



DOE-BES Project 2015-2025
“Non-equilibrium Quantum
Dynamics of Multi-qubit
Superconducting Circuits”

Sinha et al., Phys. Rev. A 106, 033714 (2022)
Pham et al., Phys. Rev. A 107, 053704 (2023)
Maldonado et al., Phys. Rev. B. **110**, 014508 (2024)
Pham et al., Phys. Rev. Appl. 23, 064009 (2025)
Bello et al., Phys. Rev. Appl. 23, 054042 (2025)
Fan et al., arxiv:2410.23116
Maldonado et al., arxiv:2411.05635
Maldonado et al., arxiv:2504.13779
Pham et al., arxiv:2504.12286

IMSI Workshop “Discrete Exterior Calculus”, U Chicago, Septemer 2025

Applications of a DEC-inspired Approach to Quantum Electrodynamics

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Department of Electrical and Computer
Engineering
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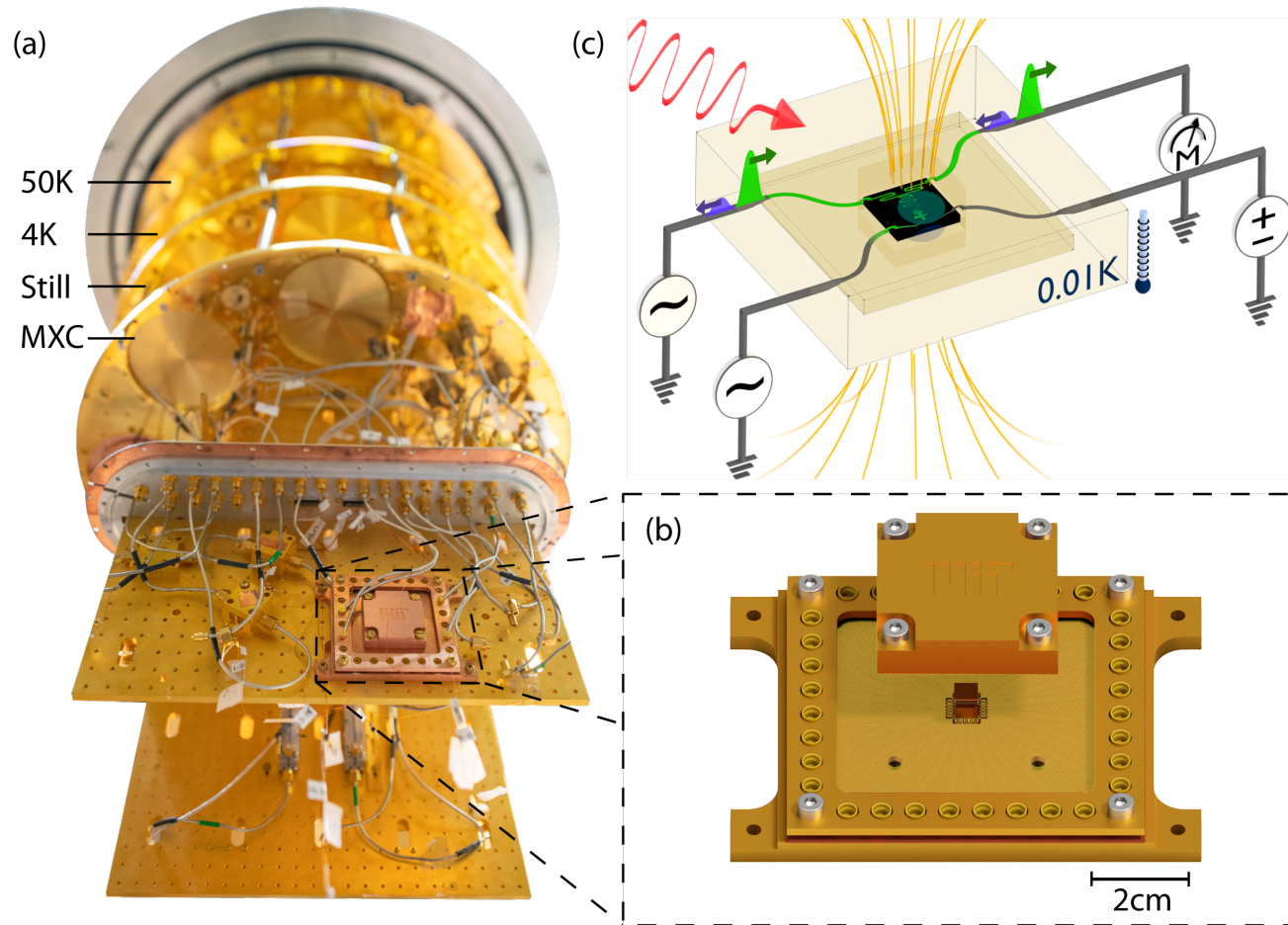
Dzung Pham

Thomas Maldonado

Wentao Fan

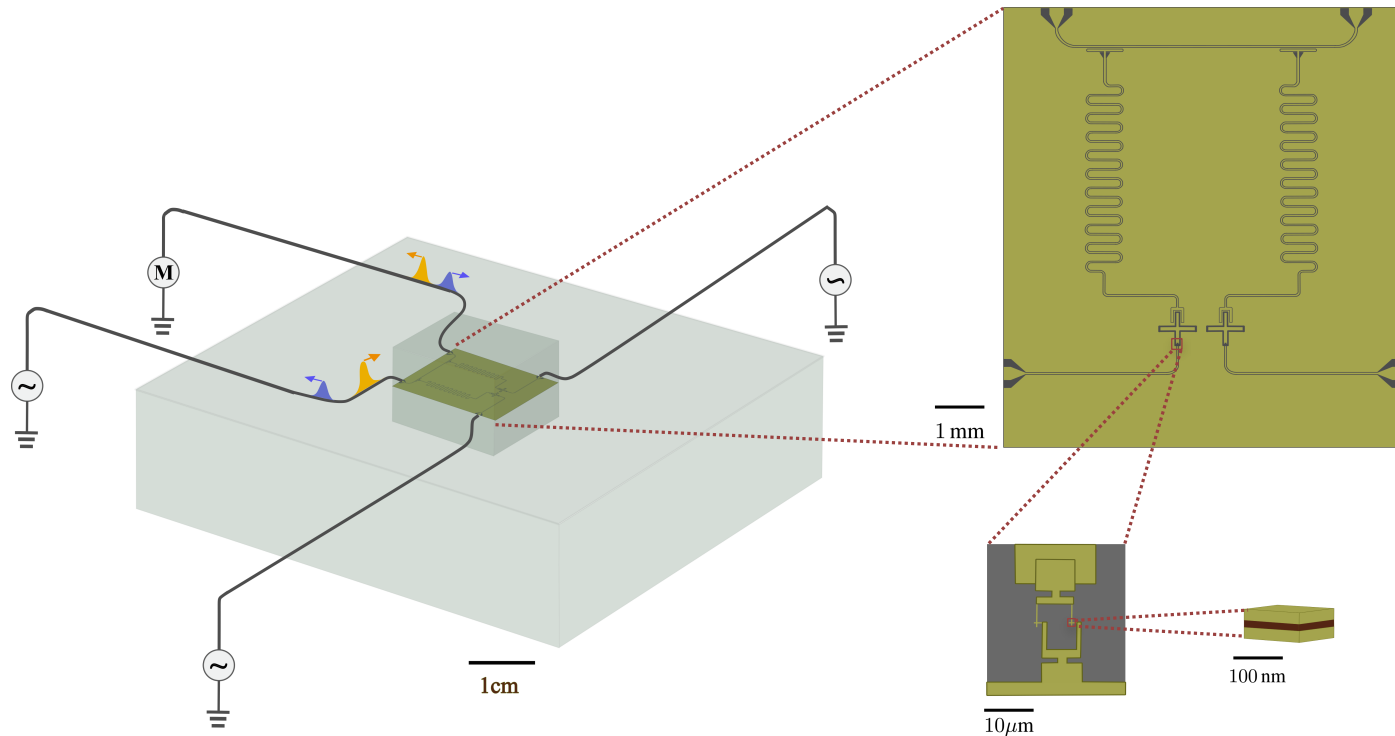
Zoe Zager

3+1D Modeling of Superconducting Quantum Chips



S. Huang et al., PRX Quantum 2, 020306 (2021) [MIT, Oliver Group]

The Problem

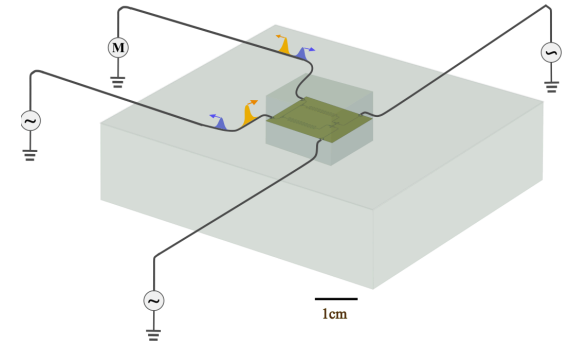
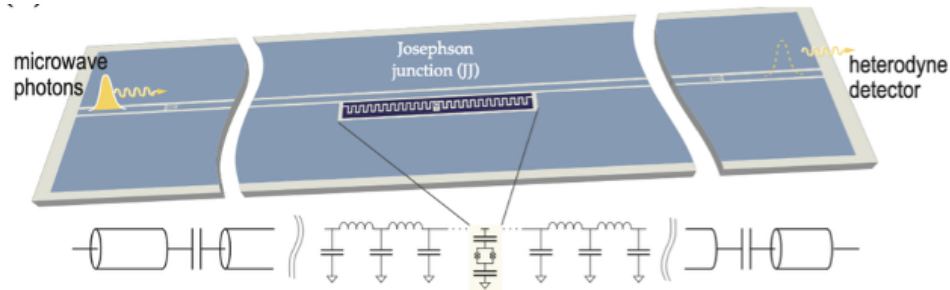


