physics”, decoupled spins



Delocalized wavefunctions (ex: LLs)

$\Longrightarrow$ **Interactions are quantum**

“Topological Bands:” $\sigma_{xy} = C\dfrac{e^2}{h}$

$$C = \frac{1}{2\pi}\int_{\text{BZ}} \Omega(\boldsymbol{k}) d^2\boldsymbol{k} = 1, 2, 3, \ \ldots. \text{ with}$$

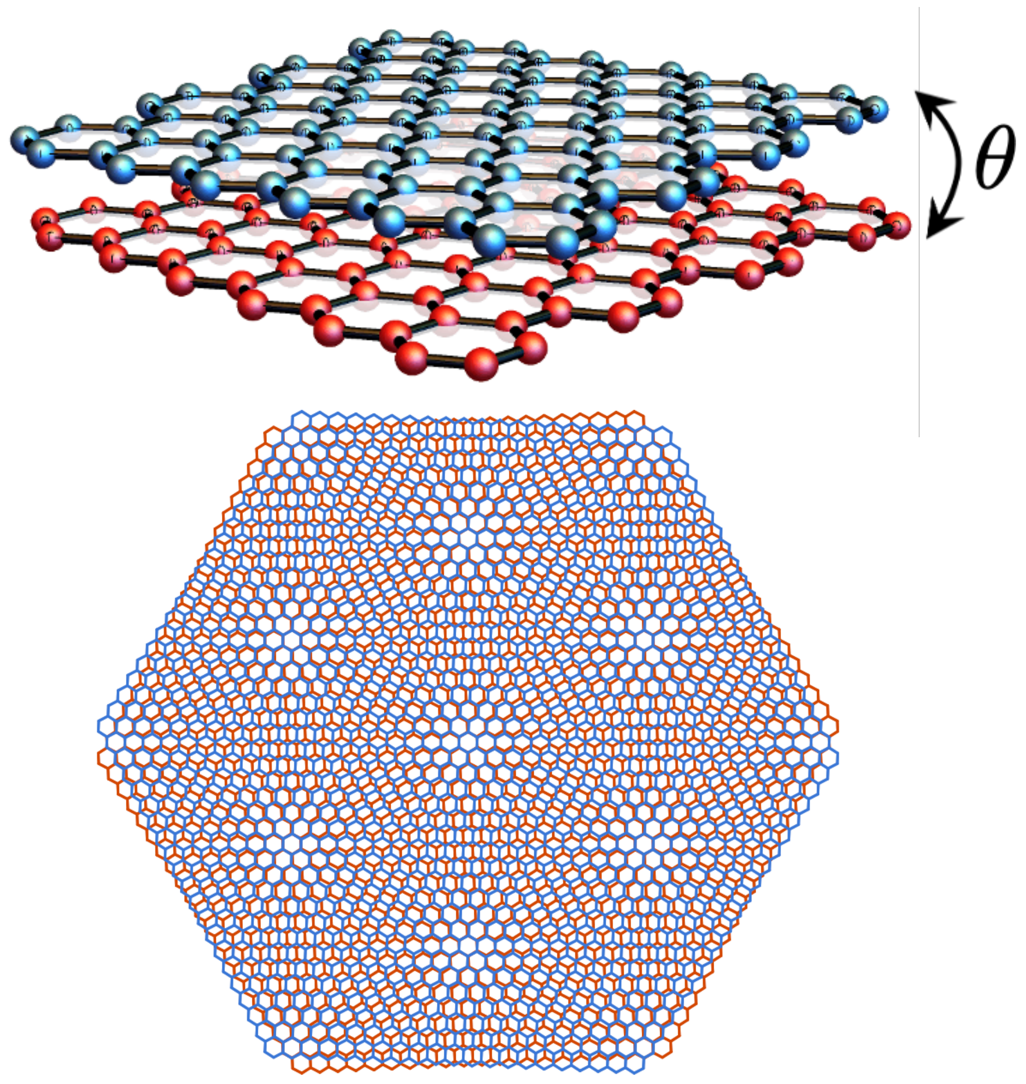
$$\Omega(\boldsymbol{k}) = -i\langle\psi_{\boldsymbol{k}}|[PxP, PyP]|\psi_{\boldsymbol{k}}\rangle$$

Still wide diversity; e.g. all LLs have same $\Omega(\boldsymbol{k})$

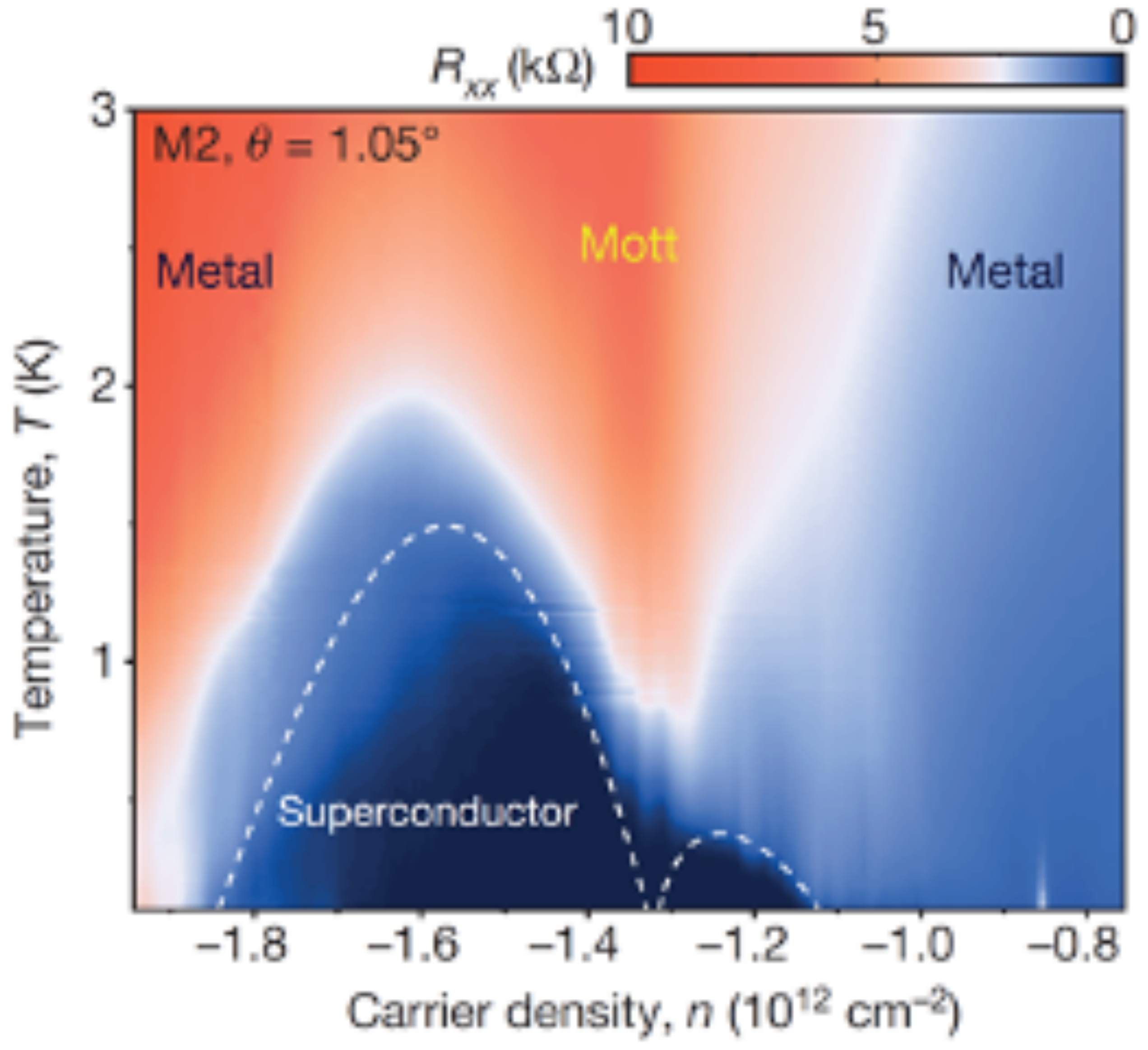
Partially encoded by “Quantum Geometry”

$$g_{\mu\nu}(\boldsymbol{k}) = \text{Re}\,\langle\psi_{\boldsymbol{k}}|Pr_\mu(1-P)r_\nu P|\psi_{\boldsymbol{k}}\rangle, \quad g_{\mu\nu}^{(n-\text{LL})} = (n+\tfrac{1}{2})\frac{\hbar}{eB}\delta_{\mu\nu}$$

Tunable strong correlations in Moiré Systems

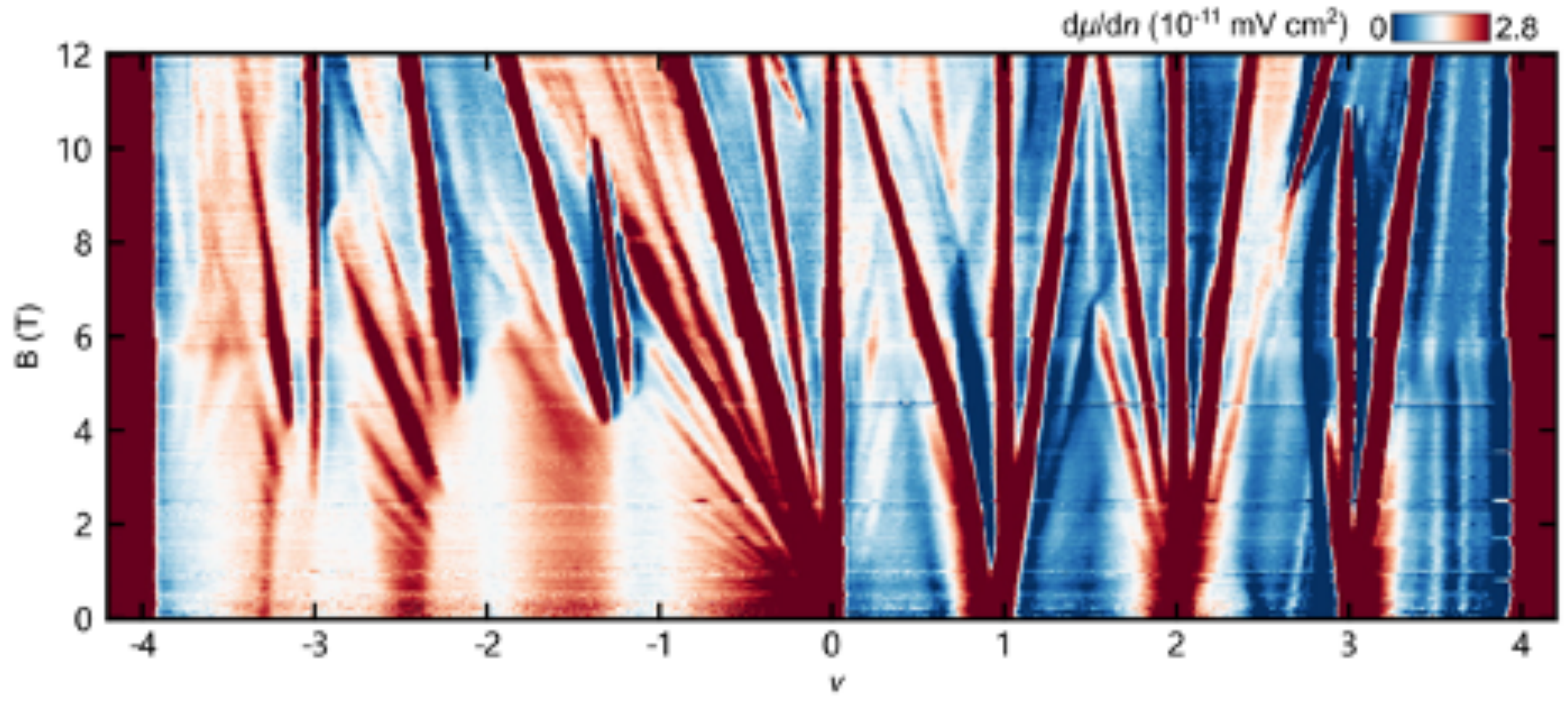


Li,…,Andrei 2010
Bistritzer, Macdonald 2011



Cao *et. al.* (Nature 2018)

(Fractional) Chern insulators

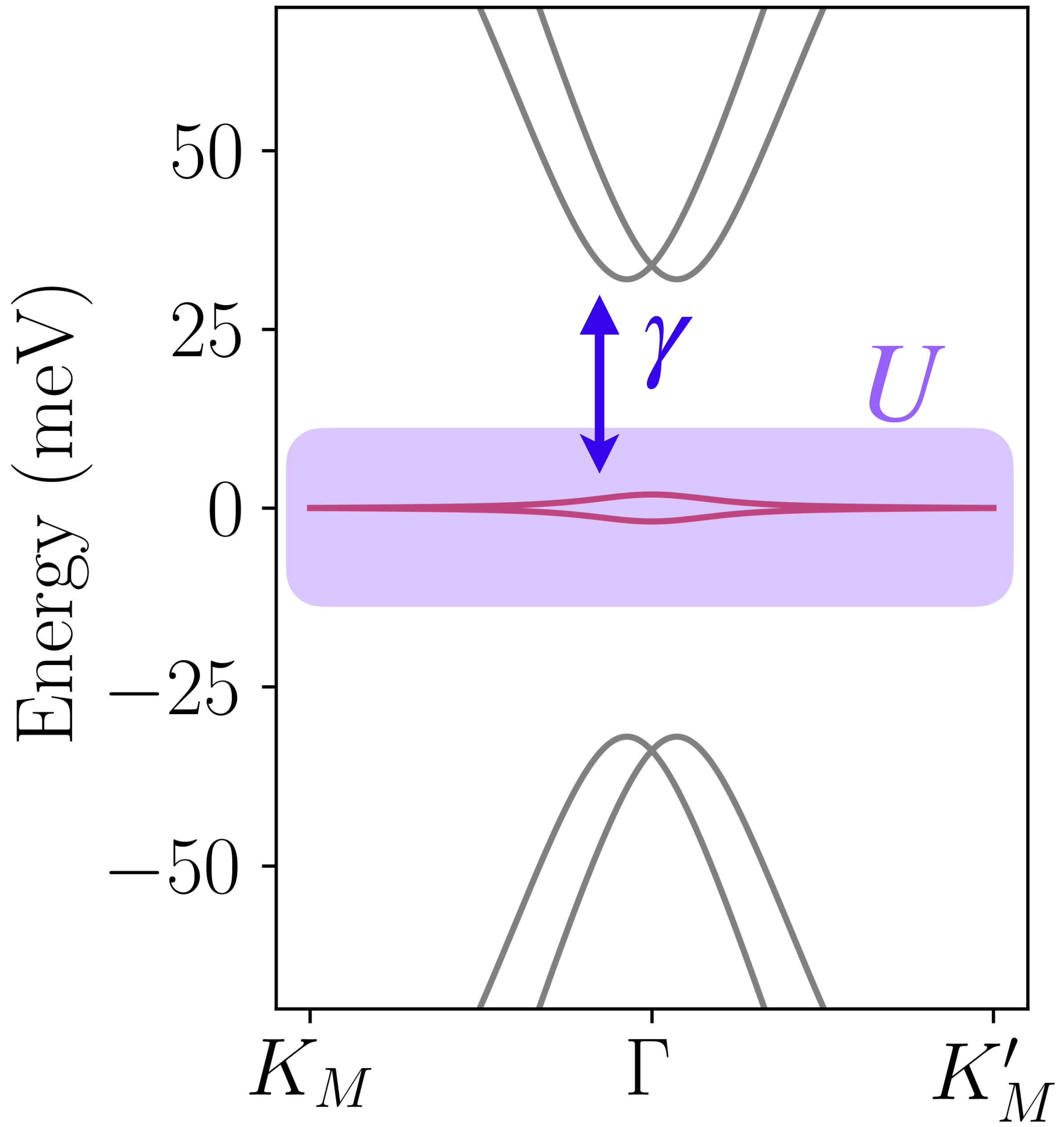


Xie++ (Nature 2021)

How to understand topological bands beyond LLs?

Mott Limit $(U \gg J_{\text{spin}})$ with topology?
What happens in it?

