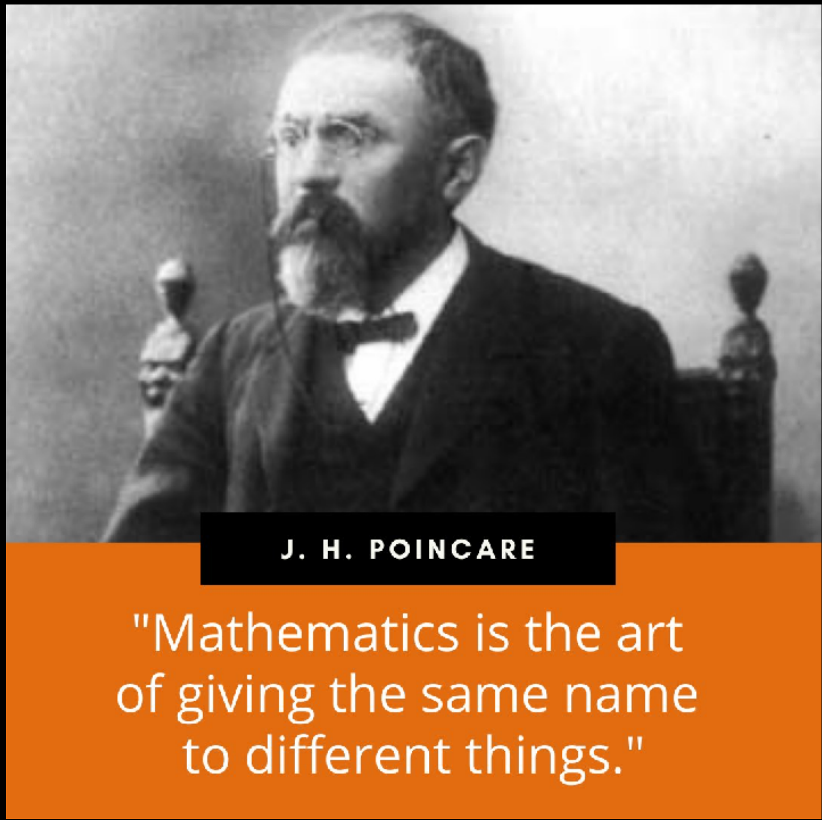
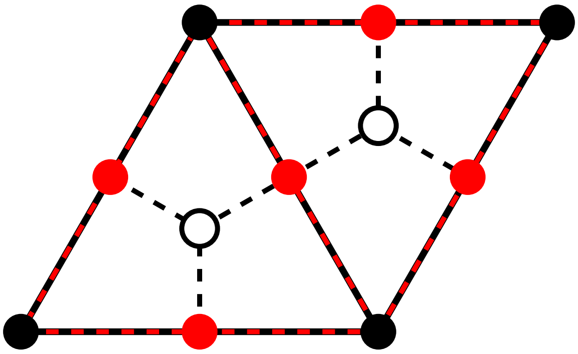
- **Exterior Calculus gives the right structure to the concept of integration** of scalar valued objects
- In Physics **bundle-valued (pseudo)-forms and Lie-algebra valued (pseudo)-forms are also fundamental**
- **There is a great need for covariant and metric independent discretization of Graded Algebras of (pseudo)-forms**.

-Stefano Stramigioli



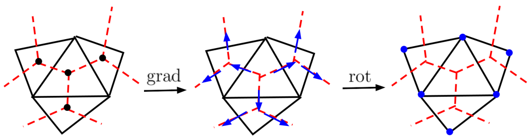
DEC hybridization and static condensation, VEEC, HDG?



- In FEEC, **hybridization** allows us to enforce continuity conditions using Lagrange multipliers on the skeleton rather than shared degrees of freedom. The Lagrange multipliers correspond to (strong or weak) **boundary traces**.
- **Static condensation** eliminates interior DOFs $\rightarrow$ smaller global problem.
- Can the same be done for DEC? In addition to the primal and dual mesh, **boundary dual** cells are already used to handle the domain boundary. This approach could be extended to the entire mesh skeleton.
- Is there a "virtual element exterior calculus" (VEEC) perspective, similar to the relationship between VEM and finite volume/difference methods?
- Can this be extended to HDG-like methods (with improved conservativity)?

FOR OPEN PROBLEM SESSION (K.HU)

1. **dualize mesh v.s. dualize functions**



KH, Lin, Zhang, SIAGA 2025

A de Rham complex starting with p.w. constants?
DEC perspective: forms on *dual mesh* (not unique)
Distributional finite element (current) perspective: Dirac deltas (unique).

2. **beyond form-valued forms**

e.g., conformal complexes and structures, Transverse-Traceless tensors (Arnold, KH, FoCM 2021)

ker of $\operatorname{dev}\operatorname{def}$: conformal Killing v.f.   Cotton-York: flatness in conformal geometry

$$0 \longrightarrow H^s(\Omega)\otimes\mathbb{V} \xrightarrow{\operatorname{dev}\operatorname{def}} H^{s-1}(\Omega)\otimes(\mathbb{S}\cap\mathbb{T}) \xrightarrow{\text{Cott}} \underbrace{H^{s-4}(\Omega)\otimes(\mathbb{S}\cap\mathbb{T})}_{\substack{\text{gravitational wave variable: transverse-traceless (TT) gauge}\\ (=\text{ symmetric, trace-free, div-free})}} \xrightarrow{\operatorname{div}} \underbrace{H^{s-5}(\Omega)\otimes\mathbb{V}}_{\substack{\text{stress like variable } \operatorname{def} u \text{ in NS}\\ \text{(Gopalakrishnan, Lederer, Schöberl, 2019)}}} \longrightarrow 0$$

$\mathbb{T}$: trace-free matrices

$\operatorname{dev} w := w - \frac{1}{n}\operatorname{tr}(w)I$, $\quad \operatorname{def} := \operatorname{sym}\operatorname{grad}$, $\quad \operatorname{cott} g := \operatorname{curl} S^{-1}\operatorname{curl} S^{-1}\operatorname{curl}$, $\quad Su := u^T - \operatorname{tr}(u)I$

Several finite element versions of form-valued forms (symmetric matrices, traceless matrices etc.).
Ideas did not work for conformal.

Discretizing vector bundle valued form

- Integration of bundle valued forms requires trivialization
- Braune et al. suggestion based on following observation:

$$d_{\nabla}\alpha = d\alpha + \omega \wedge \alpha$$

  where ω is connection 1-form

- Pick a trivialization in which ω is 0 at a point so it will be small nearby
- Questions:
  - Will this result in solutions to PDEs that are gauge invariant?
  - What are some other ideas for discretizing bundle valued forms