Dynamics of a Superconducting Condensate confined in a heterogenous environment

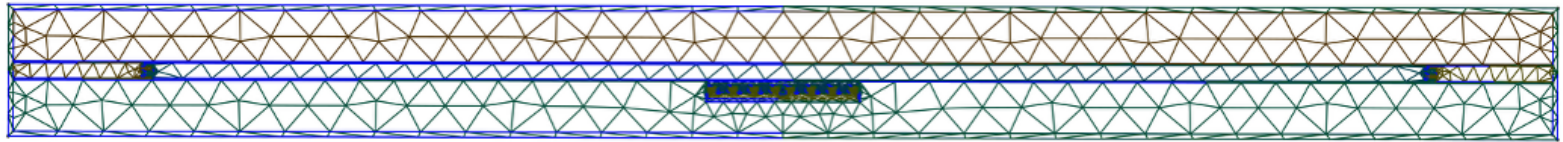
- Multiple spatial and temporal scales
- Interactions via retarded electromagnetic potentials
- Strong Hybridization, localized non-linear degrees of freedom
- Radiative Losses (Integrate out degrees of freedom outside the computational domain)
- Second quantization compatibility (“circuit QED”)

The Modeling Problem

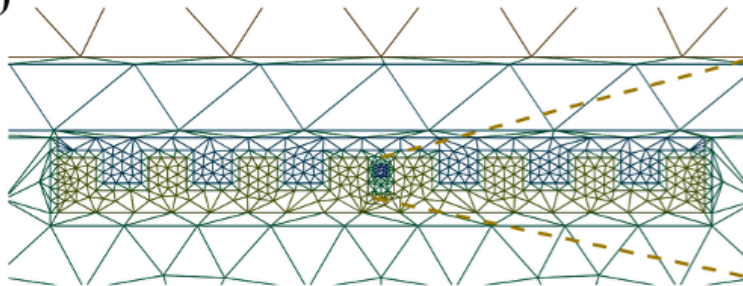
Houck, HET, Koch, Nature Physics, 2012



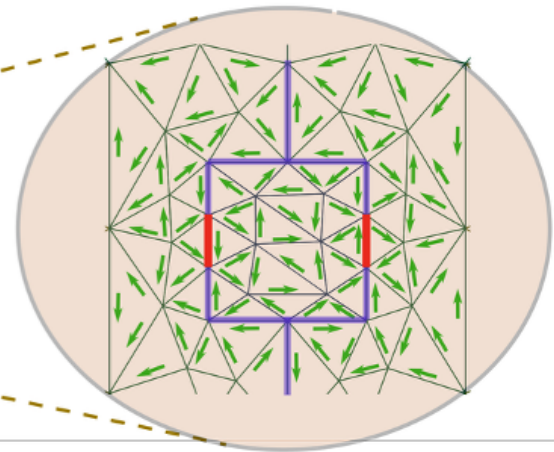
(c)



(d)

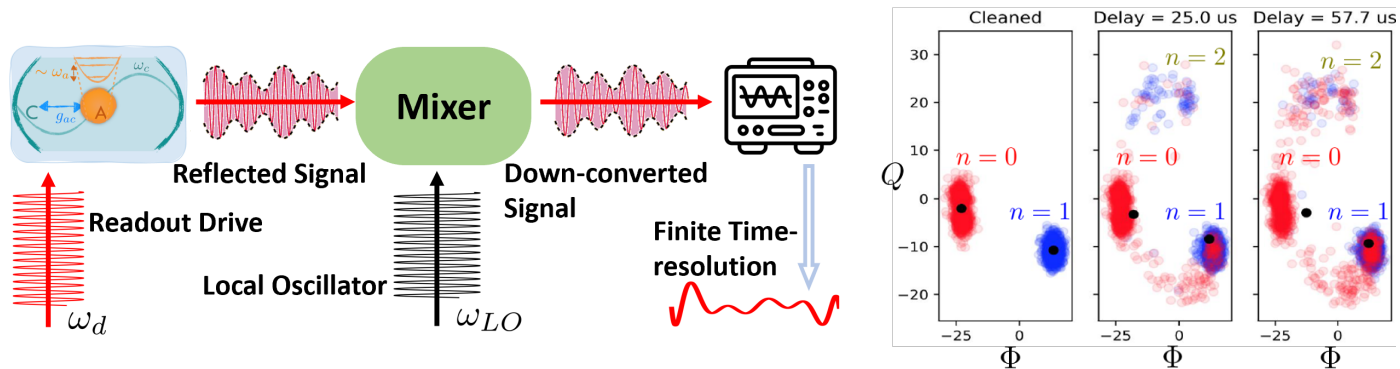


(e)

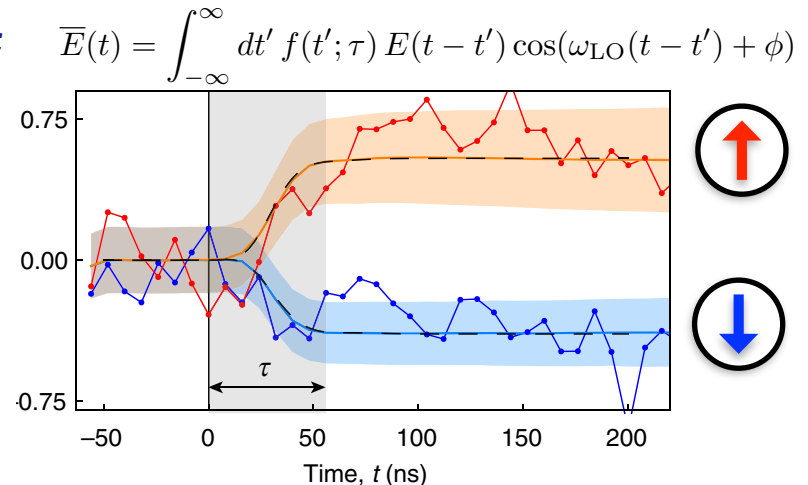
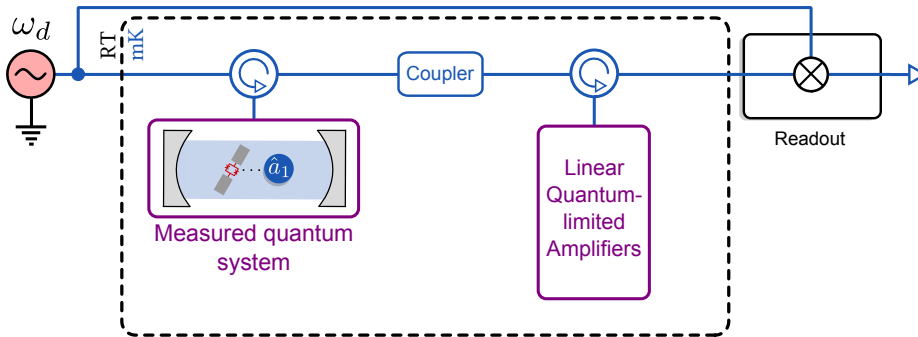


Measurement-adapted Coarse-graining of Field Theories

- Bigger Picture: Measurement-adapted coarse-graining
 - Long-term goal: Calculate only what can be resolved by measurement
 - Spatio-temporal coarse graining
 - Ab-initio determination of parameters of effective quantum Liouvillian



- Readout Detector averages observables over a period τ

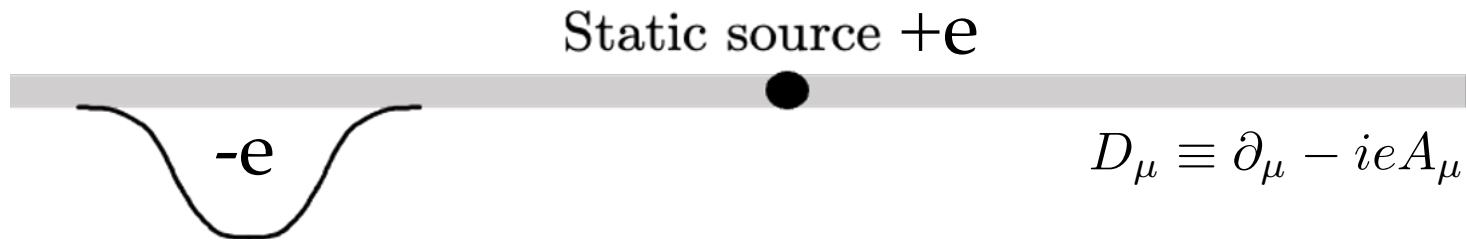


Can one directly obtain the effective quantum model governing the dynamics of coarse grained observables?