Different Possible Energetic Hierarchies

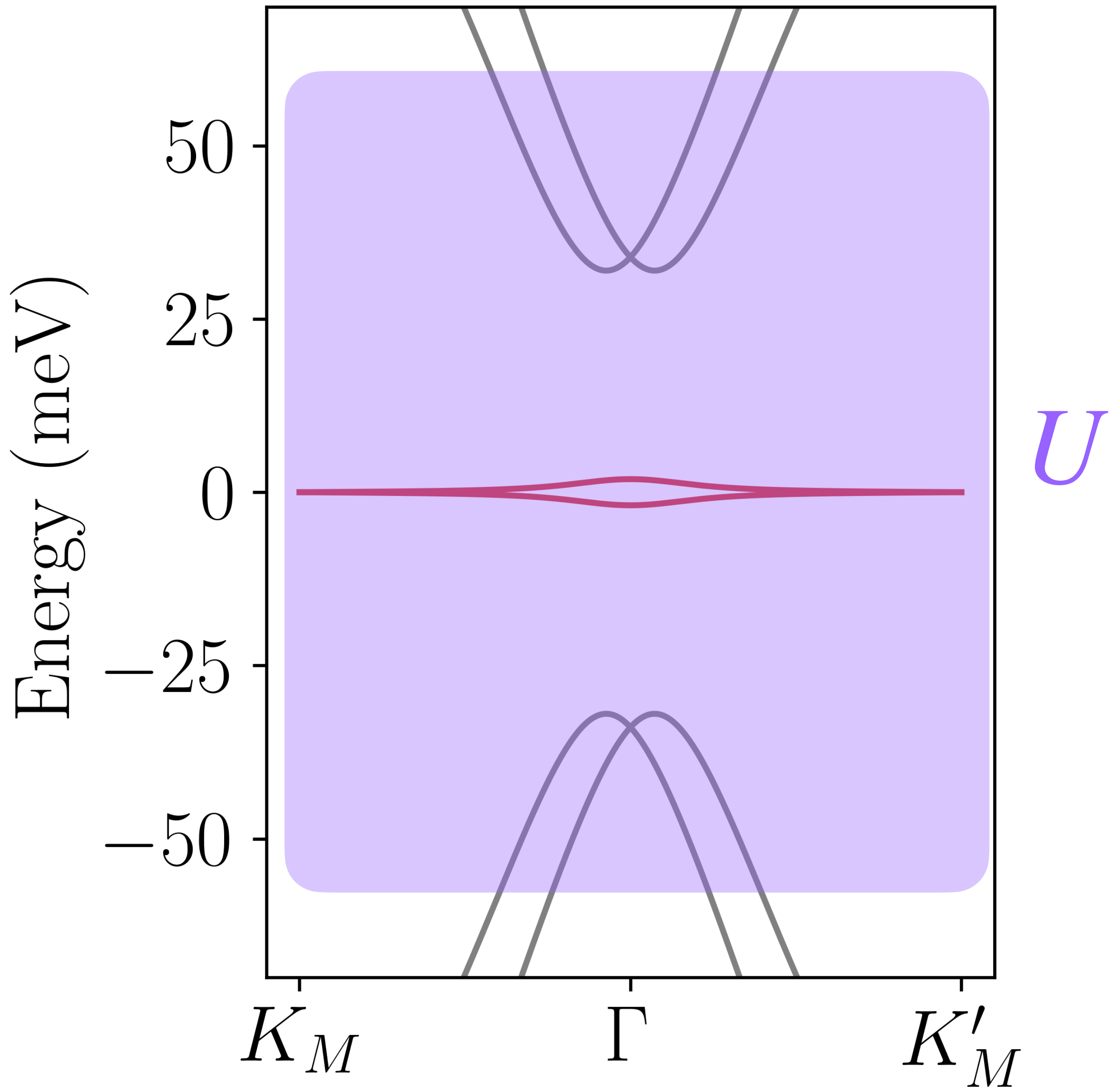


$\gamma \gg U$

Project out remote bands
Strong coupling in flat bands

No local* moments in Hilbert space

This talk: Mott in isolated topological bands



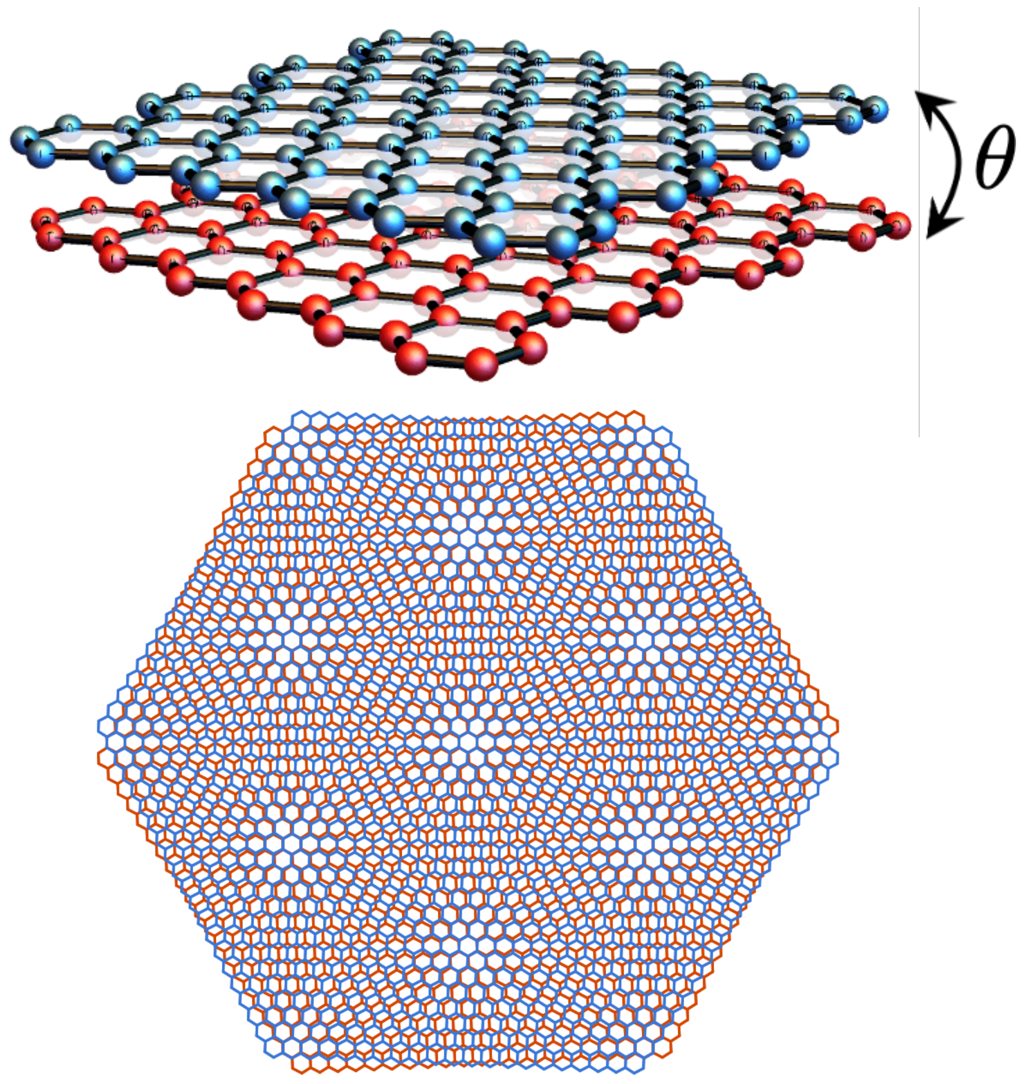
$U \gg \gamma$

Haule, Andrei, Haule arXiv:1901.09852
Song, Bernevig PRL 2022
Datta et al Nat. Comm. 2023
Lau, Coleman PRX 2025

Heavy fermion physics expected
Local moments (flat mixed with remote)
coupled through conduction electrons

*As localized as charge density

Tunable strong correlations in Moiré Systems



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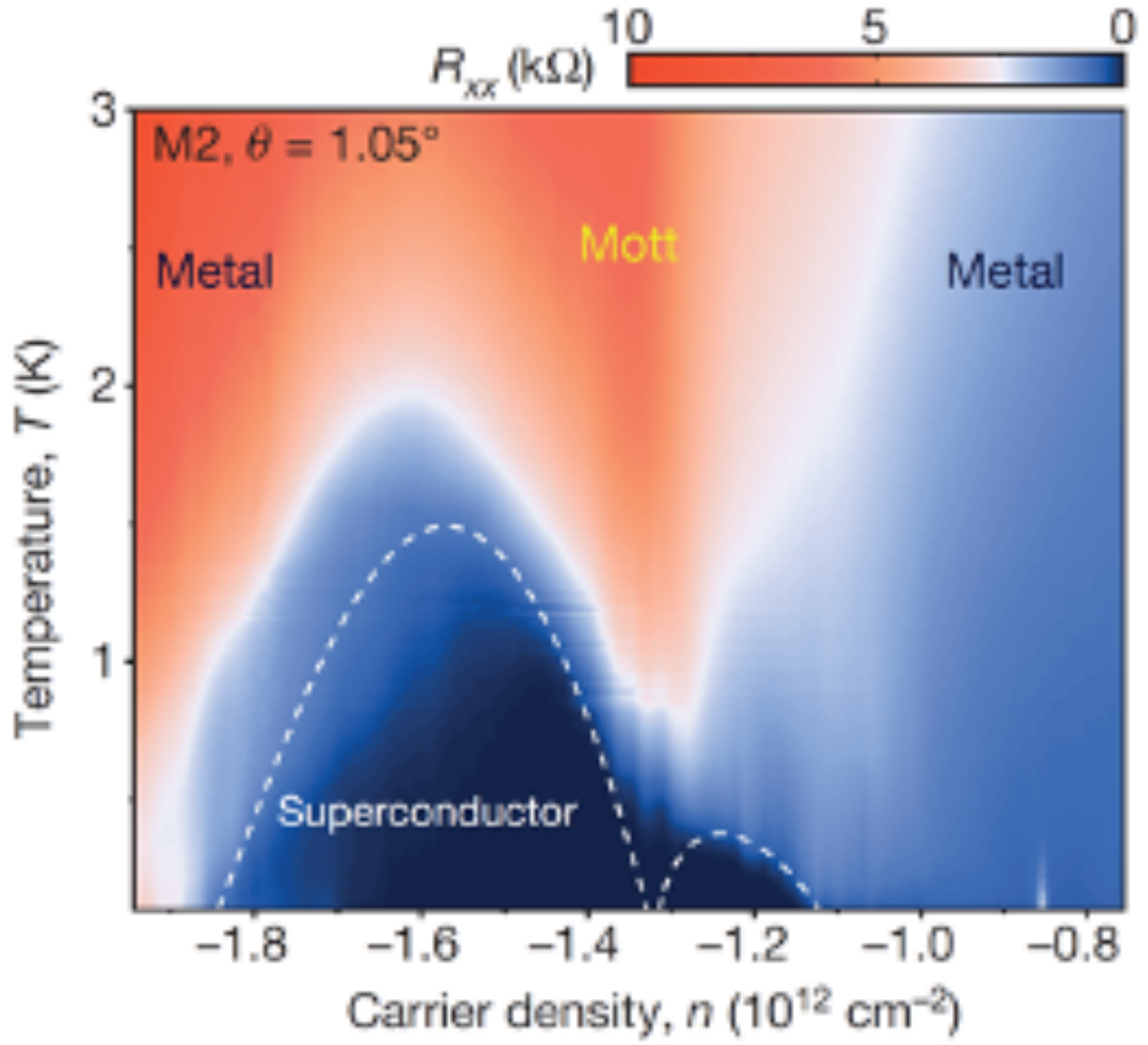

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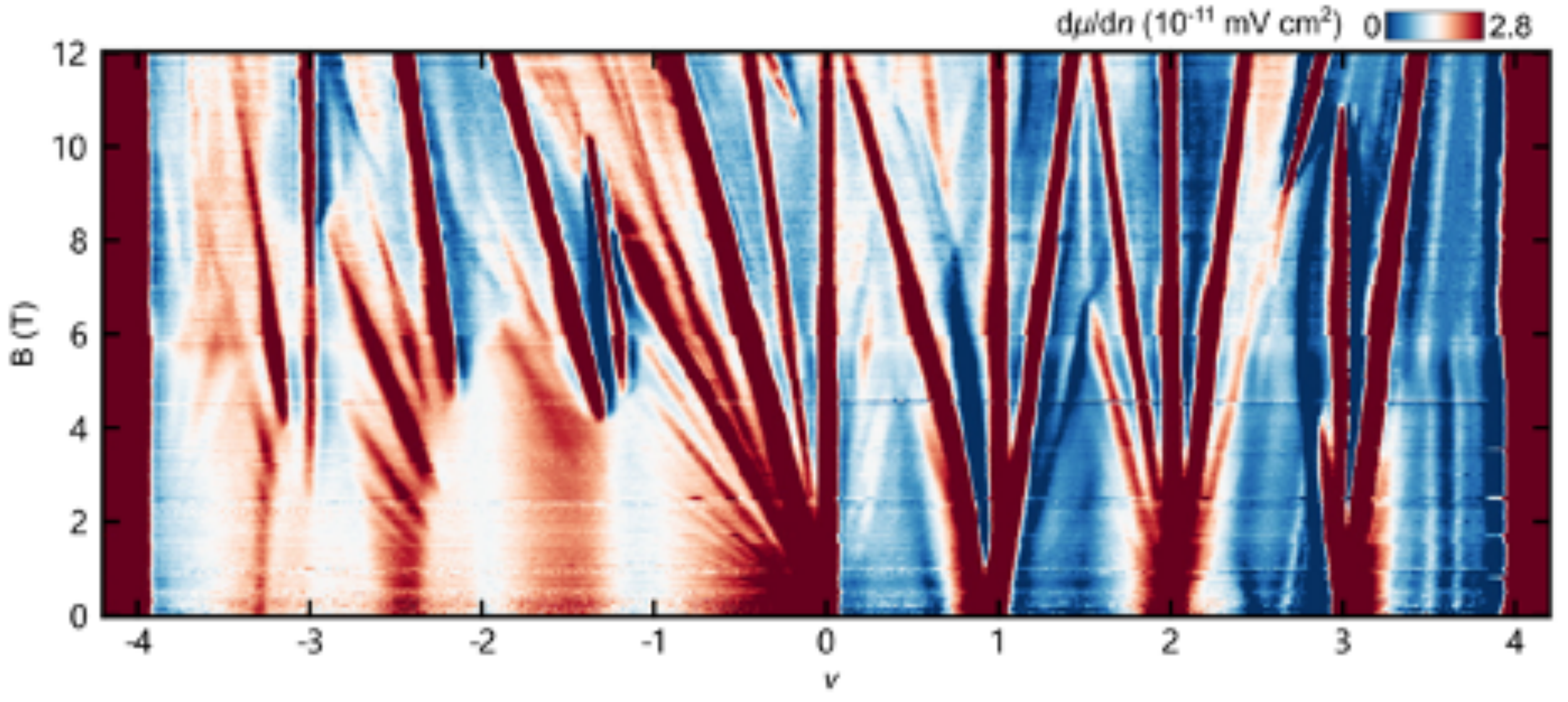


Xie++ (Nature 2021)

How to understand topological bands beyond LLs?

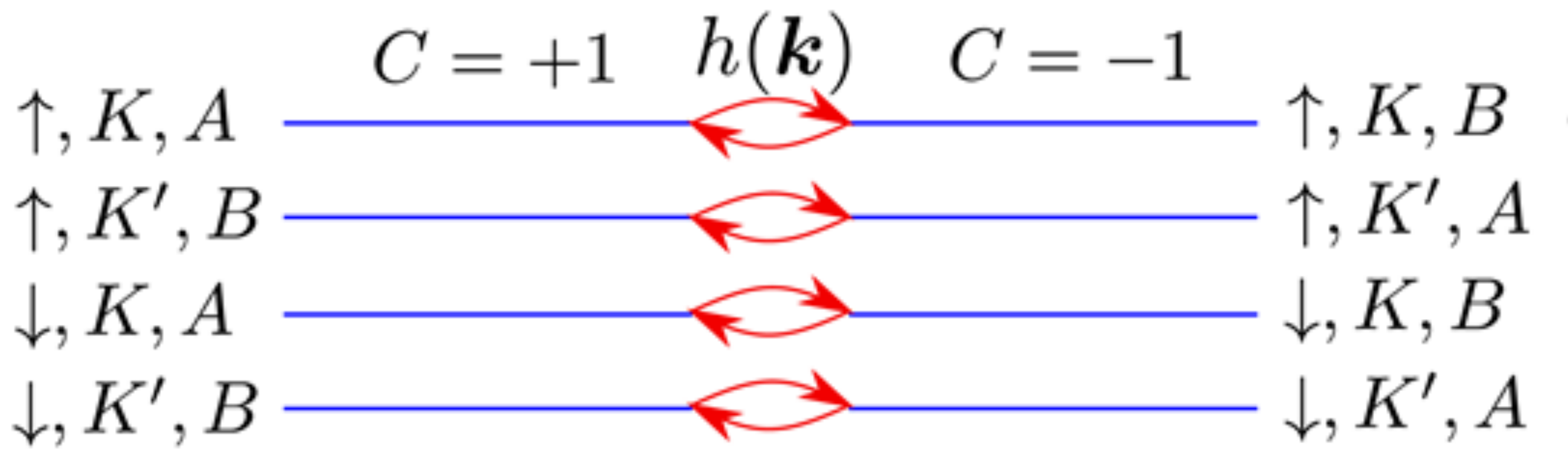
Mott Limit $(U \gg J_{\rm spin})$ with topology?
What happens in it?

Today:

1. Mott Phys in top bands far from LLL (model of TBG)
   Three heavy particles forming a massless bound state

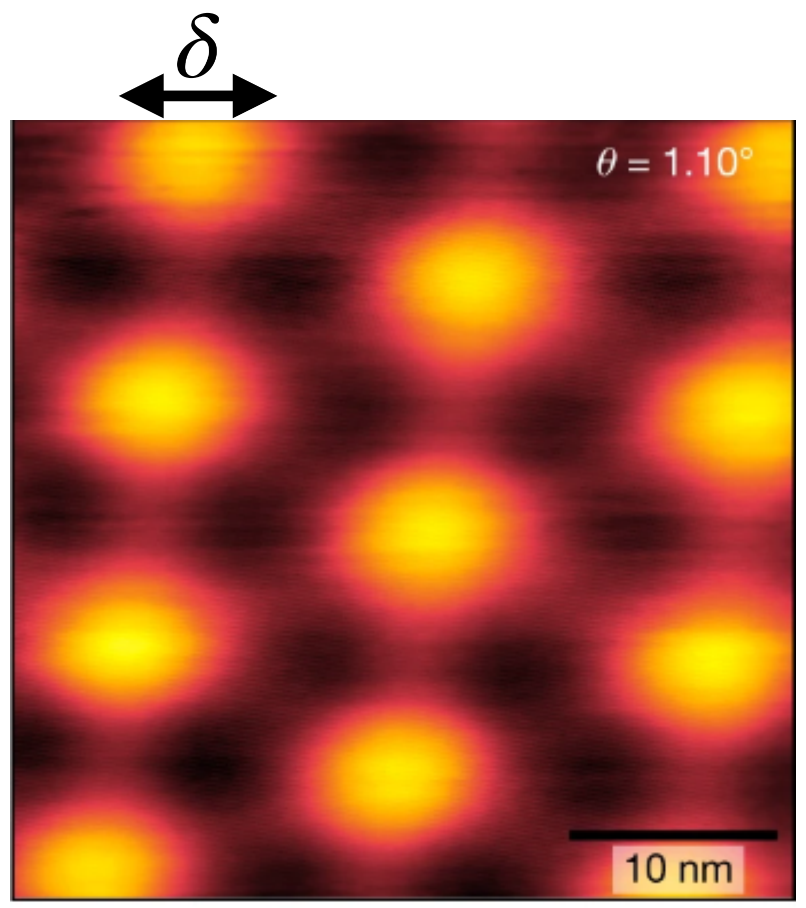
2. New analytic tool for strong correlations (+ transport!)

TBG charge, Berry curvature concentrated



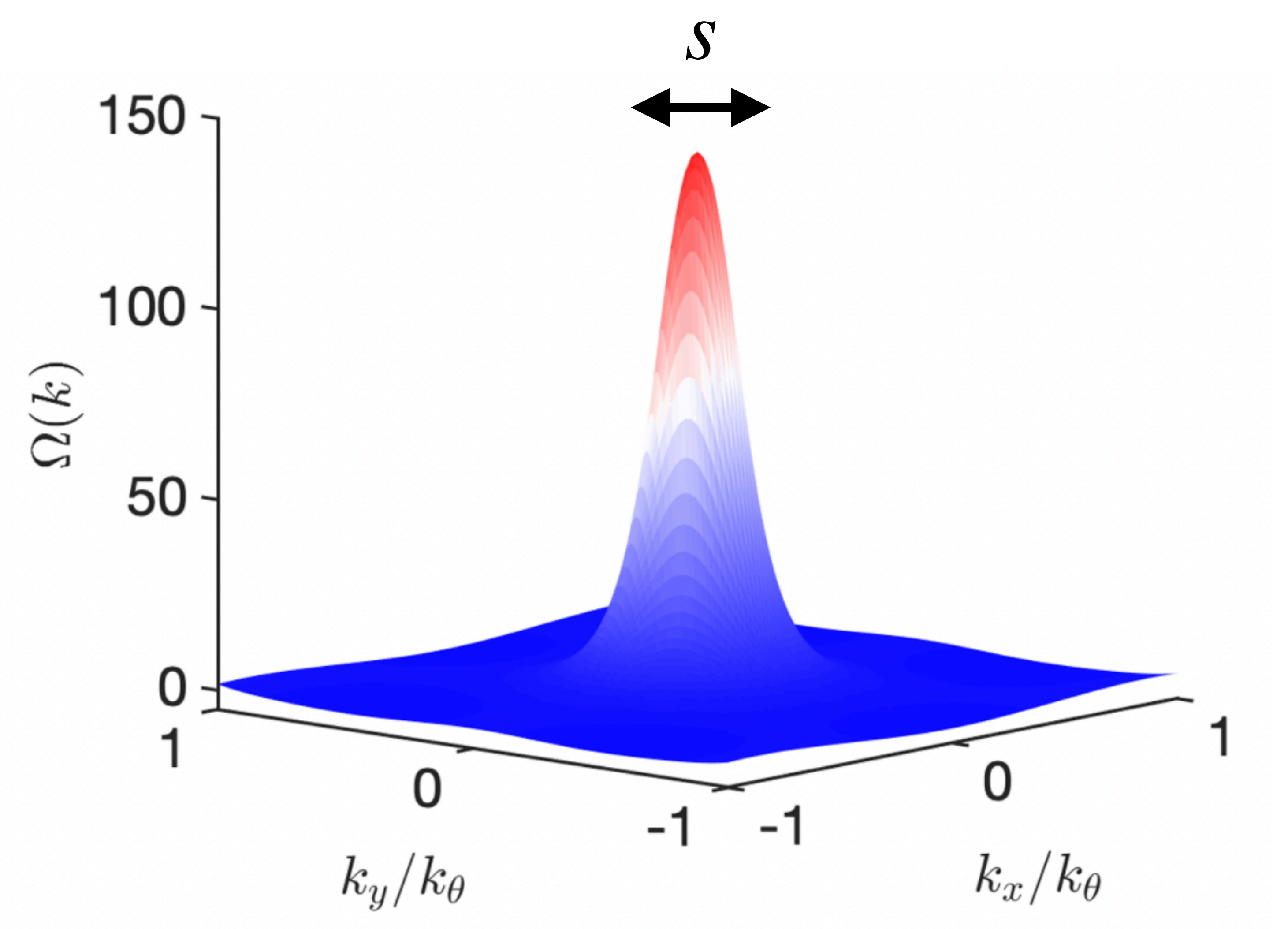
Bultinck, Khalaf++ PRX 2020

Charge Density



Pasupathy et al (Nature 2019)

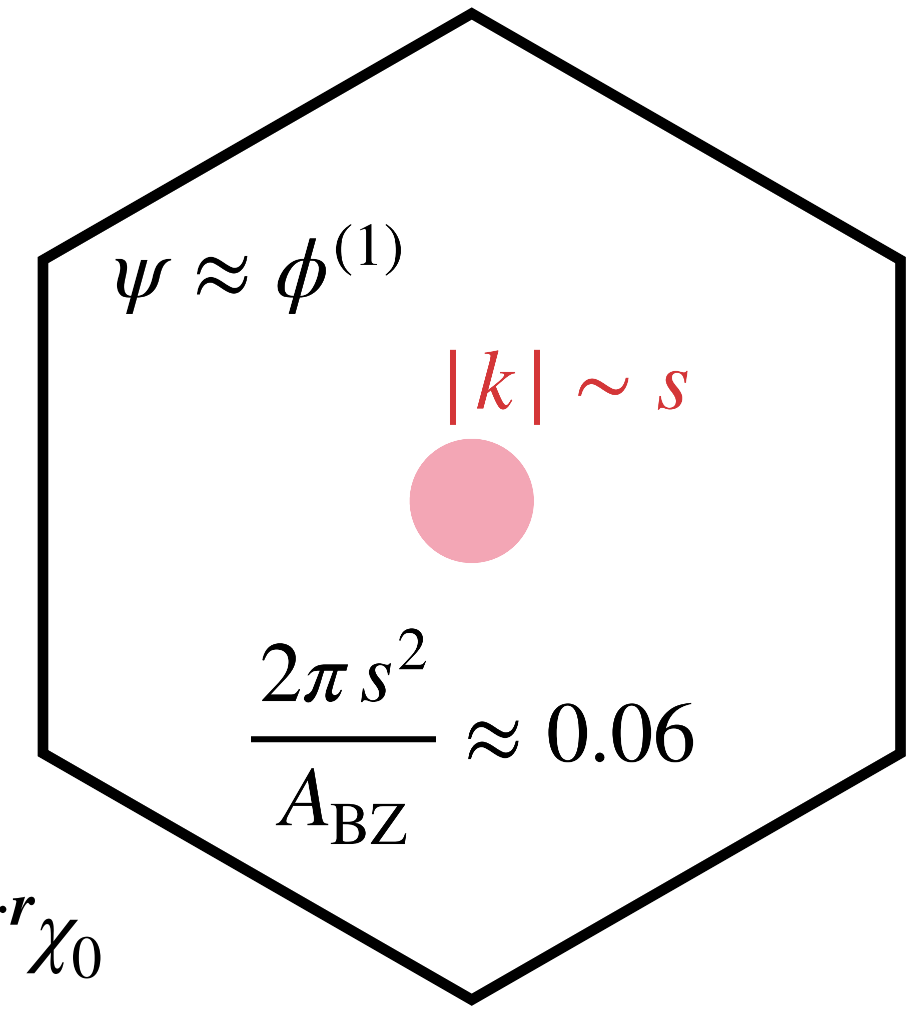
Berry Curvature



$\delta \ll L_M$ saddle point approx*

$$\psi_{\boldsymbol{k}}(\boldsymbol{r}) = \phi^{(1)}_{\boldsymbol{k}}(\boldsymbol{r}) + \frac{s}{i(k_x + ik_y)}\phi^{(z)}_{\boldsymbol{k}}(\boldsymbol{r})$$

$$\phi^{(1)}_{\boldsymbol{k}}(\boldsymbol{r}) = \sum_{\boldsymbol{R}} \frac{e^{-\frac{|\boldsymbol{r}-\boldsymbol{R}|^2}{4\delta^2}}}{2\pi\delta^2} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\chi_0 \qquad \phi^{(z)}_{\boldsymbol{k}}(\boldsymbol{r}) = \sum_{\boldsymbol{R}} \frac{z-R}{\delta}\frac{e^{-\frac{|\boldsymbol{r}-\boldsymbol{R}|^2}{4\delta^2}}}{2\pi\delta^2} e^{-i\boldsymbol{k}\cdot\boldsymbol{r}}\chi_0$$



Project Coulomb interactions:

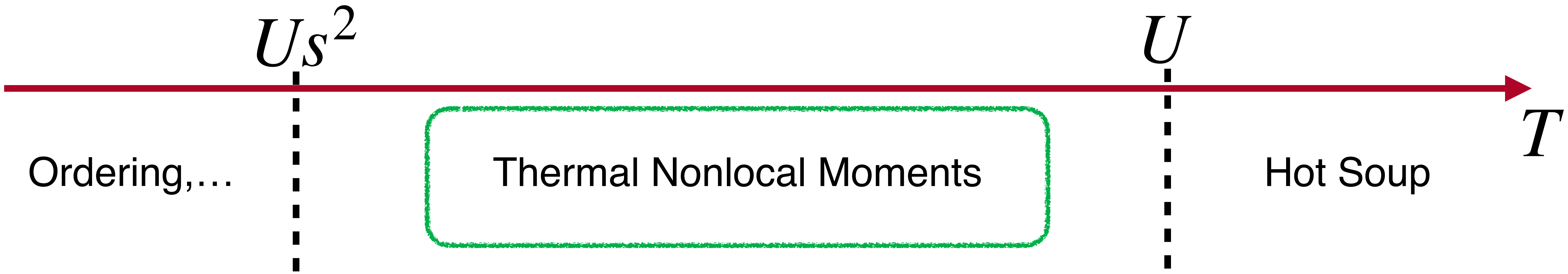
$$H_{\rm int} = \frac{1}{2A}\sum_q V_q \delta\rho_q \delta\rho_{-q}$$

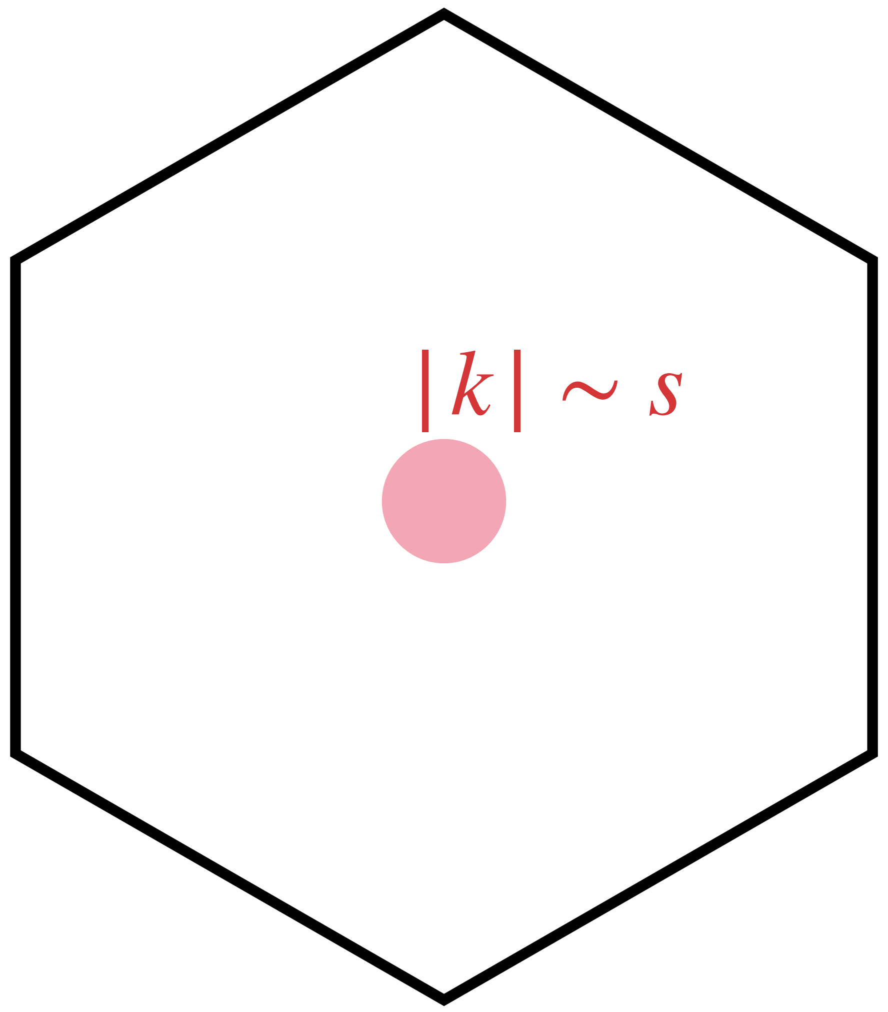
*Or Dirac particle w/ $B(\boldsymbol{r}) = B_0 + \lambda B_I(\boldsymbol{r})$, $\delta \sim s \sim \frac{1}{\sqrt{\lambda}}$,

Also related to band-projected Song-Bernevig

Matches BM at $\approx 5\,\%$ level

Mott Hierarchy, Role of Small Γ Region?





Bulk Thermodynamics (Entropy, Magnetization, ….)

Small phase space: Ignore Γ

Match TBG expts quantitatively

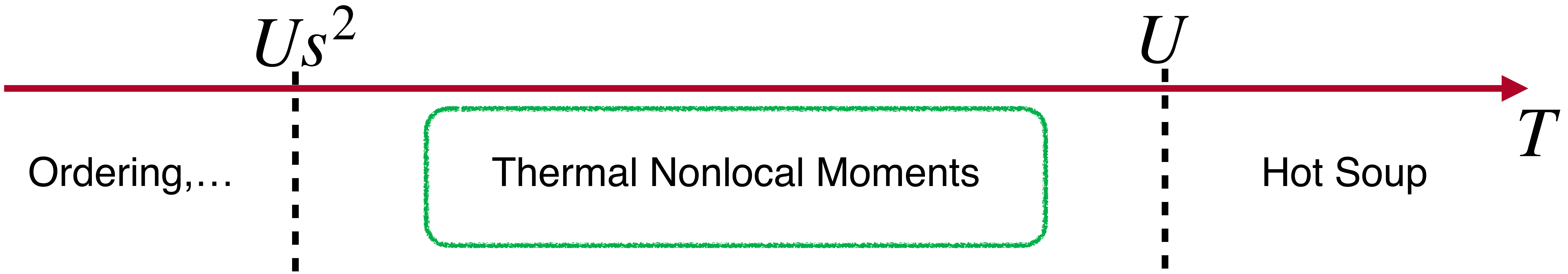
[Rozen++ Nature (2020), Saito++ Nature (2021), Zhang++ arXiv:2503.17875]

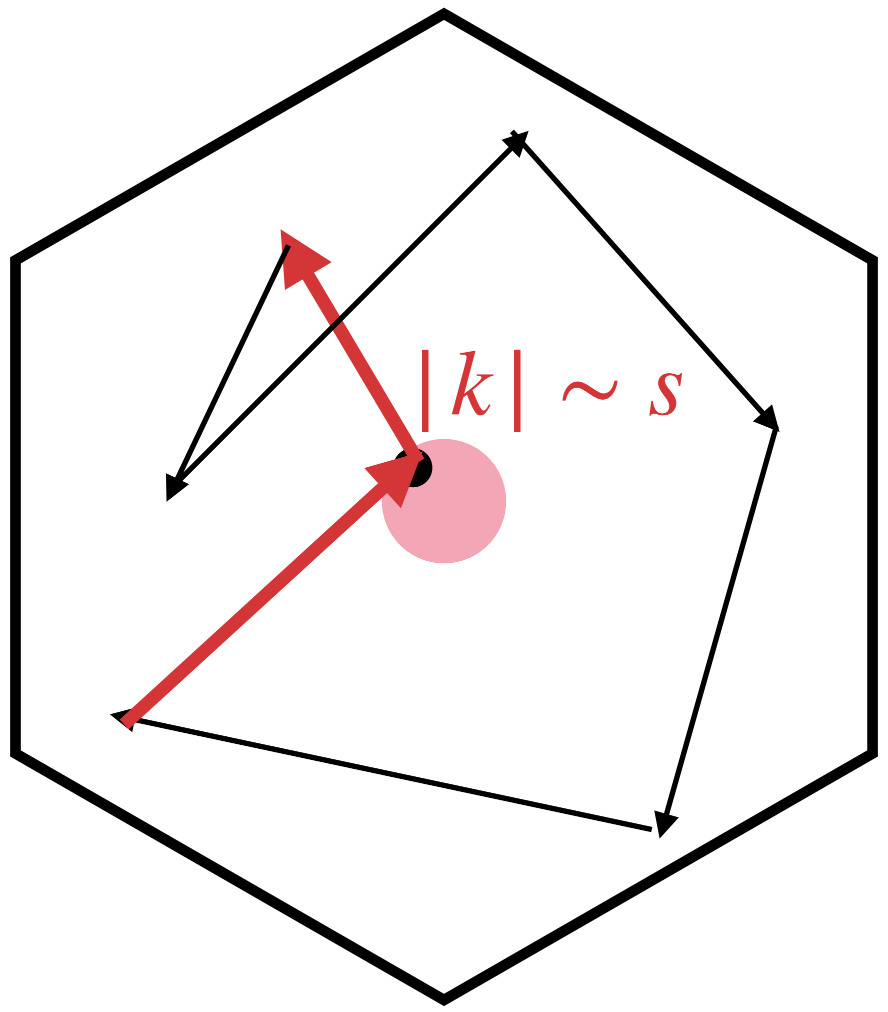
Transport, $\boldsymbol{k}$-resolved spectra,

Γ-area crucial (but solvable!)

PJL, Dong, Vishwanath, Khalaf (PRX 2025)

Mott Hierarchy, Role of Small Γ Region?