Emergence of Atomic Stability and Structure from non-perturbative QED - Case of 1+1D

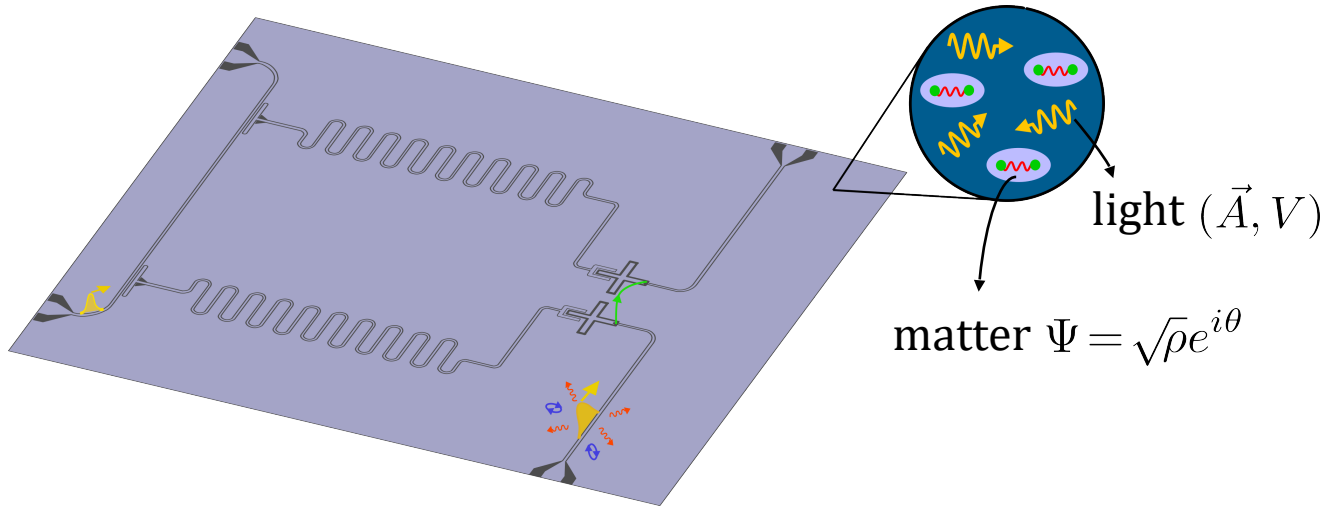
- 1+1D Quantum Electrodynamics of electrons and positrons

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu D_\mu - m) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e A^0 \delta(x - x_0)$$



- How does a stable ground state emerge?
- How does atomic structure arise?
- Can one derive a single-particle Schrödinger equation for the 1+1D Bohr atom?
- How to compute radiative losses for atomic transitions? Are they non-zero?

Electrodynamics of a Charged Condensate Field



- Order Parameter Equation

$$i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t} = \left[\frac{1}{2m} \left(-i\hbar \nabla - q\mathbf{A} \right)^2 + qV(\mathbf{r}, t) + U(\mathbf{r}) \right] \Psi(\mathbf{r}, t)$$

- Maxwell's Equations

$$\begin{aligned} \nabla \times \nabla \times \mathbf{A} + \mu_0 \epsilon_0 \ddot{\mathbf{A}} &= \mu_0 (\mathbf{J}_s + \mathbf{J}_{\text{src}}) - \mu_0 \epsilon_0 \nabla \dot{V} \\ -\nabla^2 V &= \frac{q}{\epsilon} (\rho + \rho_{\text{src}}) \end{aligned}$$

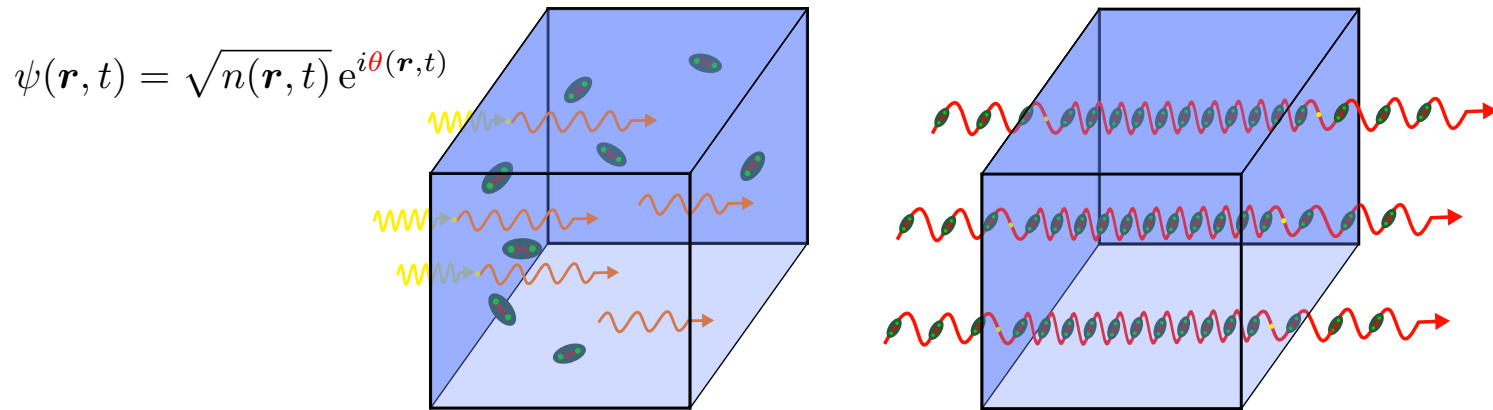
Pham, Fan, Scheer, HET, Rhys. Rev. A 107, 053704 (2023)

A Gauge-invariant formulation of SED

- The hybridized gauge-invariant field: $\mathcal{A}(\mathbf{r}, t) \equiv \mathbf{A}(\mathbf{r}, t) - \frac{\hbar}{q} \nabla \theta(\mathbf{r}, t)$

◆ Permeates all space

- Charged fluid with flow velocity $\mathbf{v} \equiv (\hbar \nabla \theta - q \mathbf{A}) / m$



Maldonado et al, Rhys. Rev. B 110, 014508 (2024)

$$A^\mu \equiv (A_0, \mathcal{A})$$

$$\mathcal{J}^\nu \equiv qn (c, \mathbf{v})^\nu$$

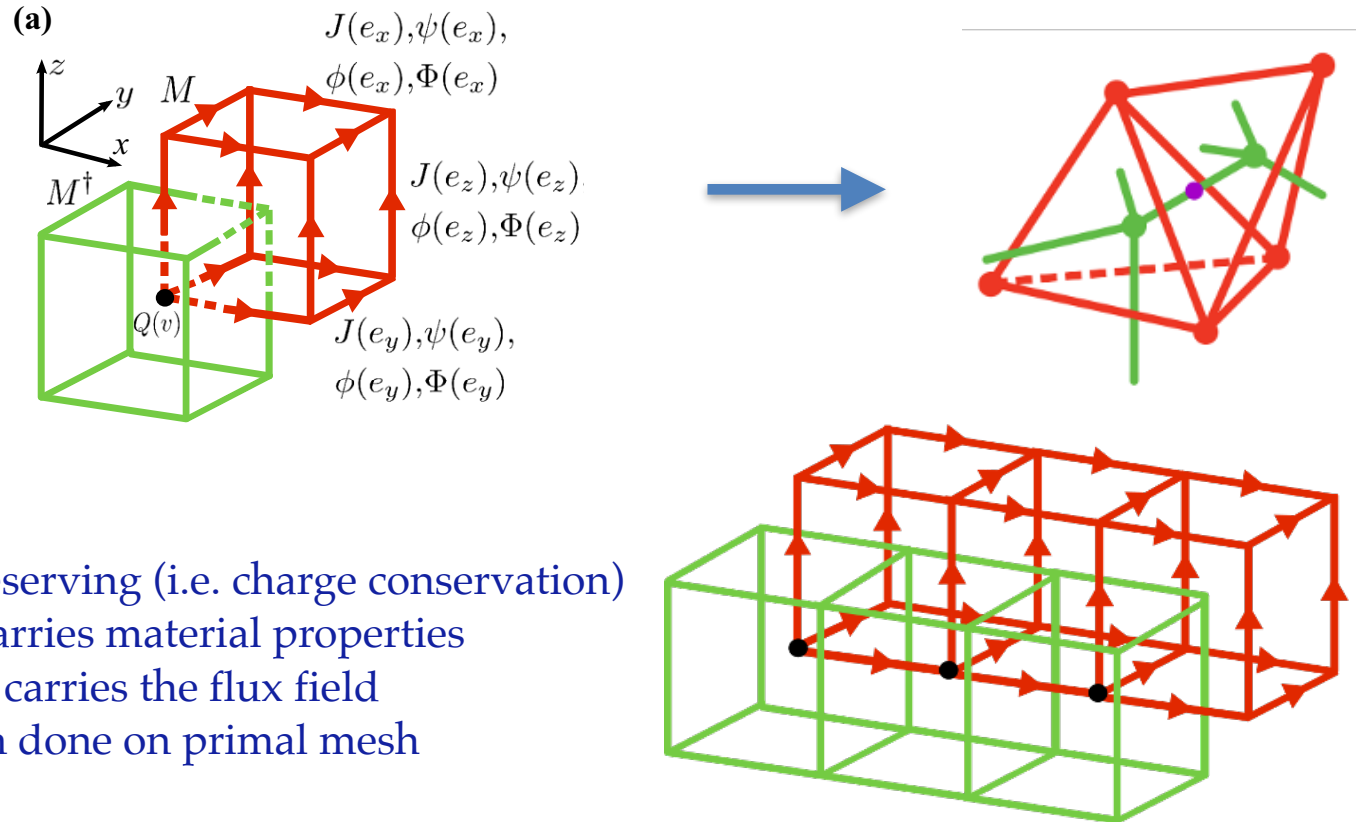
$$\partial_\mu \mathcal{F}^{\mu\nu} = \mu_0 (\mathcal{J}^\nu + j^\nu)$$