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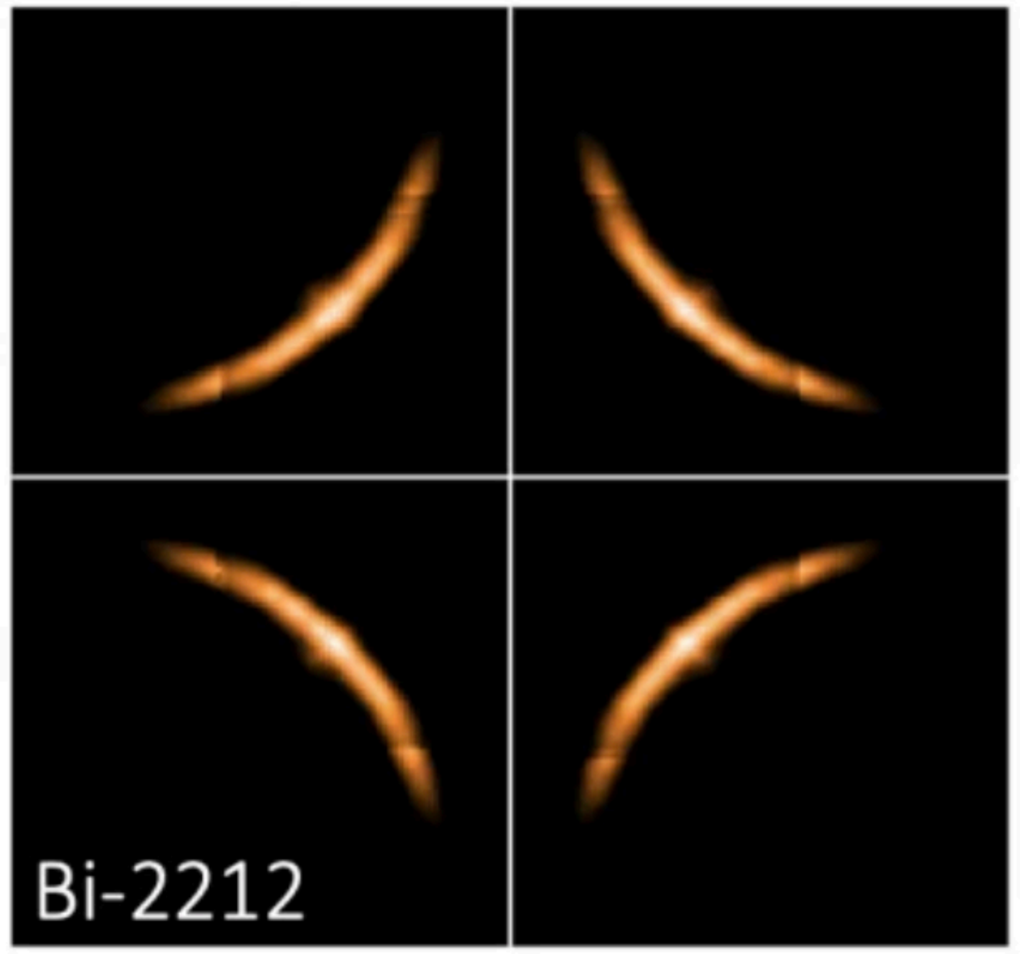
Electron Spectral Function $A(k,\omega)$



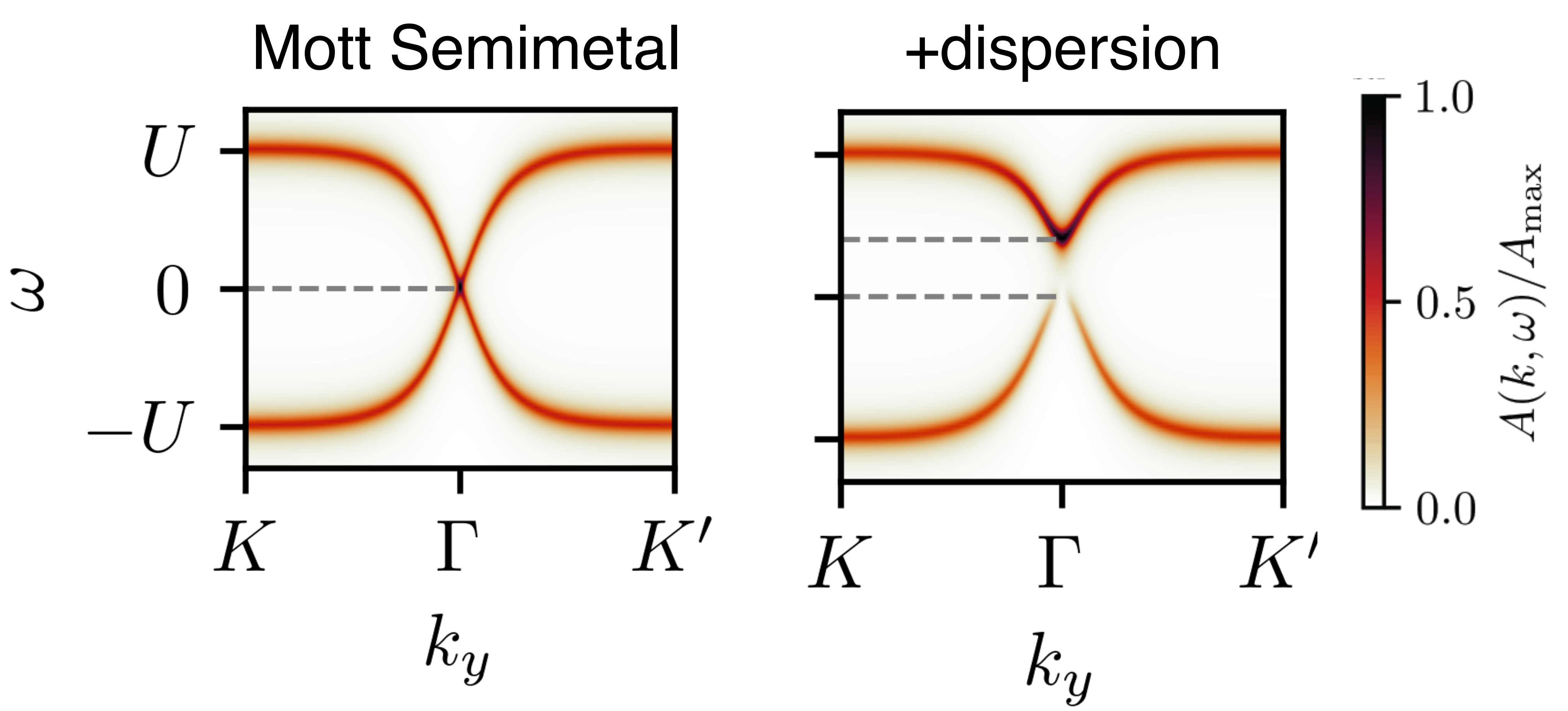
Z-X Shen
https://arpes.stanford.edu/research/quantum-materials/cuprate-superconductors

Exotic Carriers from Concentrated Topology

Mott-like $A(k,\omega)$ away from Γ, gap closes at Γ.

"Zero residue gap" from $H \to H - \sum_k h(k) n_k$;

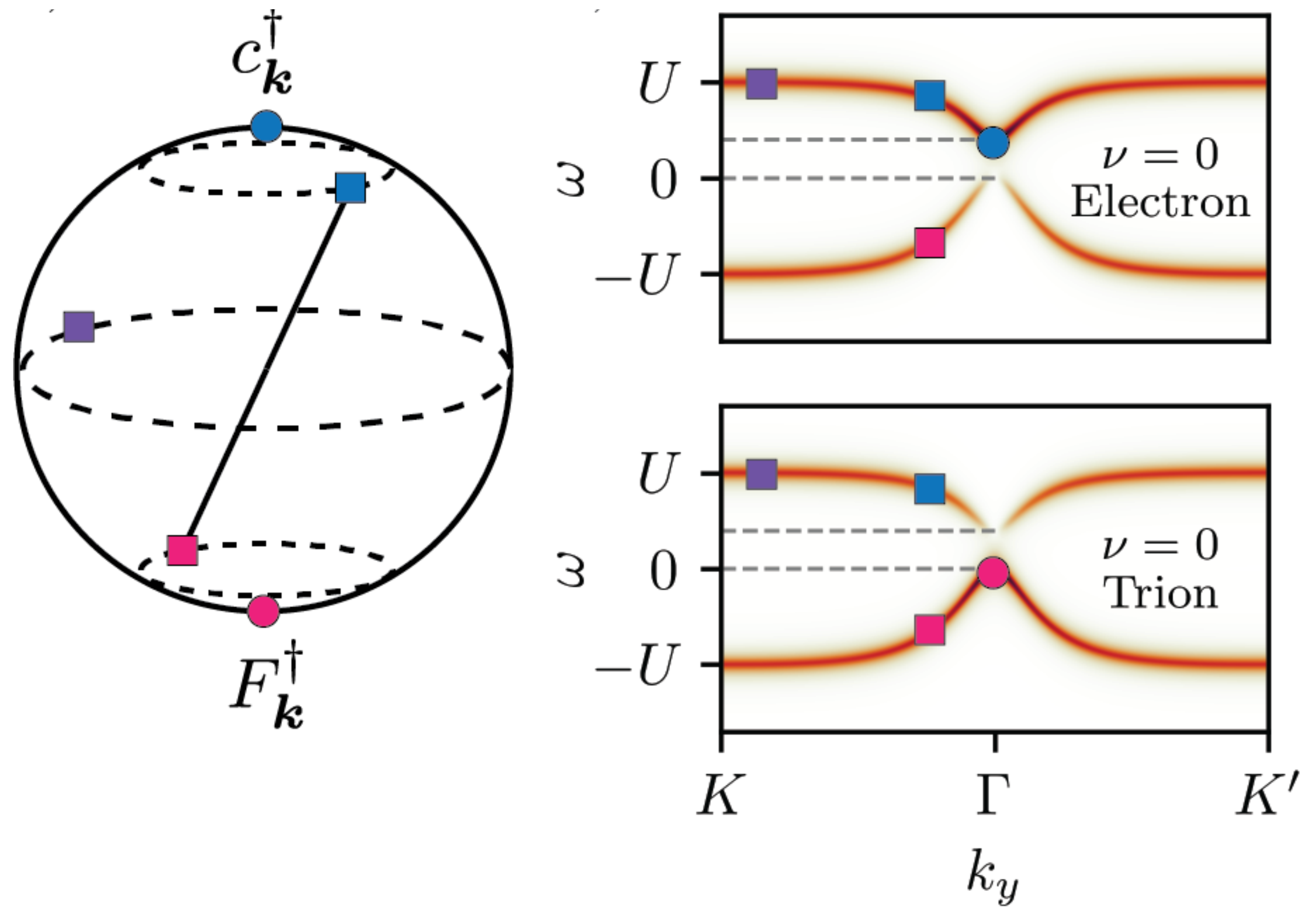
non-electron charged excitations? [Zhao, Zhou, Zhang arXiv:2502.17430]



"Dirac Trion" excitations!

$$F_{k\sigma} = \sum_R e^{ikR}\{\delta n_R, c_{R\sigma}\}, \quad \delta n_R = \sum_\sigma c^\dagger_{R\sigma} c_{R\sigma} - 1, \quad c_R = \frac{1}{\sqrt{N}} \sum_k c_k \chi_k e^{ik\cdot R}$$

- Orthogonal to electron at Γ (top. protected!)
- Light bound state of 3 heavy particles
- Carrier in the SC normal state of TBG



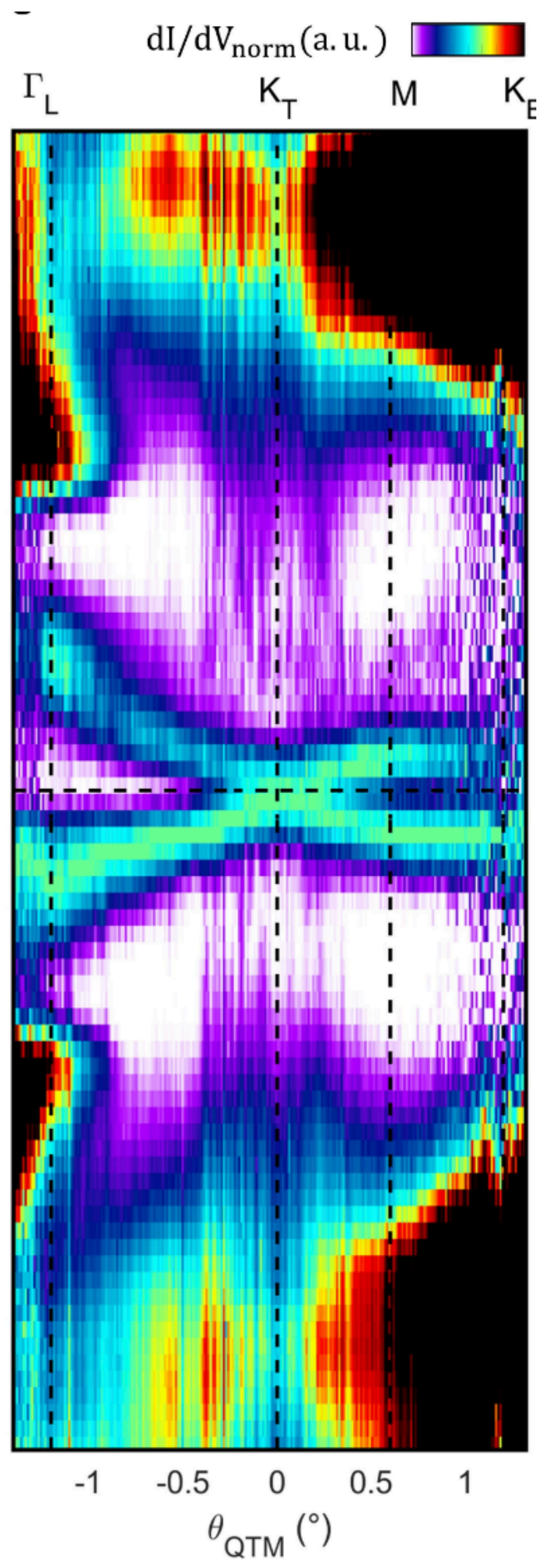
Us^2 | U | T

Ordered, … | Thermal Nonlocal Moments $S \approx N \log 2$ | Hot Soup

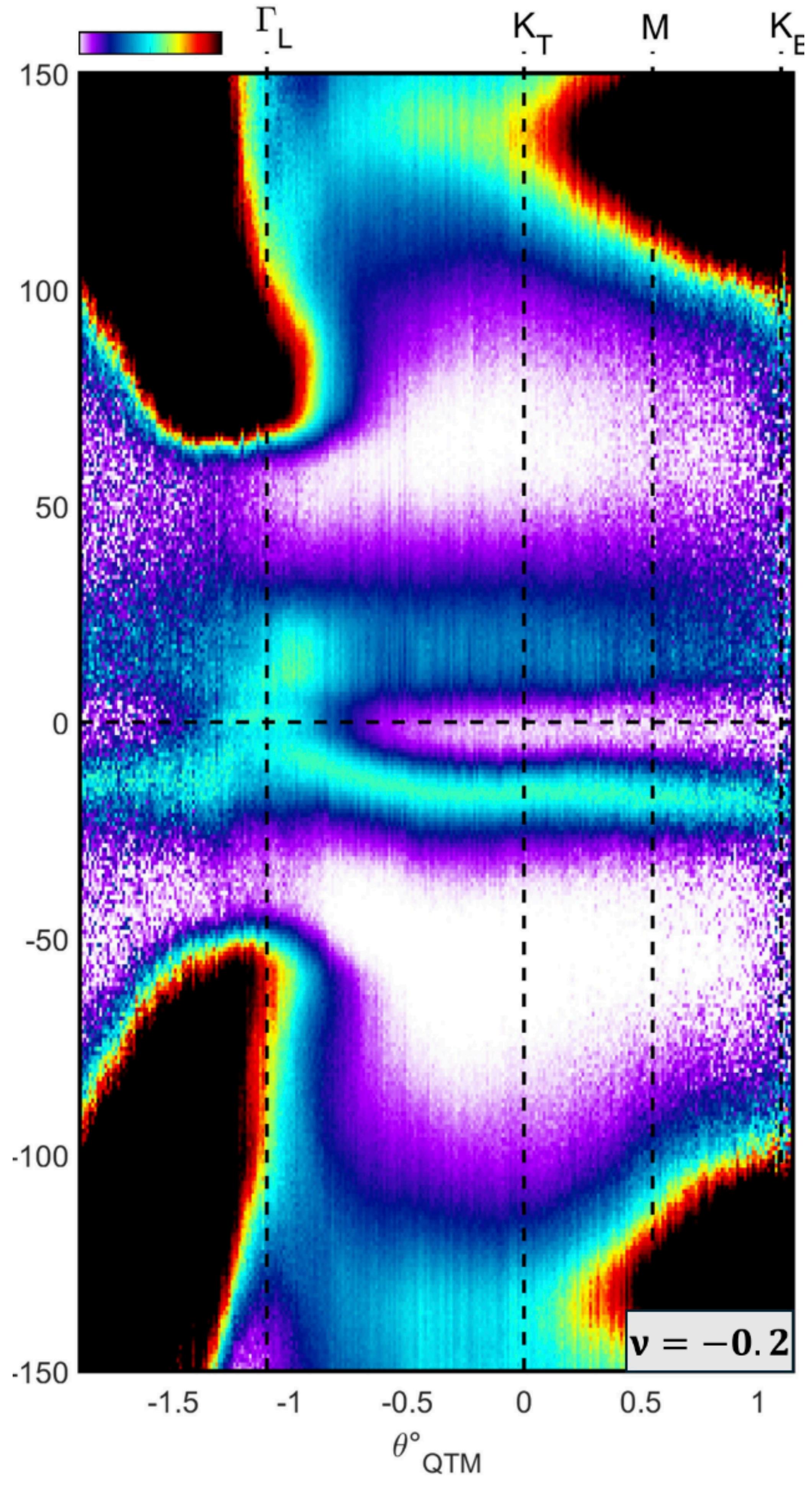
[**PJL,** Dong, Vishwanath, Khalaf PRX 2025]
[**PJL**, Vishwanath, Khalaf arXiv:2505.08779]

Experimental (QTM) Spectra: Gapless at Neutrality

Non-magic TBG
(K, K' gapless)



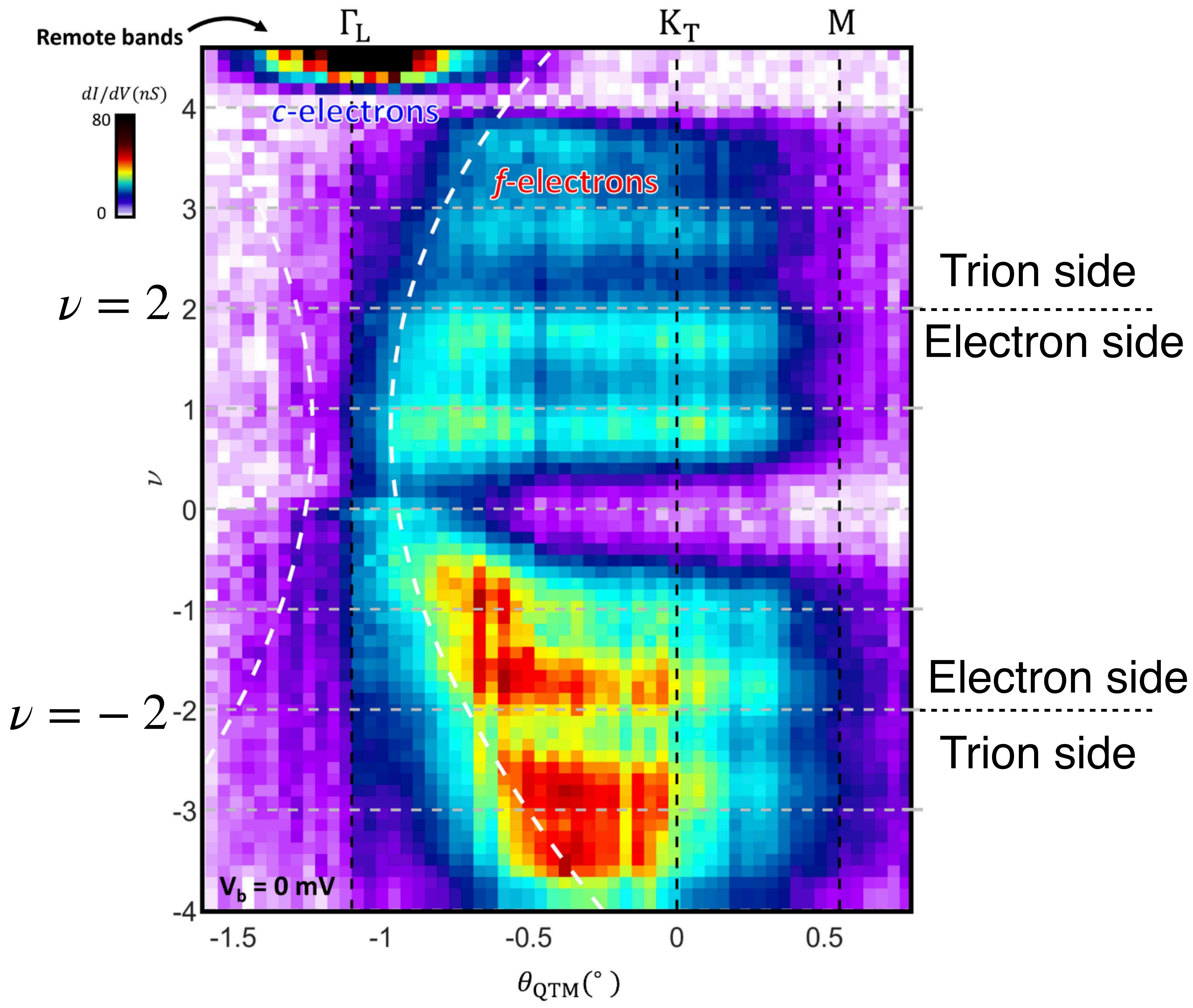
Magic angle TBG $\nu = 0$
Γ gapless

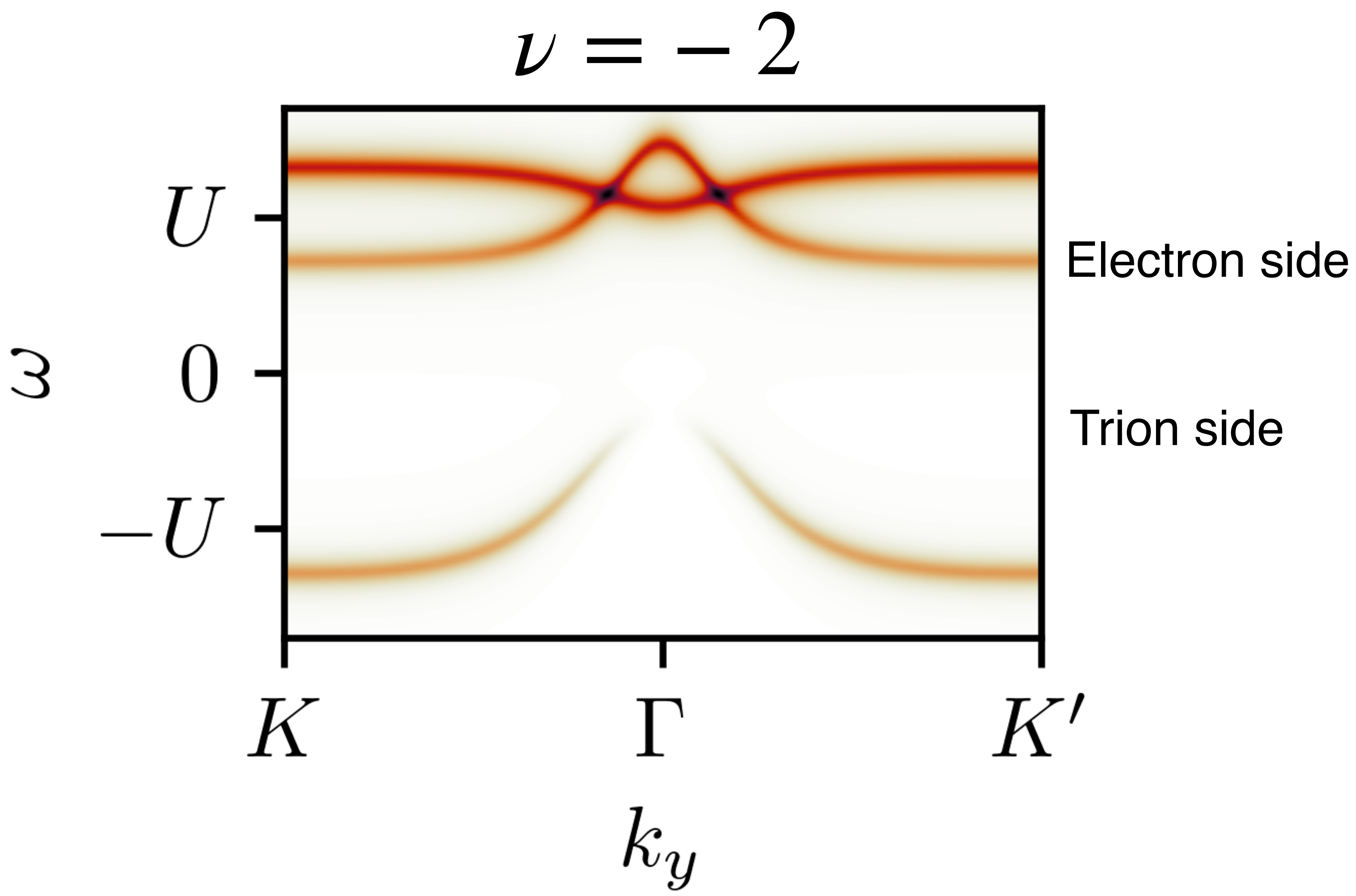


Xiao, …, Ilani arXiv:2506.20738

Experimental (QTM) Spectra: Trions in SC normal state?

MATBG, zero bias. $A(\omega = 0, \boldsymbol{k})$
Suppressed weight, no FS at $2 + \epsilon$, $-2 - \epsilon$
Compare with quantum osc: light but not electrons?



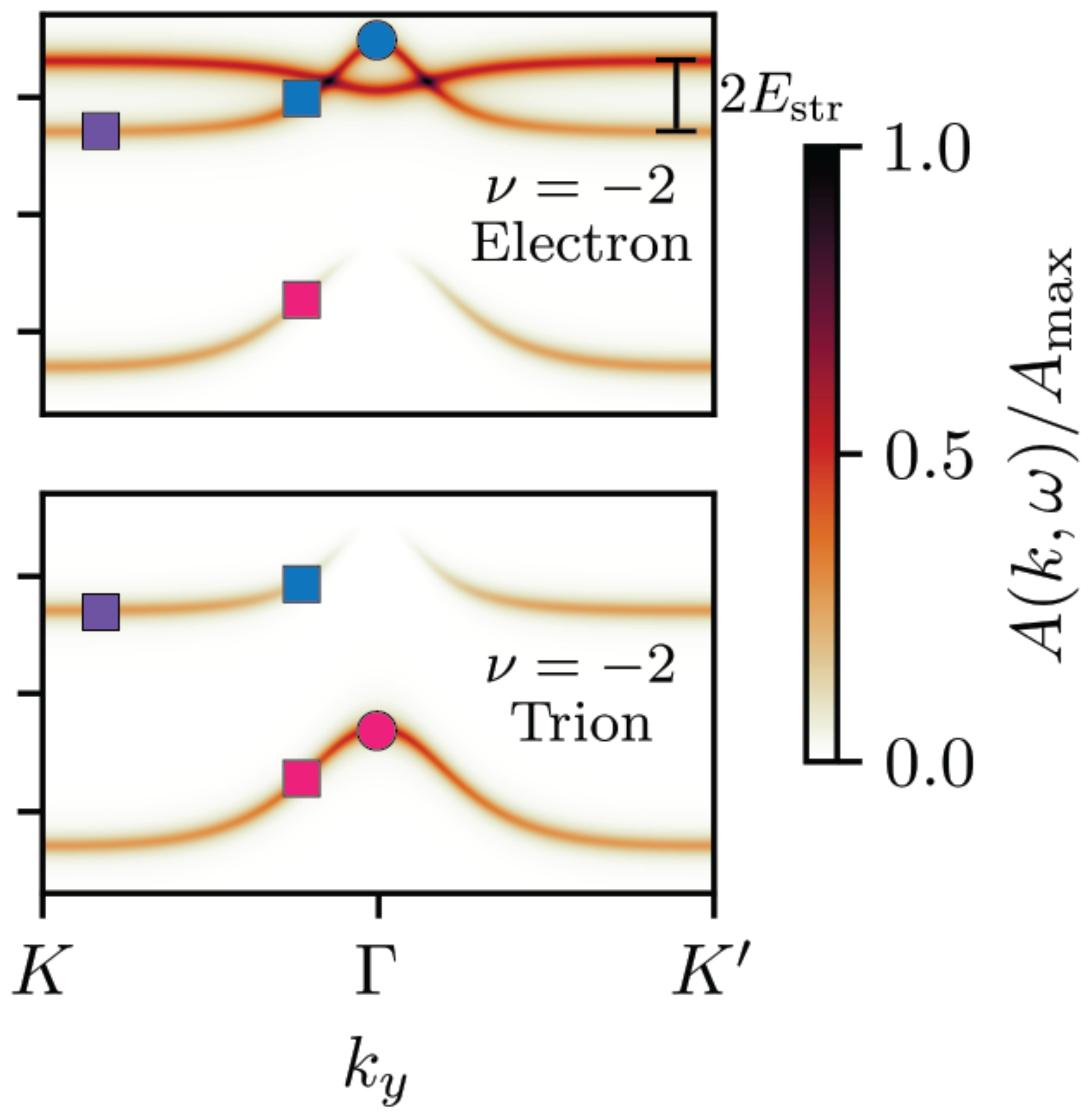


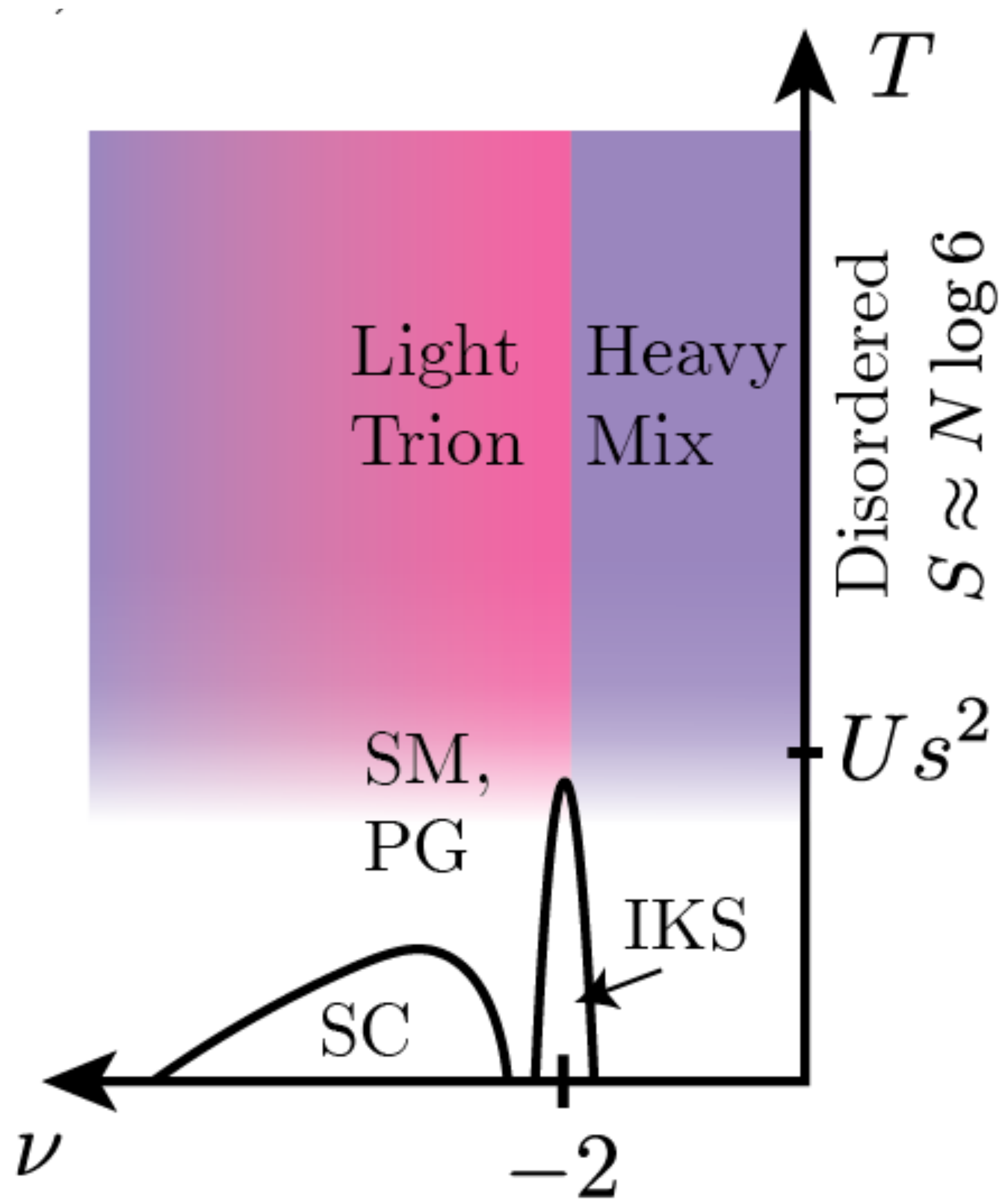
Xiao, …, Ilani arXiv:2506.20738

Mott + Topology Outlook

Concentrated charge/Berry density
Band Topology

Wide Mott-regime $Us^2 \ll T \ll U$
Orthogonal light trions





Detect through assisted tunneling?

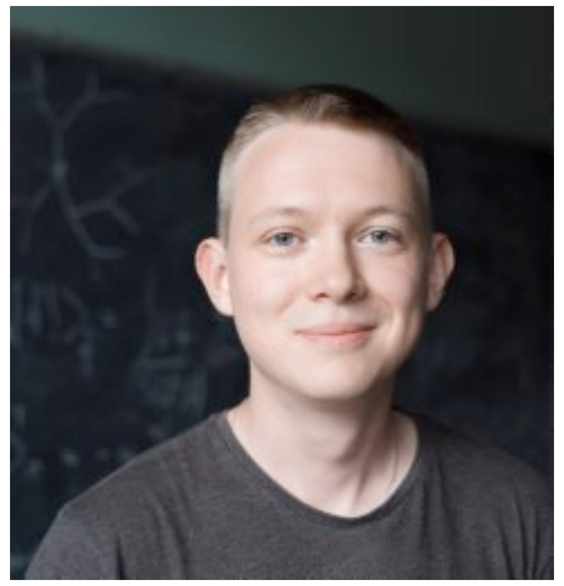
SC from trion pairing?