$$-qc\mathcal{A}_0 + \frac{q^2}{2m} |\mathcal{A}|^2 - \frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{n}}{\sqrt{n}} = 0$$

$$\dot{n} = \frac{q}{m} \nabla \cdot (n \mathcal{A})$$

A Gauge-invariant formulation of SED

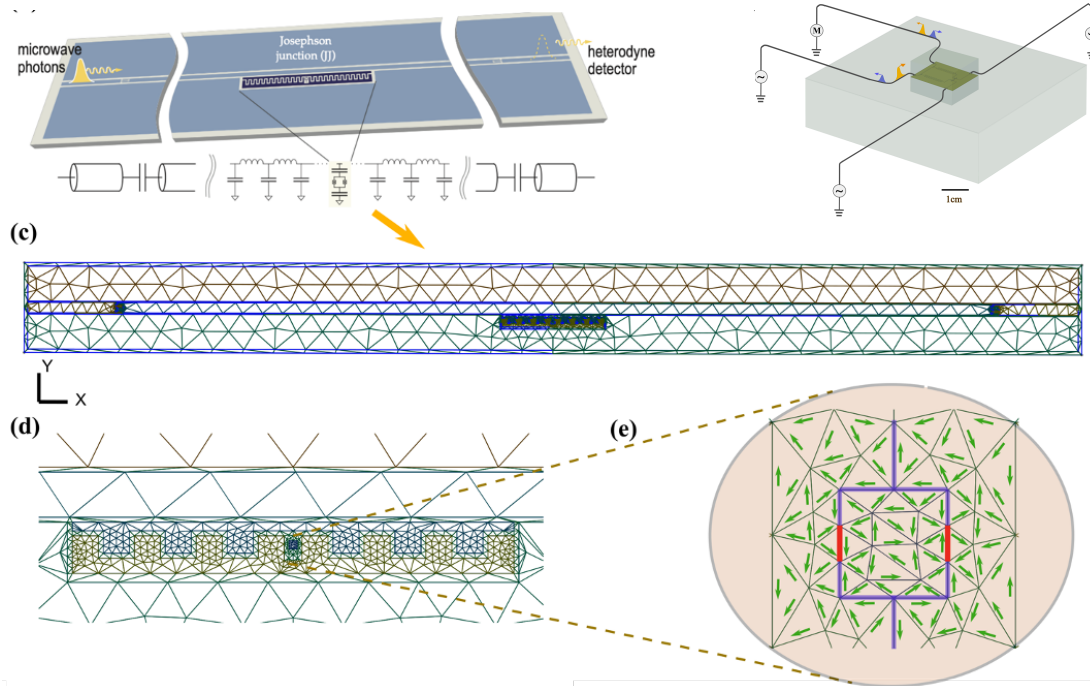
- Spatial coarse-graining (Discrete Exterior Calculus): $\Phi(e) = \int_e d\ell \cdot \mathcal{A}$



- Meshing:
 - Structure Preserving (i.e. charge conservation)
 - Dual mesh carries material properties
 - Primal mesh carries the flux field
 - Computation done on primal mesh

Anil N. Hirani, PhD Thesis, Caltech (2003)
 Chen, Chew, J. Comp. Physics 350, 668 (2017)
 Wang et al, J. Comp. Physics 488, 112245 (2023)
 Pham, Fan, Scheer, HET, Rhys. Rev. A 107, 053704 (2023)

DEC-QED: Some details



- QED of multi-scale superconducting devices
- Non-linear PDEs integrated in time-domain with long-time stability
- 1+1D: Reproduces *circuit QED* Framework, quantization ready
- Ab-initio determination of parameters of effective quantum Liouvillian

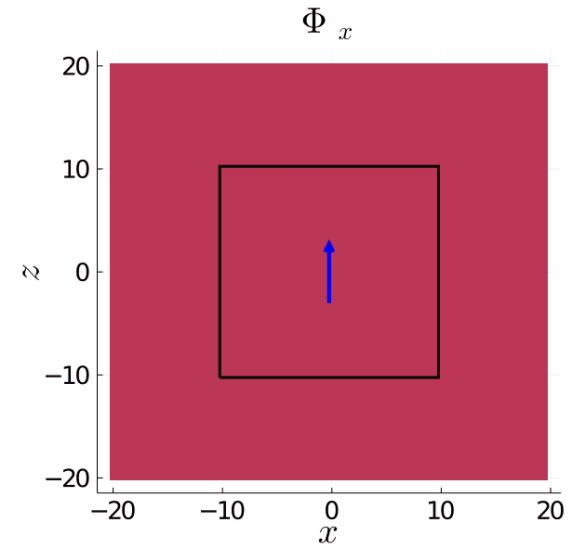
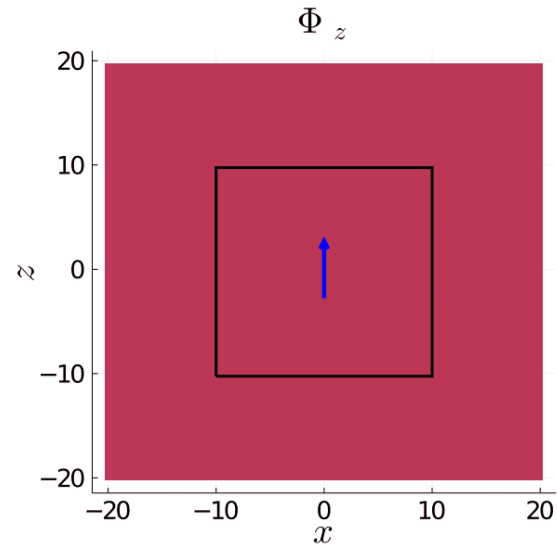
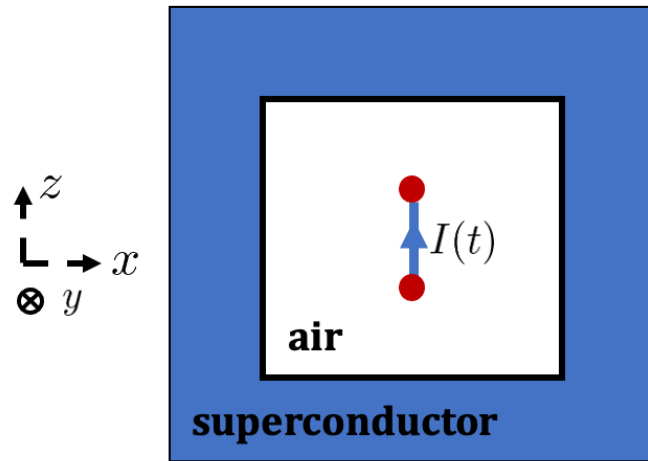
Pham, Fan, Scheer, HET, Rhys. Rev. A 107, 053704 (2023)

- Second quantization (“*circuit QED*”): $\Phi(e) = \int_e d\ell \cdot \mathcal{A} \rightarrow \hat{\Phi}(e)$