Other systems?
Narrow top band with conc. charge

Order-by-Order Expansion in s^2

Models beyond TBG (e.g. trivial Hubbard)

Analytic DC Transport in fluctuating moment regime



Pavel Nosov

Eslam Khalaf

Nosov, Khalaf, PJL arXiv:2605.20171

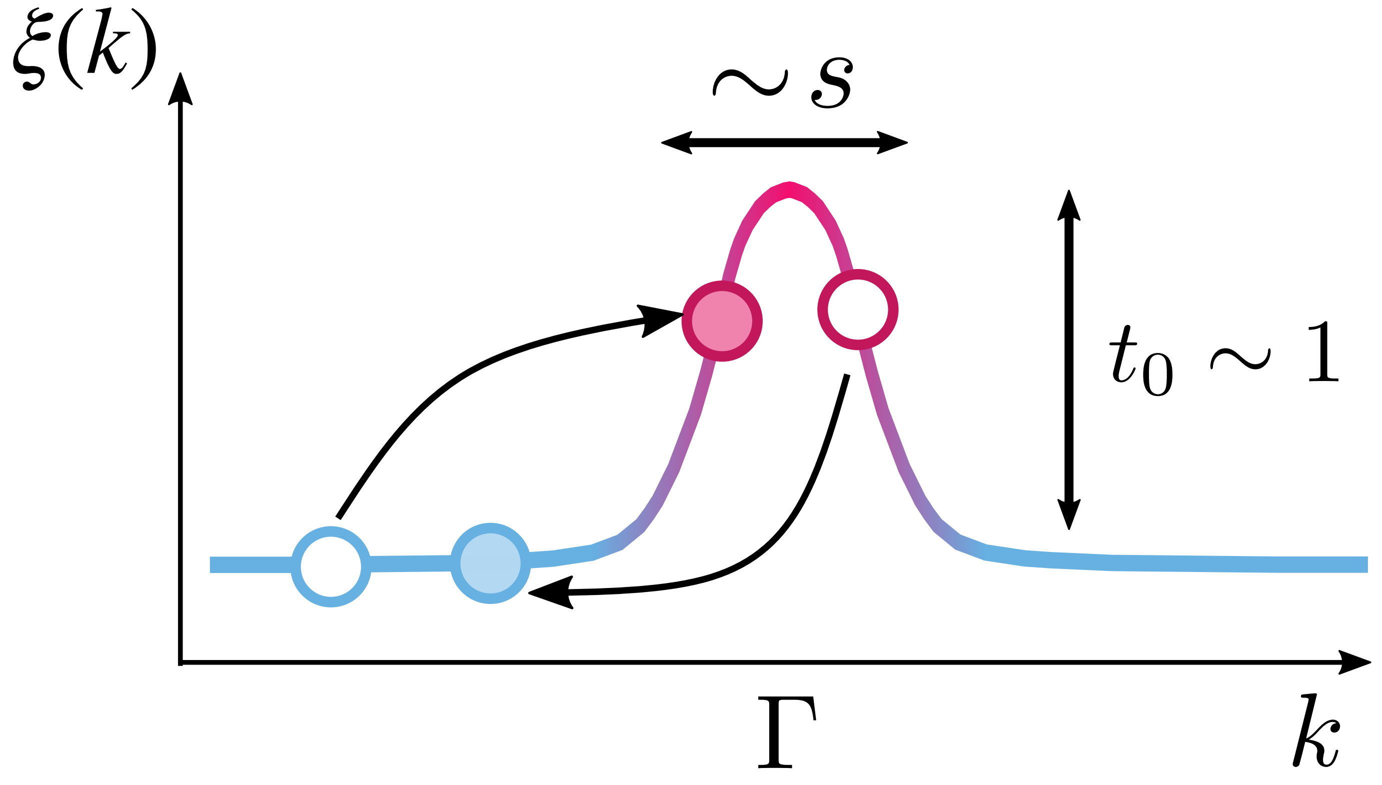
TBG/Song-Bernevig small s^2 expansion:

Vituri, Berg arXiv:2604.14278
Wei, von Oppen, Glazman arXiv:2604.14369
Hu++ arXiv:2604.14303

Systematic Small s^2 Expansion

Simplest case: Hubbard with **concentrated dispersion**

$$H + \mu N = U \sum_i n_{i\uparrow} n_{i\downarrow} + \sum_{\boldsymbol{k}} \xi(k/s)\, c^\dagger_{\boldsymbol{k}\sigma} c_{\boldsymbol{k}\sigma}$$



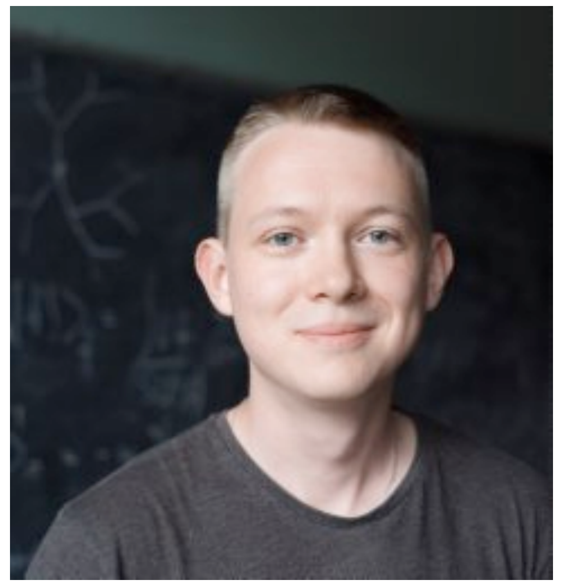
Expand around H_U

Linked cluster expansion (Metzner 1991).
Perturbation theory in t_0 singular due to local moment degeneracy

Expand in s^2: resum hopping lines

Each hopping loop $\rightarrow s^2$ suppression

Decay rate $\mathrm{Im}\Sigma \sim |t_0| s^2$.



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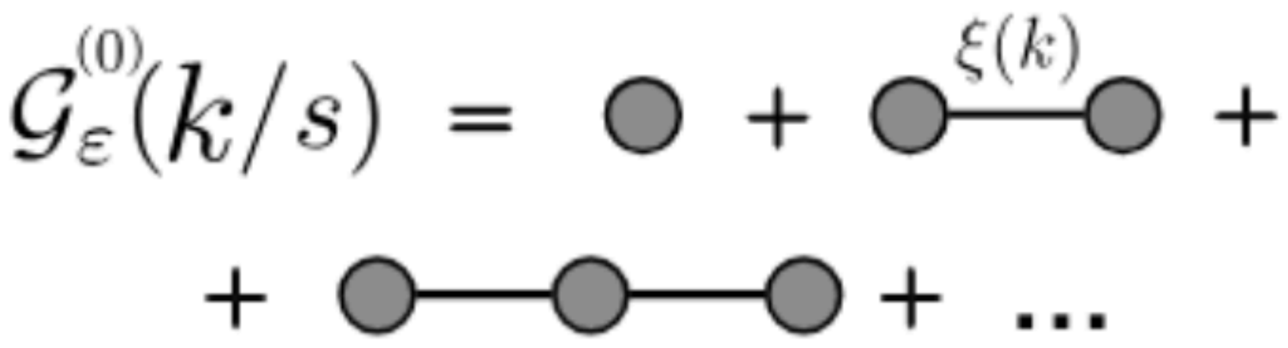
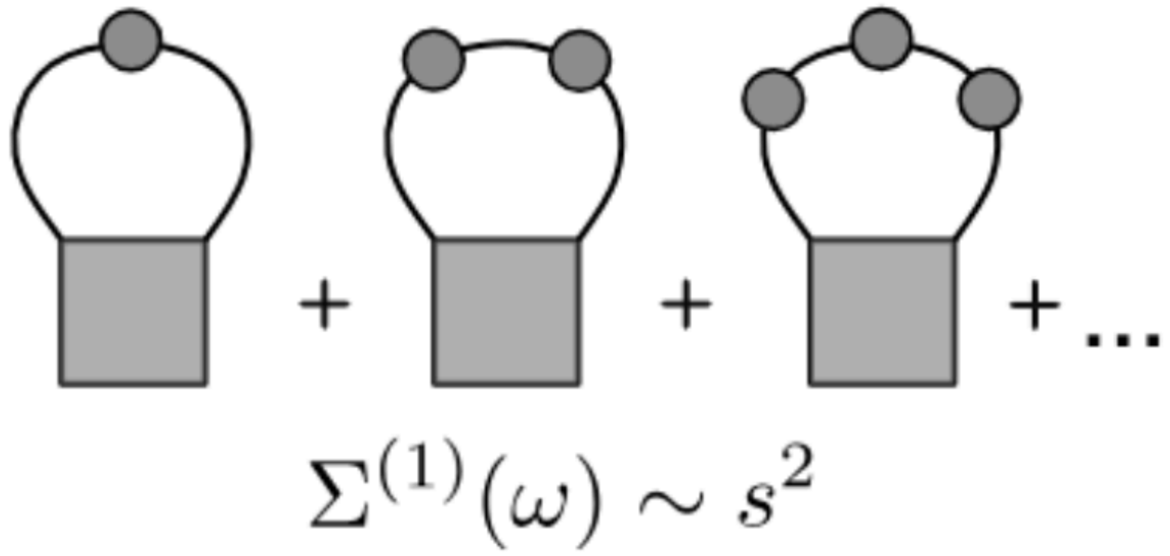
Nosov, Khalaf, PJL arXiv:2605.20171





TBG/Song-Bernevig small s^2 expansion:

Vituri, Berg arXiv:2604.14278
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Hu++ arXiv:2604.14303

Analytic Controlled DC Transport!

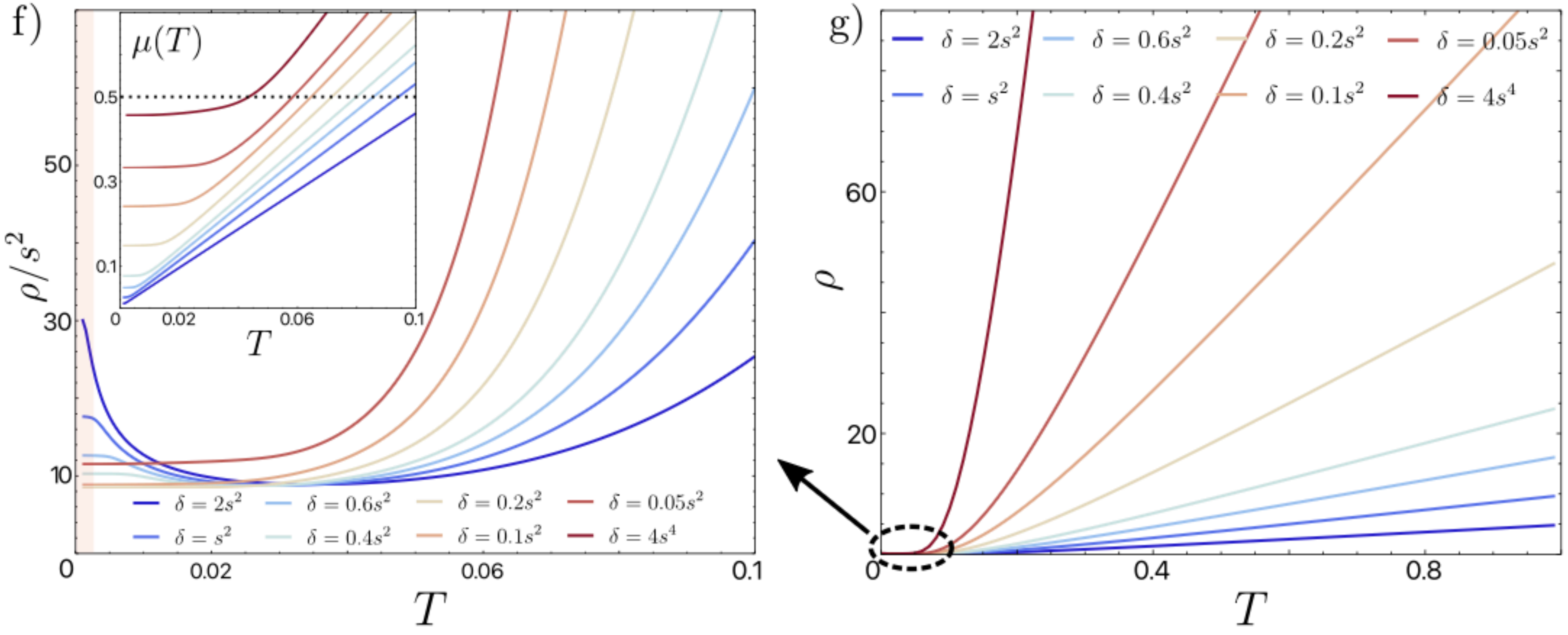
Dope lower Mott band → Variety of Transport regimes $T \gtrsim t_0 s^2$

Doped holes scatter elastically off of fluctuating moments (act like disorder)

$T \ll T_*$: Form small FS (volume δ). “Thermal FL*”

$T \gtrsim 0.1 t_0$: Holes long lived but thermally incoherent, bad metal $\rho \propto T$

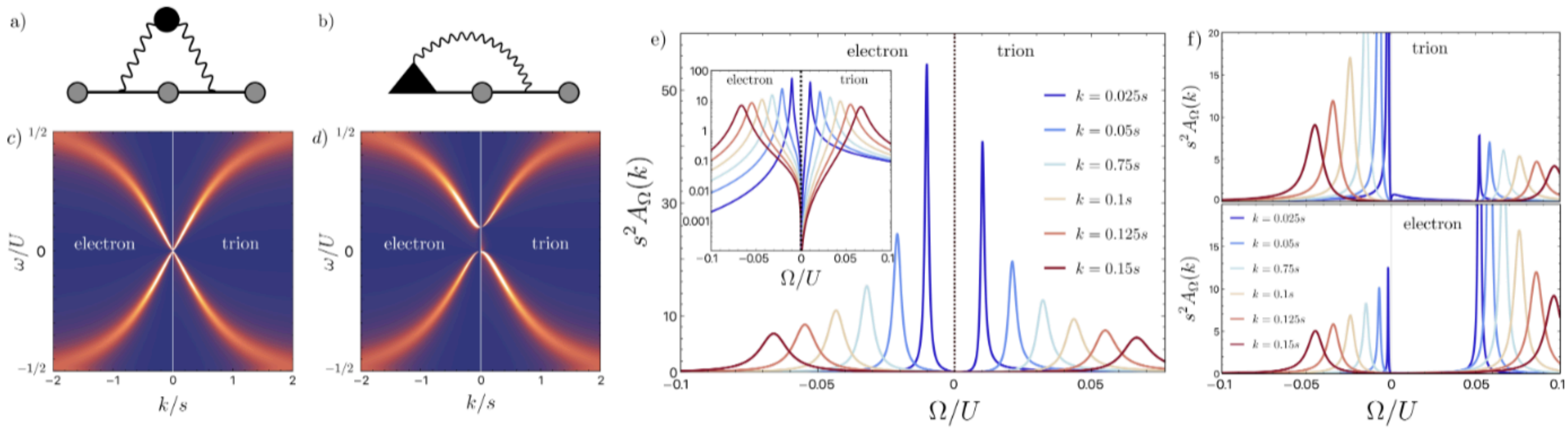
Thermal FL*
$\rho \sim (\text{const.})s^2$



Bad Metal
$\rho \propto Ts^2$

Nosov, Khalaf, PJL arXiv:2605.20171

Applies equally well to Top bands, Mott SM



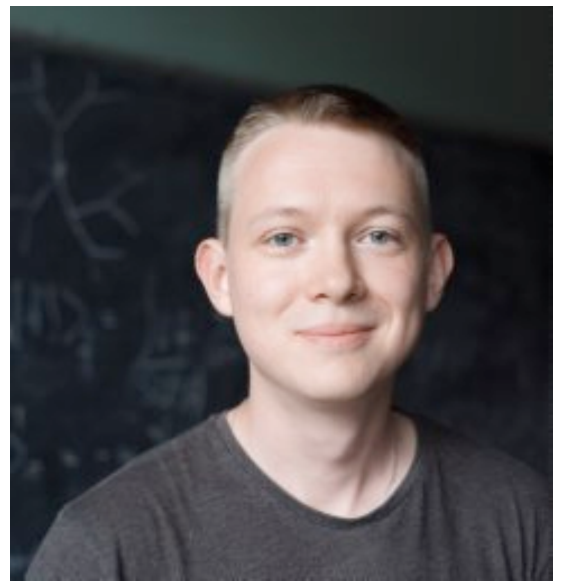
Reproduces TBG results and includes nonzero lifetime

Lifetime $\rightarrow 0$ as $\Omega \rightarrow 0$ for electrons and trions (low DOS)

Outlook for small s^2

Systematic expansion possible, with analytic DC transport

Rare for Mott correlations!



Pavel Nosov

Insight into enduring puzzles?

Strange metals, Pseudogap, Small-Large FS transition

Can a clean system have $\rho \propto T$ at low T?

Correlations between moments for $T = O(s^2)$: resum ladder diagrams $\propto \left(\frac{t_0 s^2}{T}\right)^n$

Inelastic scattering from fluctuating moment exchange splitting

Derive long wavelength EFT

Transport in TBG? Pairing?

Many possible model extensions

Nosov, Khalaf, PJL arXiv:2605.20171

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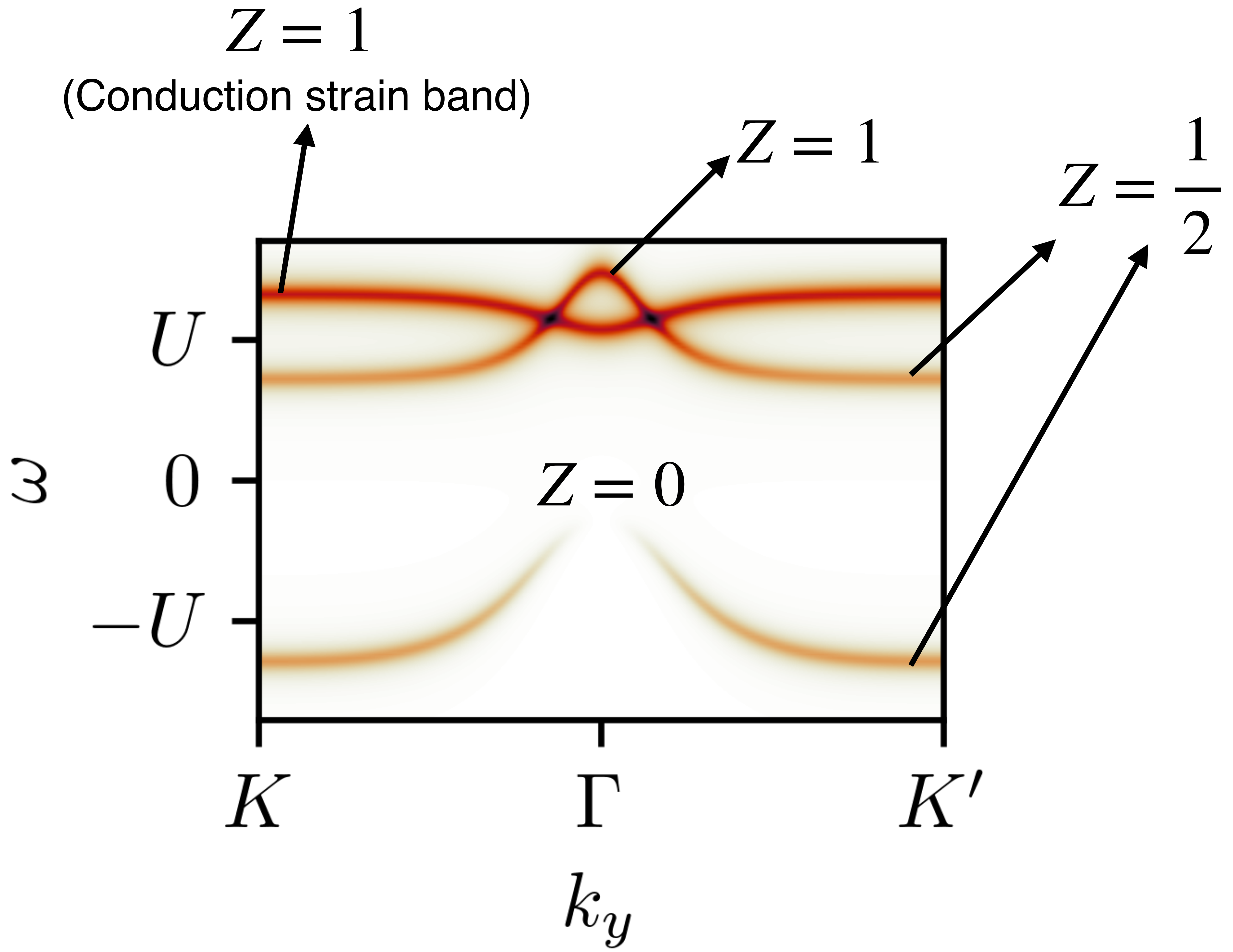
What is the missing weight? A hint from Hubbard