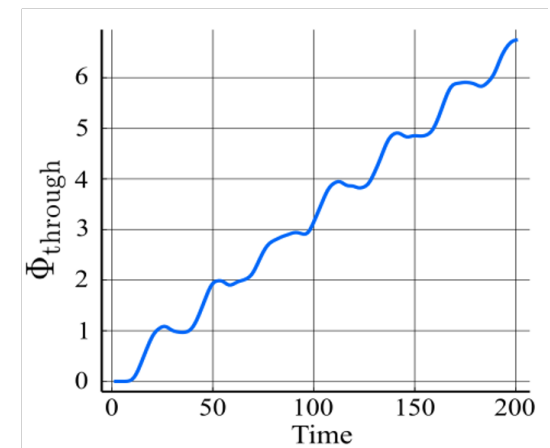
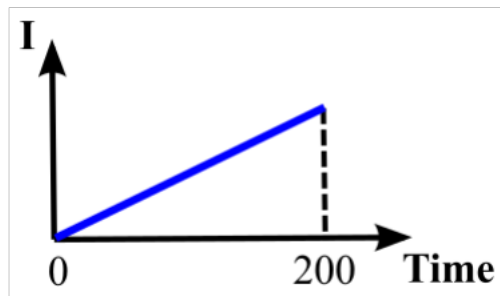
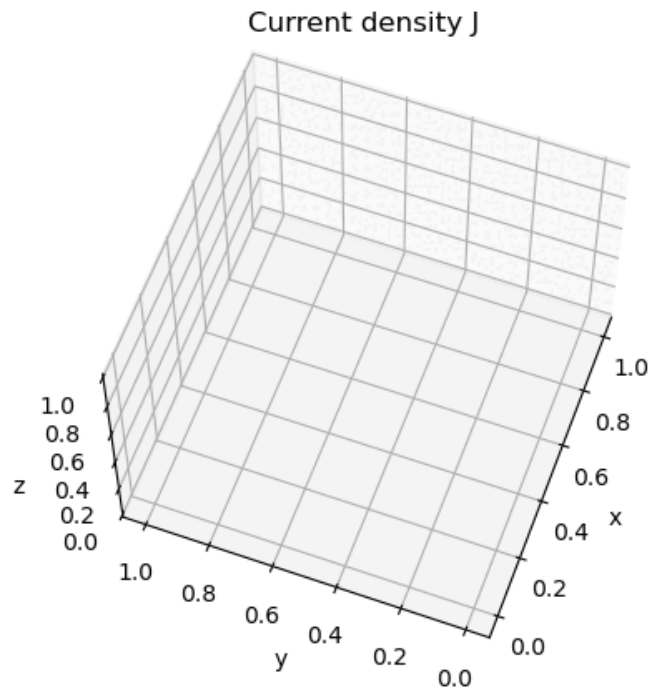
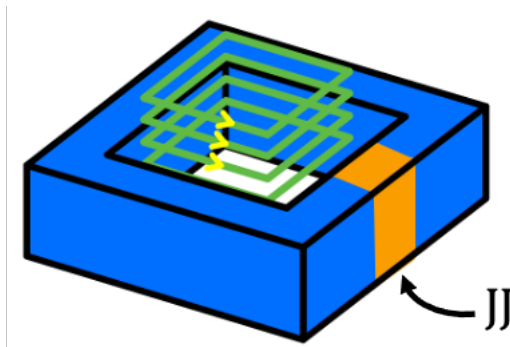
Maldonado et al., Rhys. Rev. B 110, 014508 (2024)

Maldonado, Rodriguez, HET, arxiv:2504.13779

DEC-QED Demo 1: Cavity + Josephson Junction



DEC-QED Demo 2: Flux Quantization



The spectral problem

Linearization:

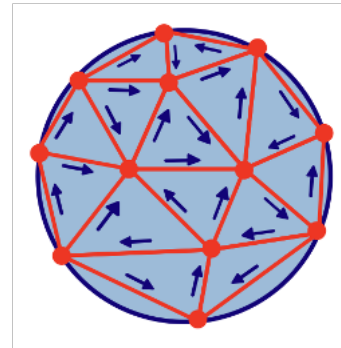
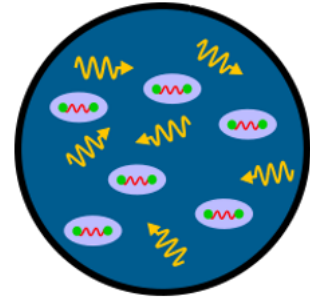
$$\nabla \times \nabla \times \mathcal{A} + \left(\frac{1}{\lambda_L^2(\mathbf{r})} - n^2(\mathbf{r})k^2 \right) \mathcal{A} = 0$$



Solve equation for $\Phi(e)$ on every edge e

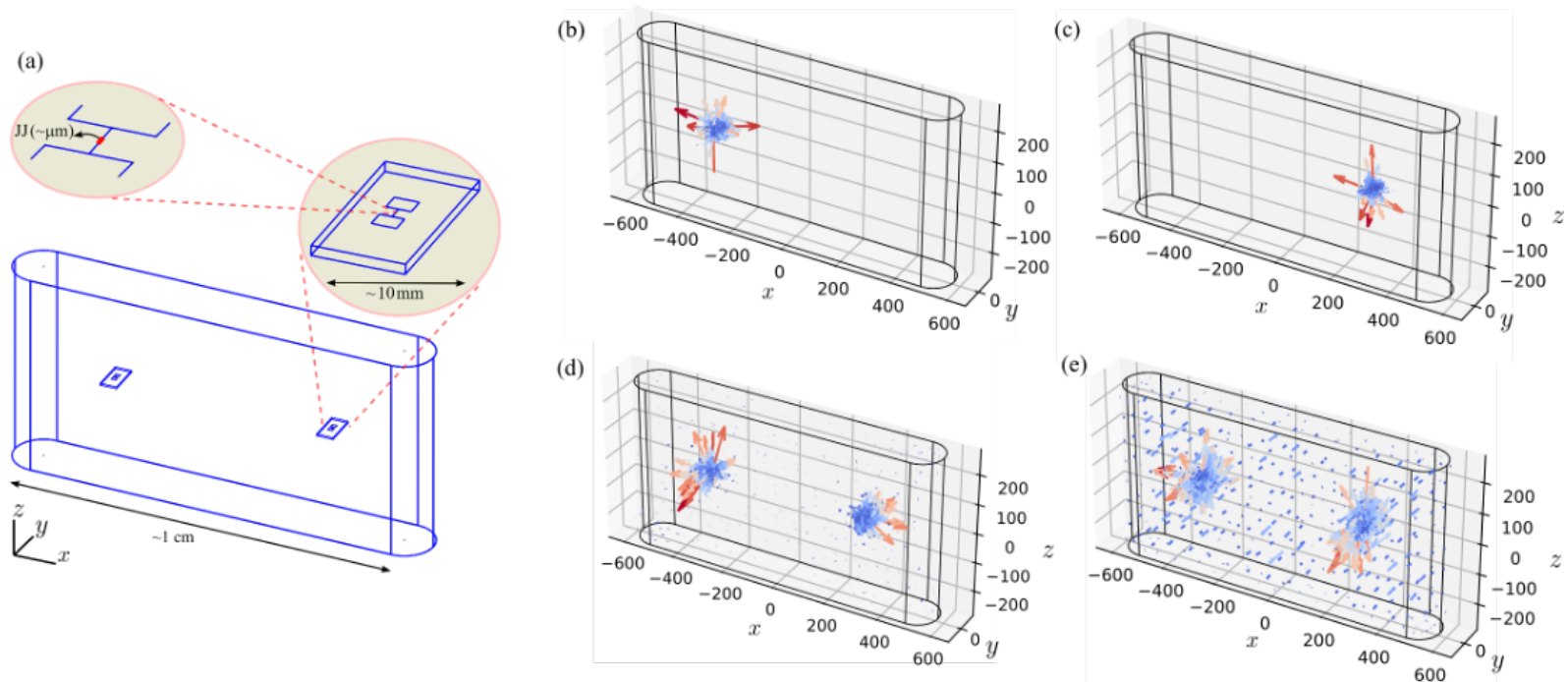


Extract relaxation rates, mode hybridization, etc.

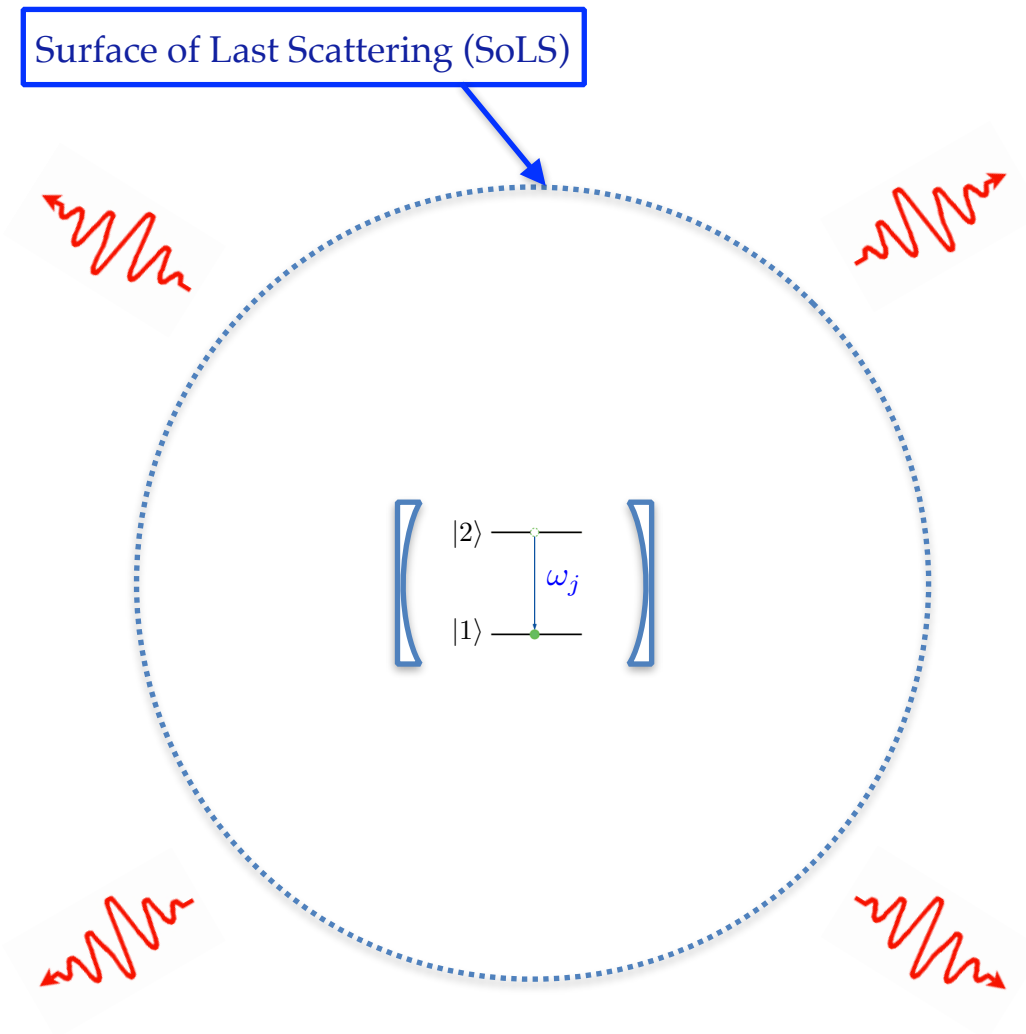


Electrodynamics in Finite Volume

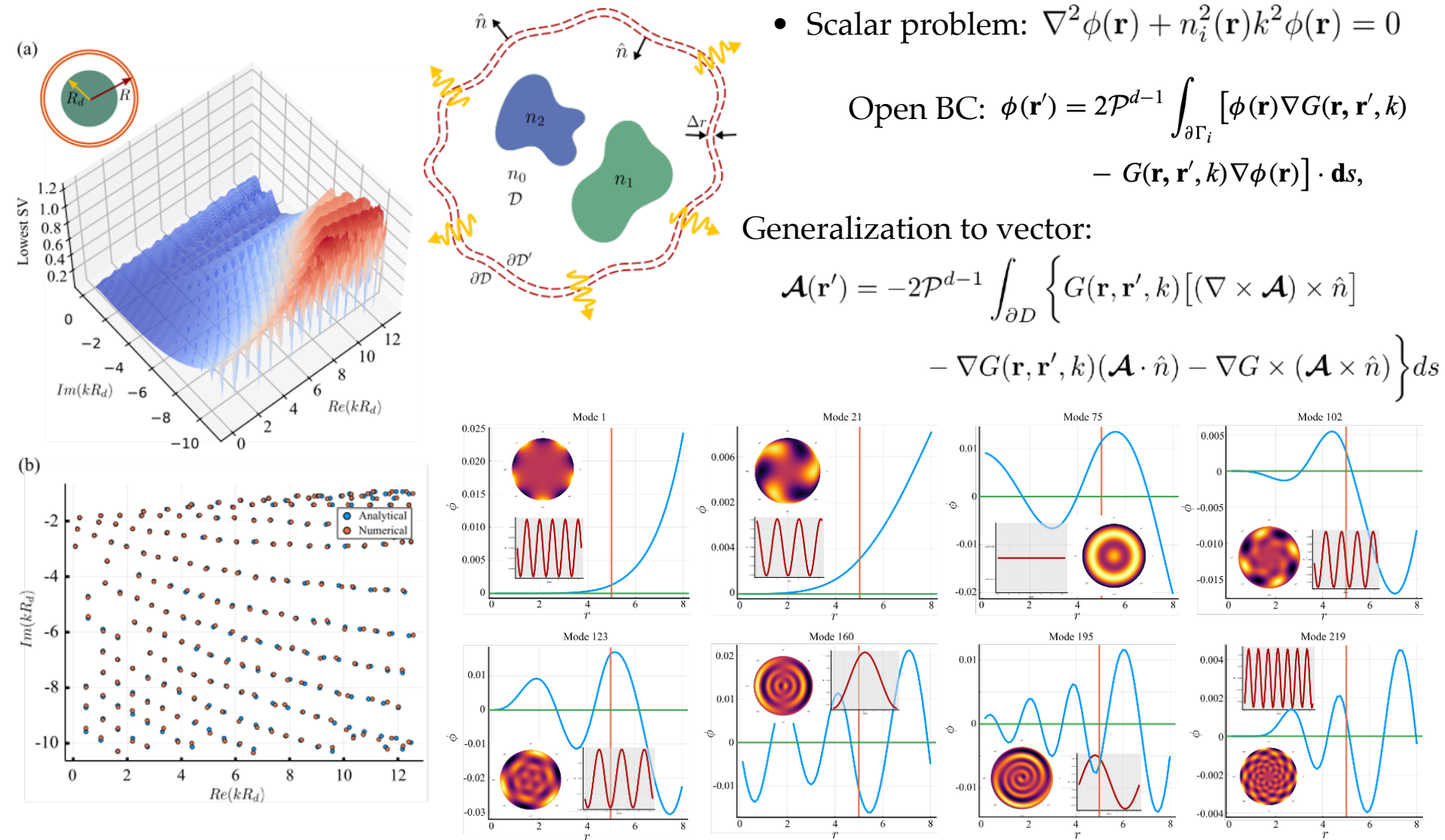
- Example of a closed system: a 3D cavity with two substrates



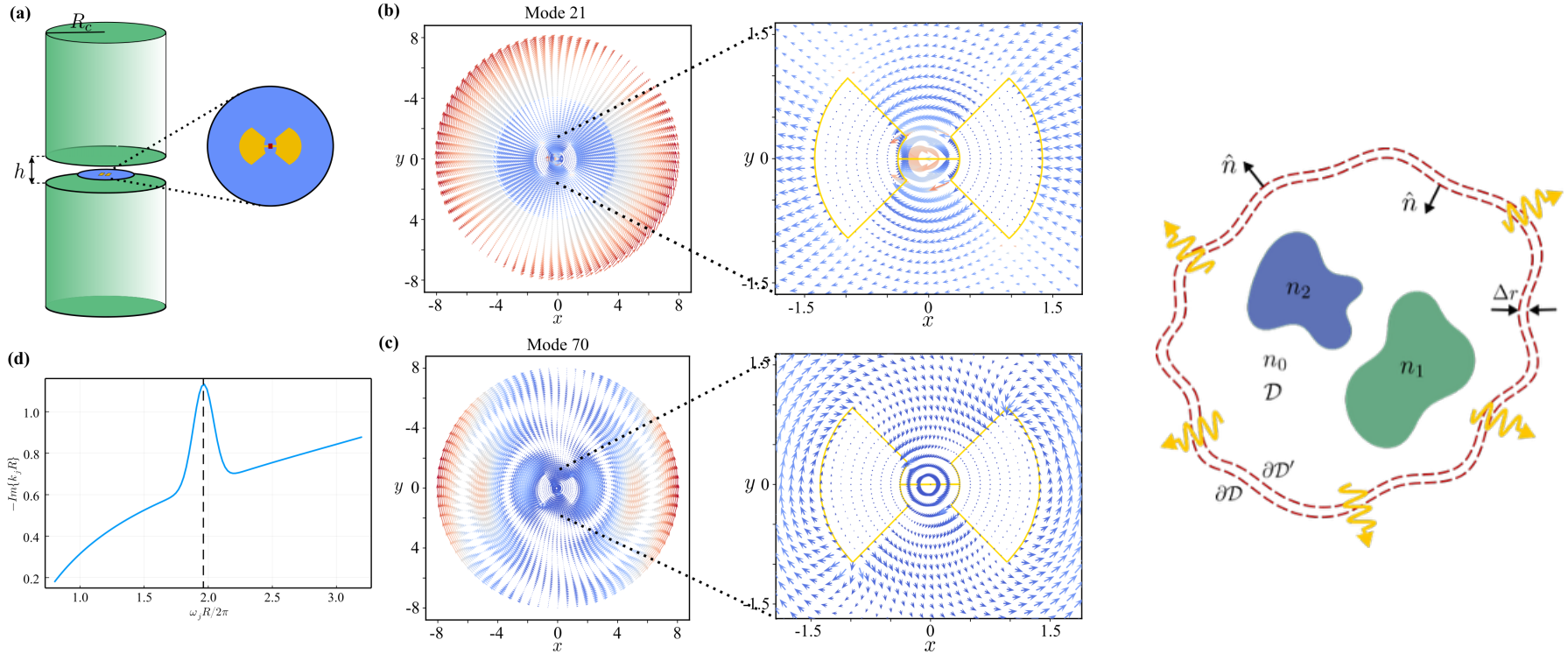
Quantum Electrodynamics in Finite Volume