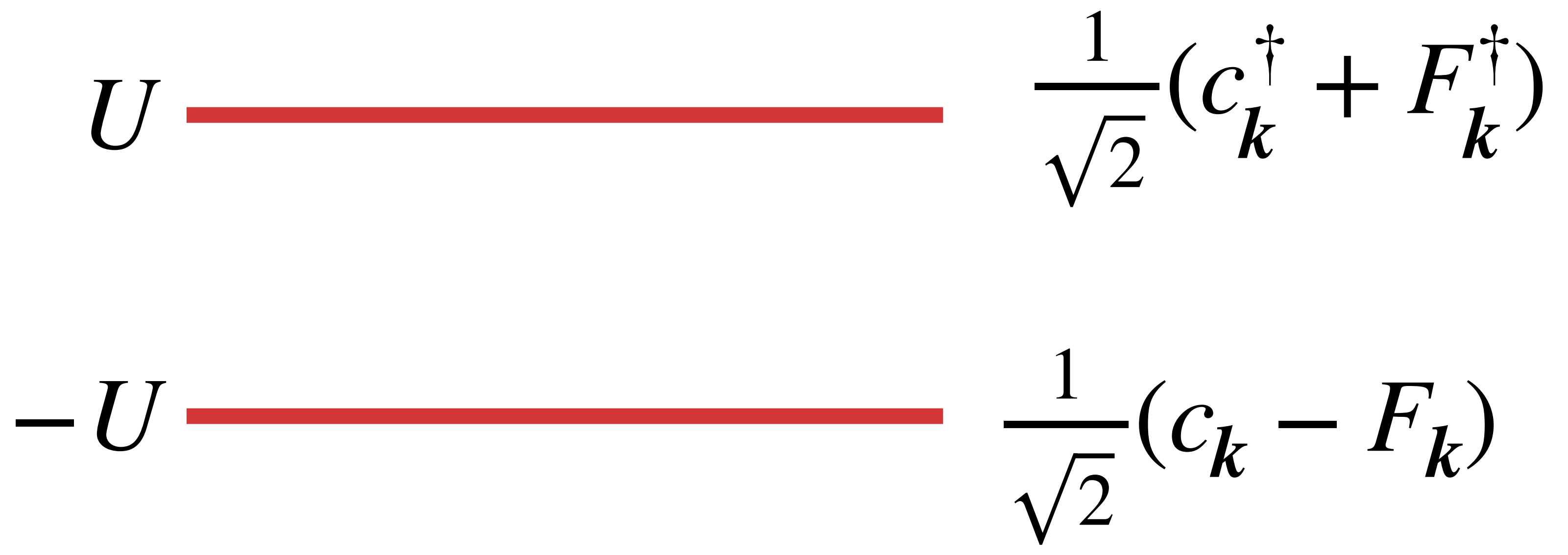


Missing excitation = Half of Hubbard Band

Hubbard (1963): $H_U = U \sum_R \delta n_R \delta n_R$

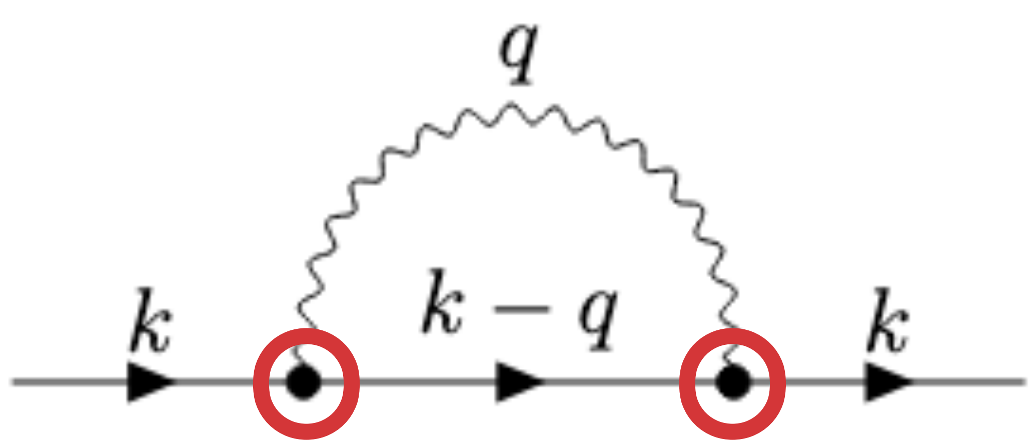
Small $T > 0$: $e^{-\beta H} = | \uparrow \rangle \langle \uparrow | + | \downarrow \rangle \langle \downarrow |$

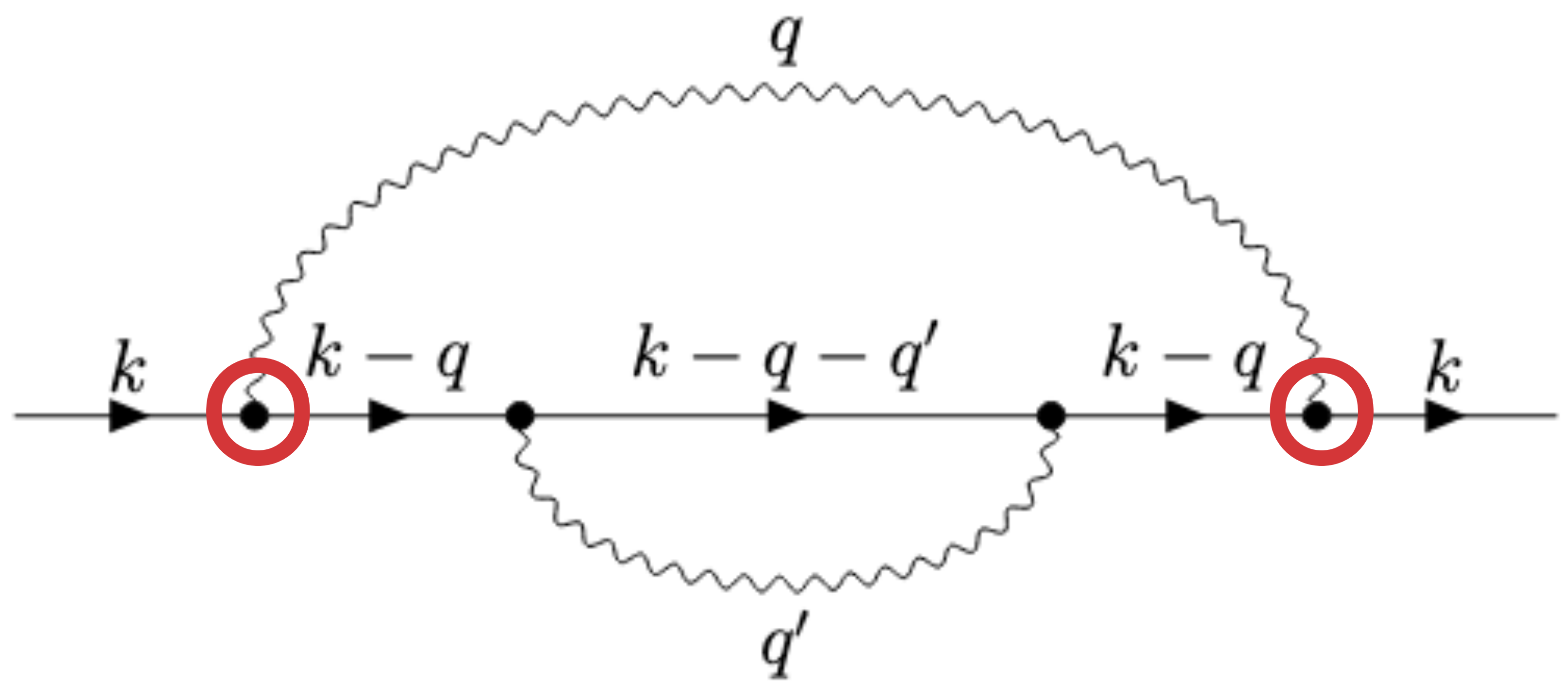
$$[H_U, c_{R\alpha}] = -UF_{R\alpha} = -U\{c_{R\alpha}, \delta n_R\}$$
$[H_U, F_{R\alpha}] \to -Uc_{R\alpha}$ (for SP excitation)

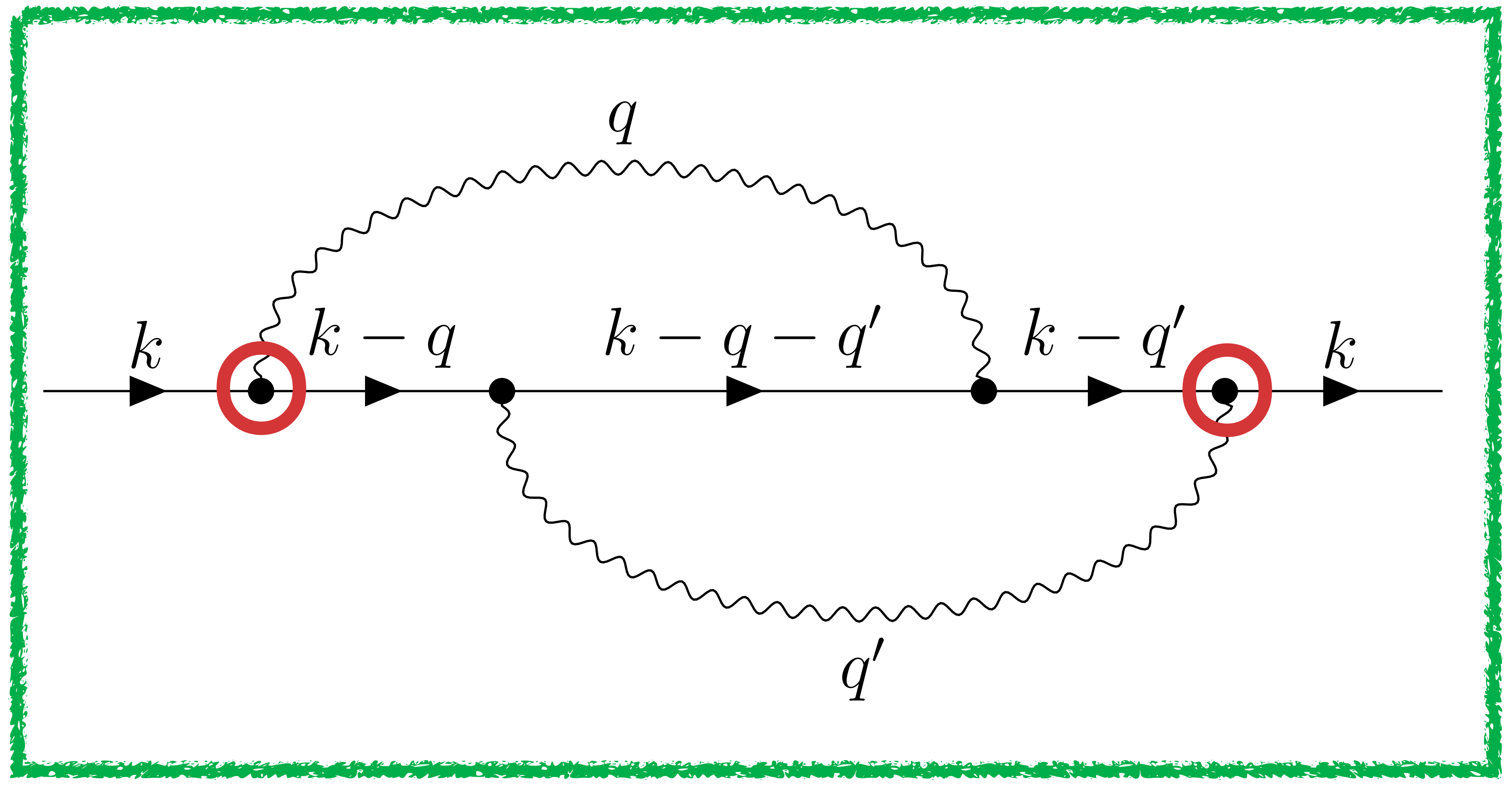


Mott band is electron-trion mix. Near Γ Berry phase $\implies \frac{1}{\sqrt{2}}(c_{\boldsymbol{k}} - e^{i\theta_k}F_{\boldsymbol{k}}) \implies$ Dirac cone hybridization!

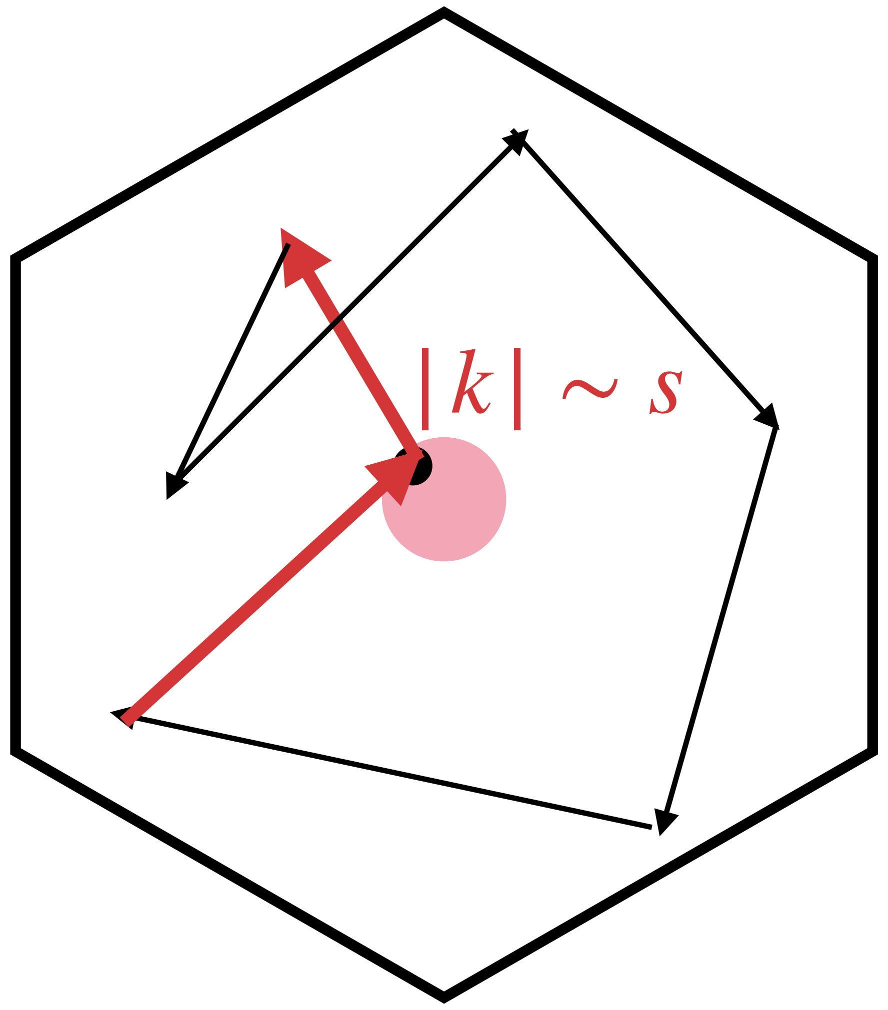
Why is $A(k,\omega)$ computable? Every diagram simplifies