Computation of open modes



DEC-QED Implementation of Radiative Boundary Conditions



Vision for the DEC-QED Program

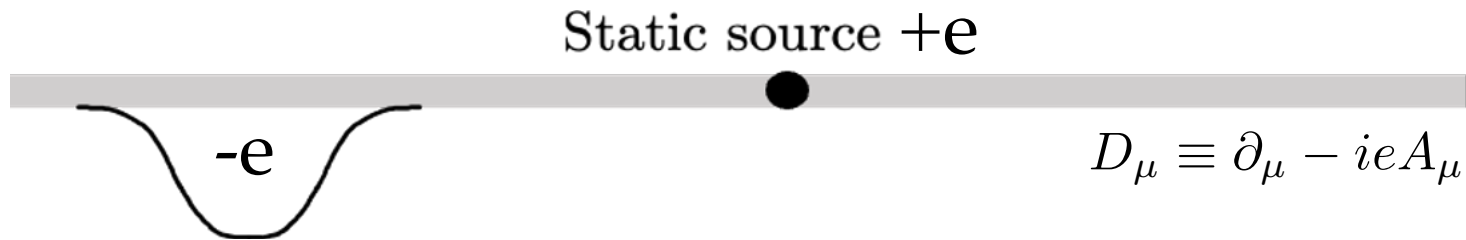
Bigger picture: **Measurement-based** coarse-graining (Adaptive space-time mesh?)

- Long-term goal: Calculate accurately only what's measurable (resolution-dictated calculations)
- **Spatio-temporal** coarse-graining Pham, Zager, Fan, HET, arxiv:2504.12286
- Public Packages:
 - DEC-QED, <https://github.com/dnpham23/DEC-QED>
 - Time-coarse graining: <https://github.com/leonbello/QuantumGraining.jl>

Emergence of Atomic Stability and Structure from non-perturbative QED - Case of 1+1D

- 1+1D Quantum Electrodynamics of electrons and positrons

$$\mathcal{L} = \bar{\psi} (i\gamma^\mu D_\mu - m) \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - e A^0 \delta(x - x_0)$$

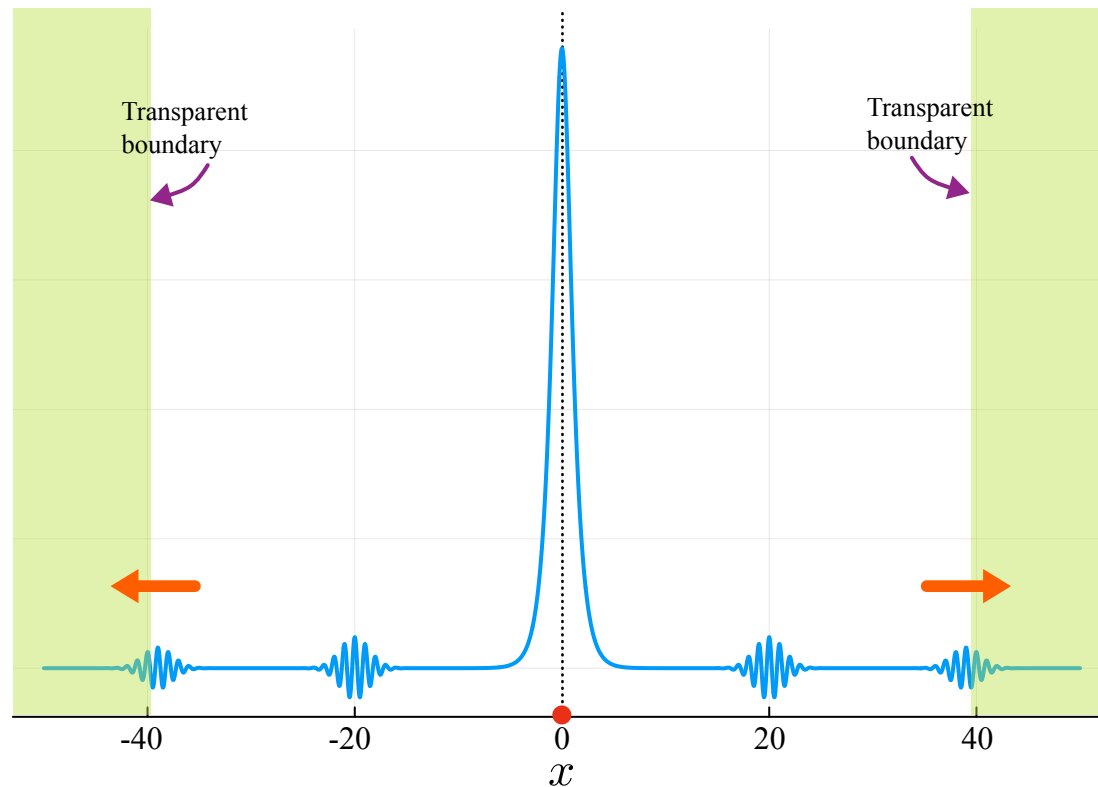


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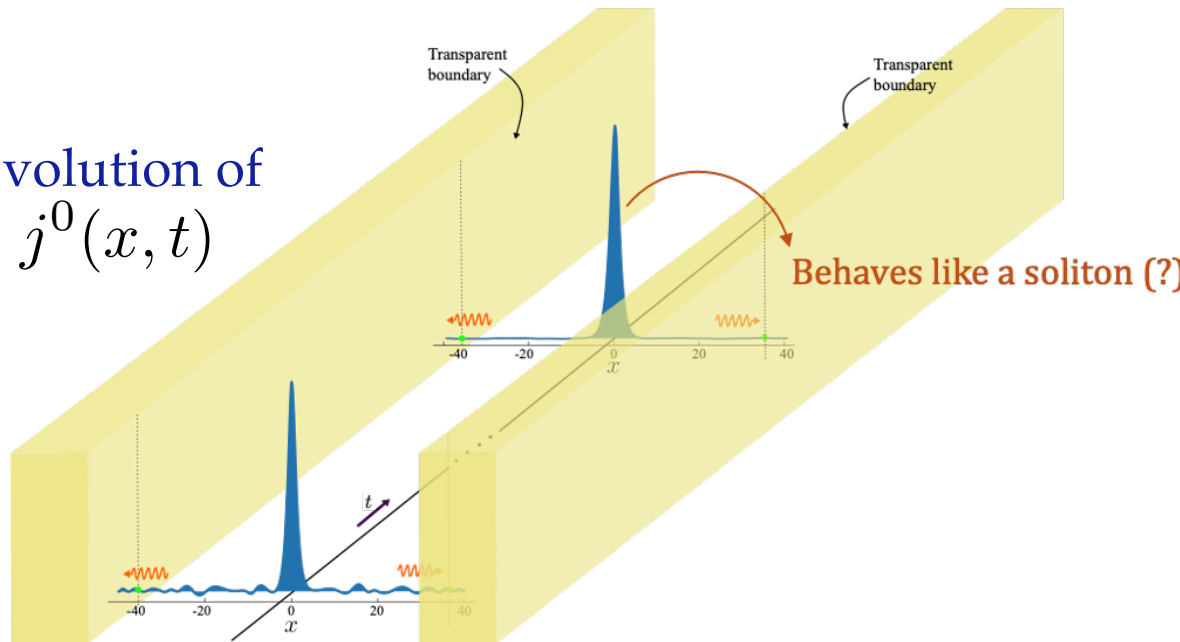


charge:	$j^0 = -\bar{\psi}\gamma^0\psi = -g\partial_x\varphi$
current:	$j^1 = -\bar{\psi}\gamma^1\psi = g\partial_t\varphi$

Bosonic EoM:

$$(\partial_t^2 - \partial_x^2) \varphi(t, x) + g^2 \varphi(t, x) + \sqrt{\pi} \kappa \sin(2\sqrt{\pi} \varphi(t, x)) = -g^2 \sqrt{\pi} \Theta(x - x_0)$$