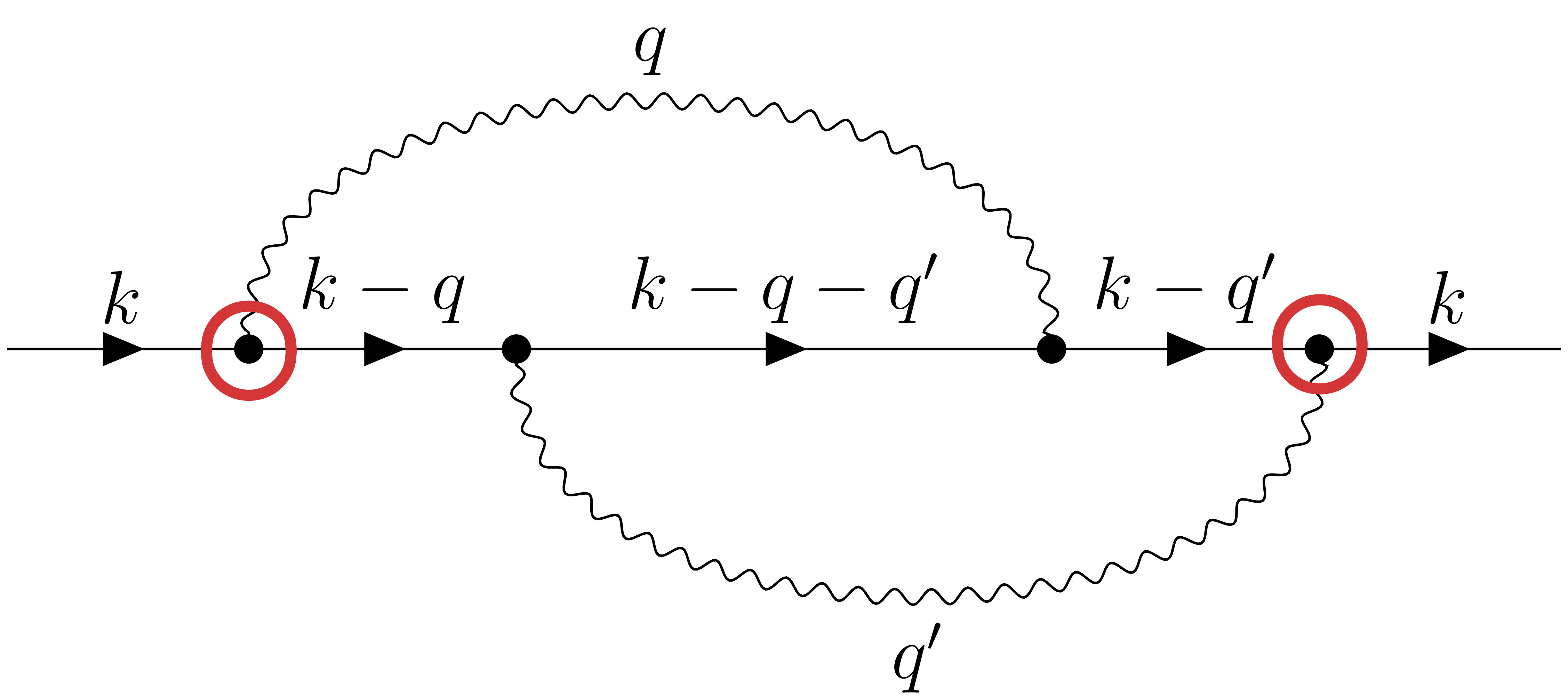
Representative example

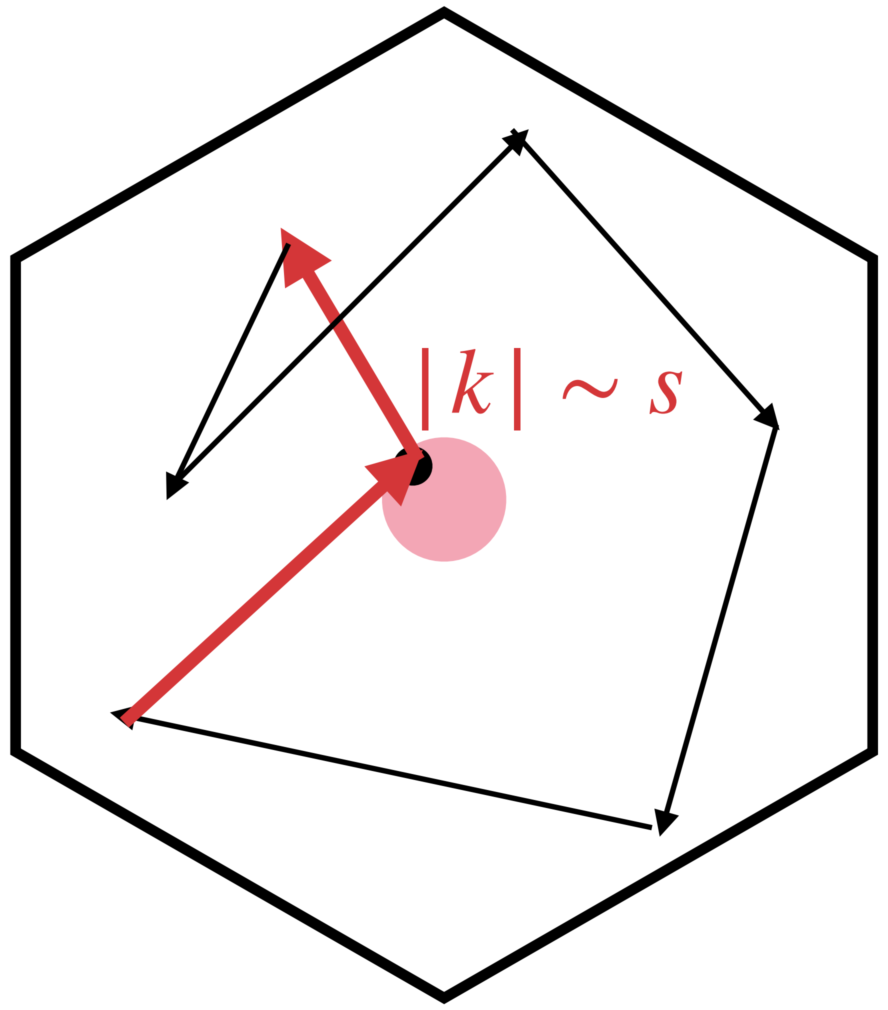


Internal parts: same as trivial model

External vertices → overall factor

Representative Diagram





$$D(\boldsymbol{k}) = \frac{1}{A^2} \sum_{\boldsymbol{q},\boldsymbol{q}'} \boxed{\Lambda_{-\boldsymbol{q}'}(\boldsymbol{k})} \left[G^{(0)}_{\boldsymbol{k}-\boldsymbol{q}'} V_{\boldsymbol{q}'} \Lambda_{-\boldsymbol{q}}(\boldsymbol{k}-\boldsymbol{q}') G^{(0)}_{\boldsymbol{k}-\boldsymbol{q}-\boldsymbol{q}'} \Lambda_{\boldsymbol{q}'}(\boldsymbol{k}-\boldsymbol{q}-\boldsymbol{q}') V_{\boldsymbol{q}} G^{(0)}_{\boldsymbol{k}-\boldsymbol{q}} \right] \boxed{\Lambda_{\boldsymbol{q}}(\boldsymbol{k}-\boldsymbol{q})}$$

$$D(\boldsymbol{k}) = \frac{1}{A^2} \sum_{\boldsymbol{q},\boldsymbol{q}'} \boxed{\Lambda_{-\boldsymbol{q}'}(\boldsymbol{k})} \left[G_* V_{\boldsymbol{q}'} \xi_{-\boldsymbol{q}} G_* \xi_{\boldsymbol{q}'} V_{\boldsymbol{q}} G_* \right] \boxed{\Lambda_{\boldsymbol{q}}(\boldsymbol{k}-\boldsymbol{q})}$$

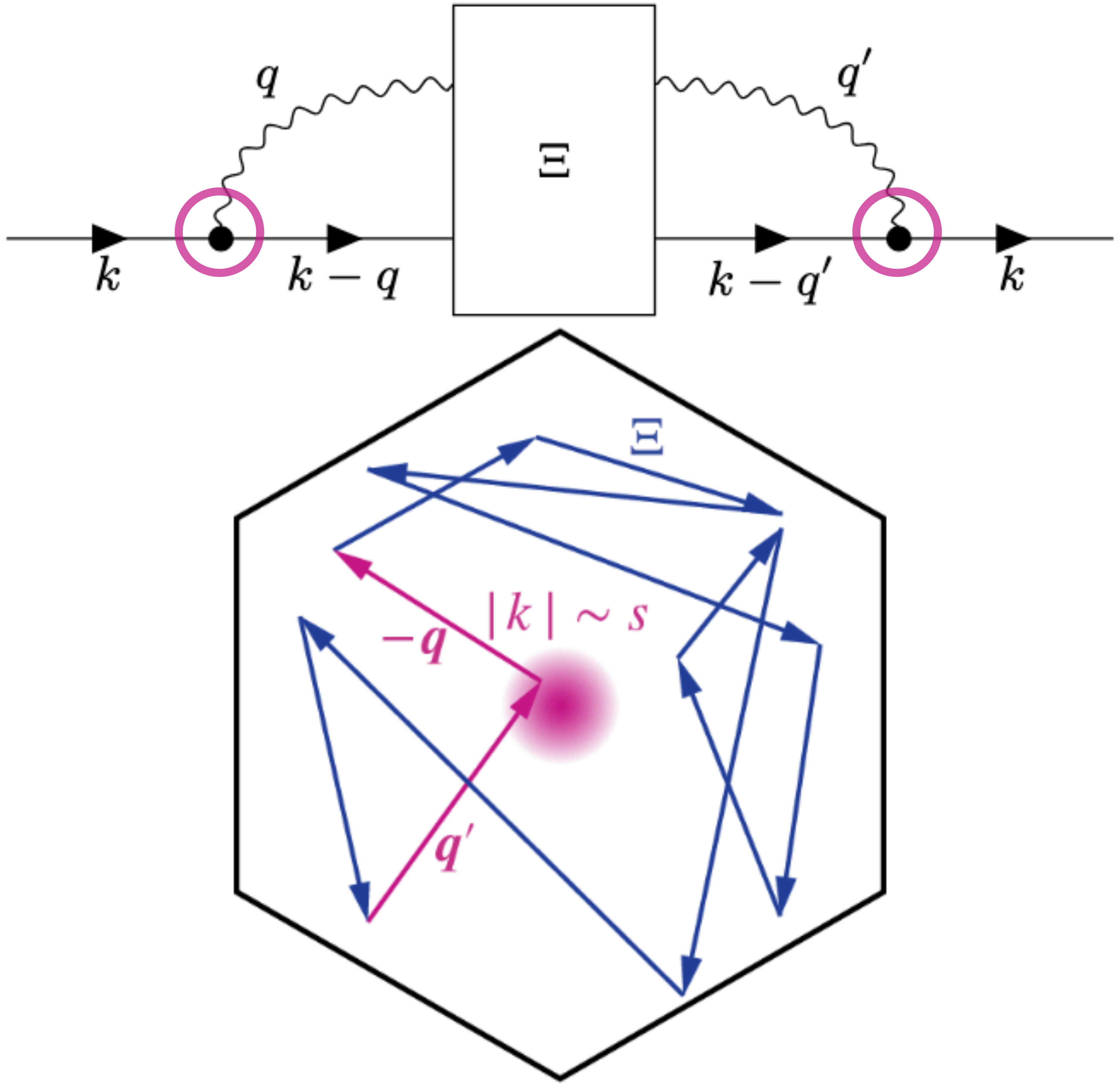
$$D(\boldsymbol{k}) = \boxed{\frac{|k|^2}{|k|^2 + 2s^2}} D(*)$$

$\boldsymbol{k} \to *$ means $\boldsymbol{k}$ far from Γ

$\xi_{\boldsymbol{q}}$ is $\langle u_{\boldsymbol{k}} | u_{\boldsymbol{k}+\boldsymbol{q}} \rangle$ where $\boldsymbol{k}, \boldsymbol{k}+\boldsymbol{q} \to *$

Controlled for $T \gg Us^2$

Formalism for k-bath Self Energy



Solve EOM, identify dynamical bath correlator

$$-\partial_\tau G^{cd}_{k\tau} = \delta(\tau)\langle\{c_k, d^\dagger_k\}\rangle + \langle\mathscr{T}[H, c_{k\tau}]d^\dagger_k\rangle\,.$$

$$\vdots$$

$$\Sigma_{k\omega} = h^{\mathrm{mf}}_k + [G^{OO}_{k\omega}]_{1\mathrm{PI}} \qquad [G^{OO}_{k\omega}]_{1\mathrm{PI}} = G^{OO}_{k\omega} - G^{Oc}_{k\omega}[G^{cc}_{k\omega}]^{-1}G^{cO}_{k\omega}$$

Internal states away from Γ

$$O_k = -[H_{\mathrm{int}}, c_k] = \frac{1}{2A}\sum_q V_q \boxed{\Lambda_{k,k+q}}\,\boxed{\{\delta\rho_q, c_{k+q}\}}$$

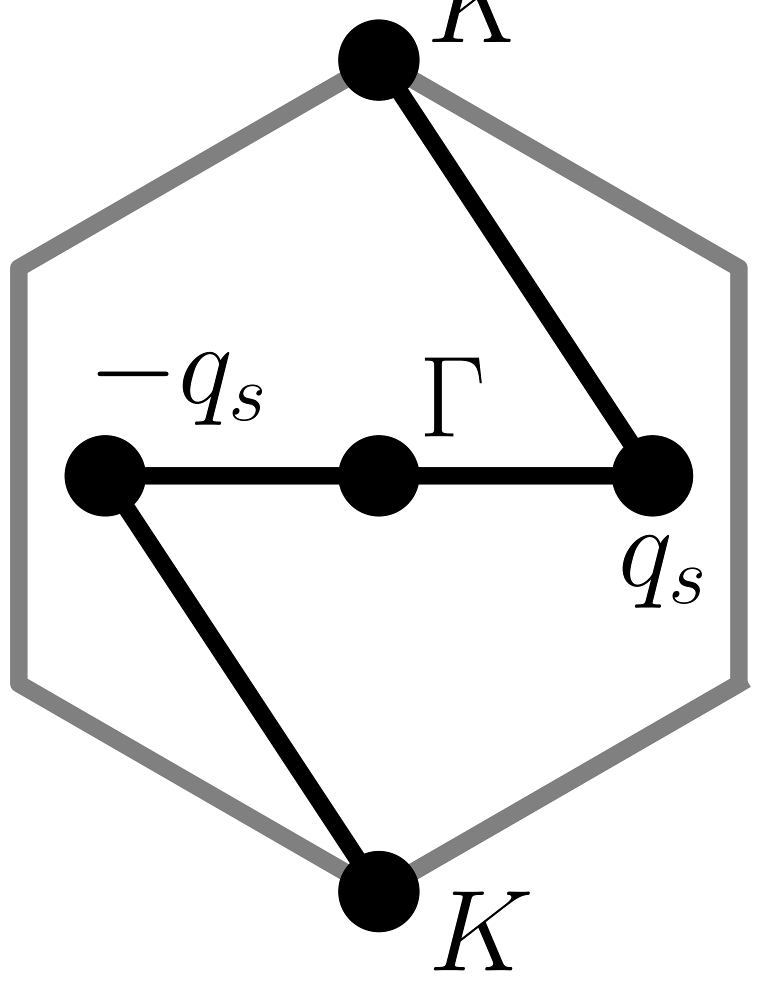
Topology/Berry phase

External fermions far from Γ

$$[G^{OO}_{k\omega}]_{1\mathrm{PI}} = \boxed{\frac{|k|^2}{|k^2| + 2s^2}}\,\boxed{\Xi_\omega}$$

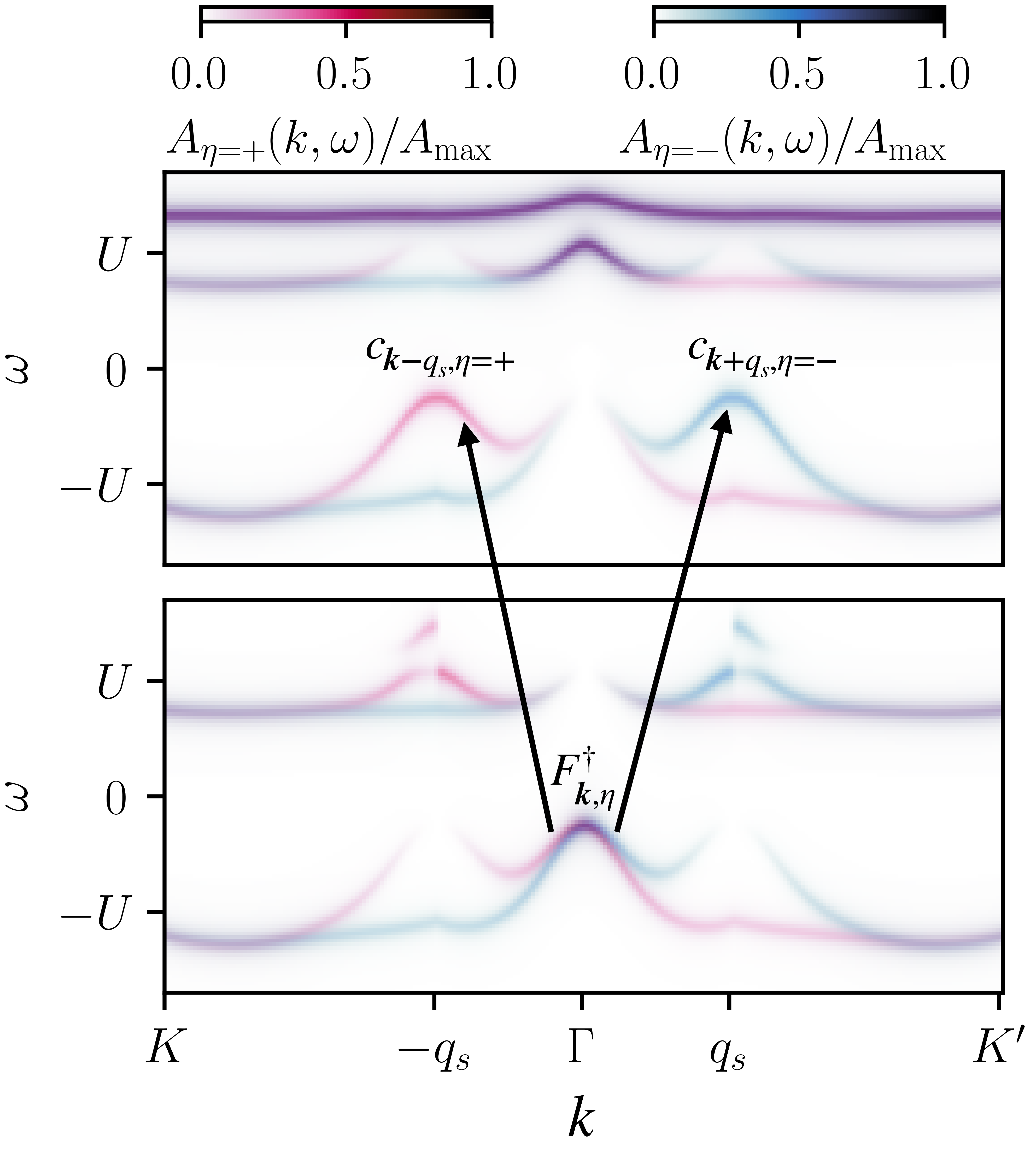
Fate of Trions after IKS order

$\boldsymbol{q}_s$ Spiral order in intervalley phase
Hybridize Γ with $\pm\boldsymbol{q}_s$



$$F_{\boldsymbol{k}\eta} \to O_{\text{IKS},\boldsymbol{\eta q_s}} c_{\boldsymbol{k}-\eta\boldsymbol{q}_s,-\eta} = c_{\boldsymbol{k}-\eta\boldsymbol{q}_s,-\eta}$$

$$\Sigma^{IKS}_{\boldsymbol{k}\approx\Gamma,\boldsymbol{k}+\eta q_s} = \begin{pmatrix} 0 & U\bar{\lambda}_{\boldsymbol{k}} \\ U\lambda_{\boldsymbol{k}} & 0 \end{pmatrix} = \Sigma_{c-F}$$



*(Kwan et al PRX 2021, Nuckolls et al Nature 2023, Kim et al Nature 2023, Wang et al PRB 2023, Herzog-Arbeitman et al 2025 arXiv:2502.08700)