Evolution of
 $j^0(x, t)$

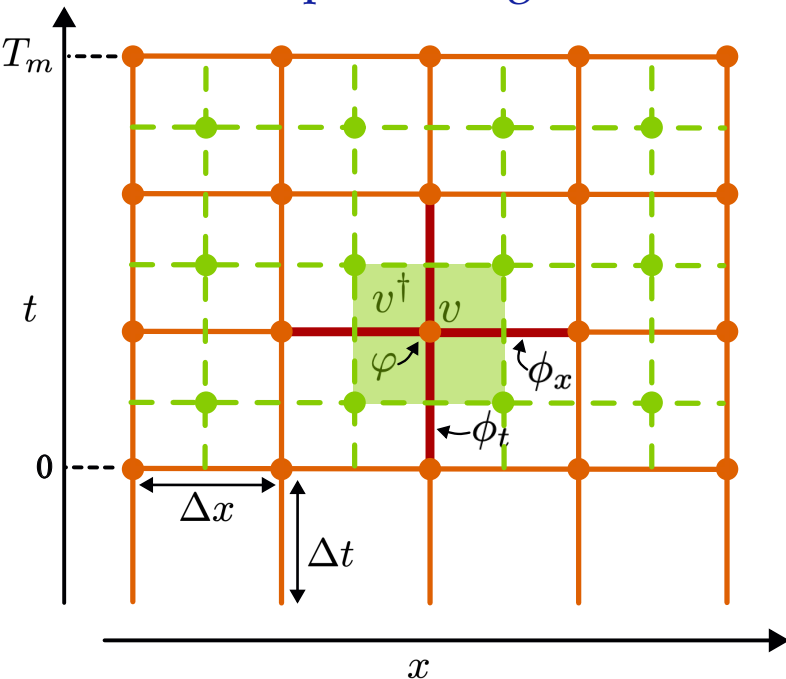


Emergence of Atomic Stability and Structure from non-perturbative QED - Case of 1+1D

$$g^{\mu\nu} \partial_\mu \partial_\nu \varphi + g^2 \varphi + \sqrt{\pi} \kappa \sin(2\sqrt{\pi} \varphi) = -g^2 \sqrt{\pi} \Theta(x - x_0)$$

where $g_{\mu\nu}$ is the 1+1D Minkowski metric tensor.

Spacetime grid



$$\phi_x(e_x) = \int_{\Delta x} \nabla_x \varphi dx$$

$$\propto j^0$$

$$\phi_t(e_t) = \int_{\Delta t} \nabla_t \varphi dt$$

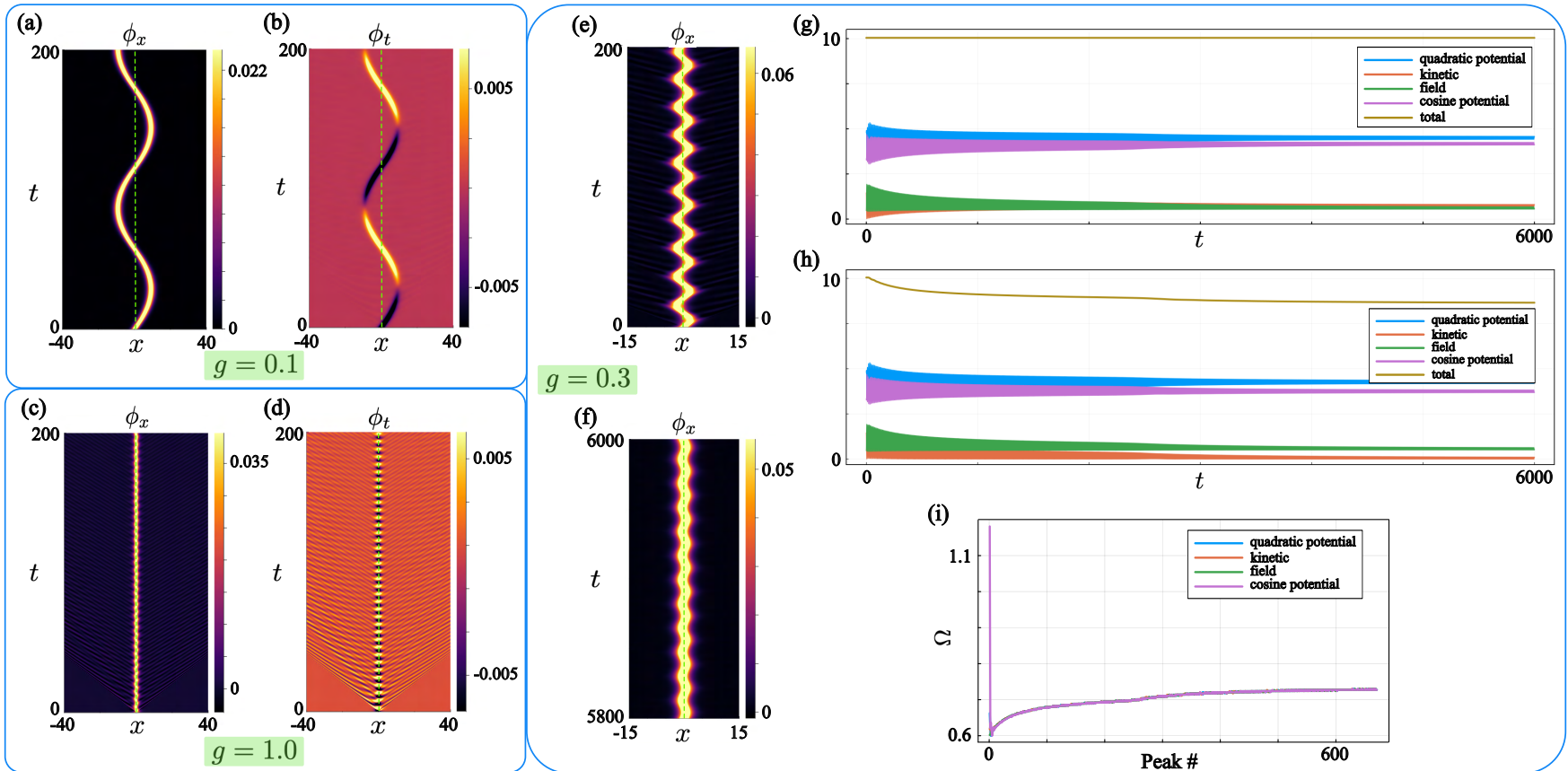
$$\propto j^1$$

$$-\sum_{\partial_t v^\dagger} \frac{\phi_x}{\Delta x^2} + \sum_{\partial_x v^\dagger} \frac{\phi_t}{\Delta t^2} + g^2 \varphi + \sqrt{\pi} \kappa \sin(2\sqrt{\pi} \varphi) = -g^2 \sqrt{\pi} \Theta(x)$$

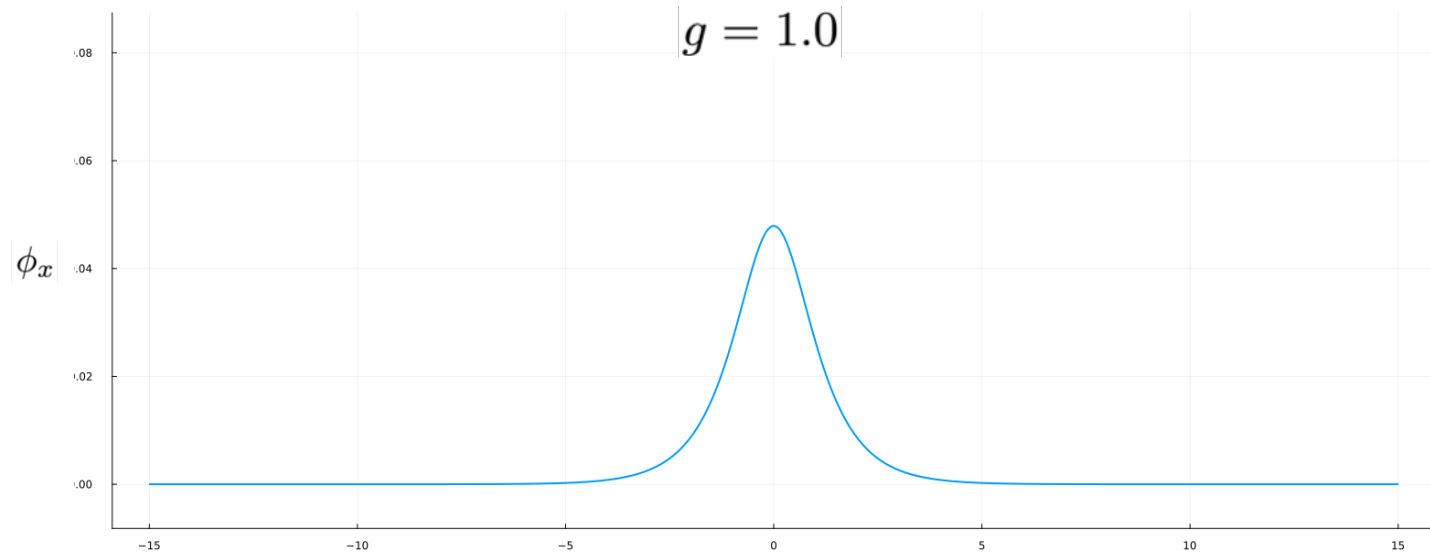
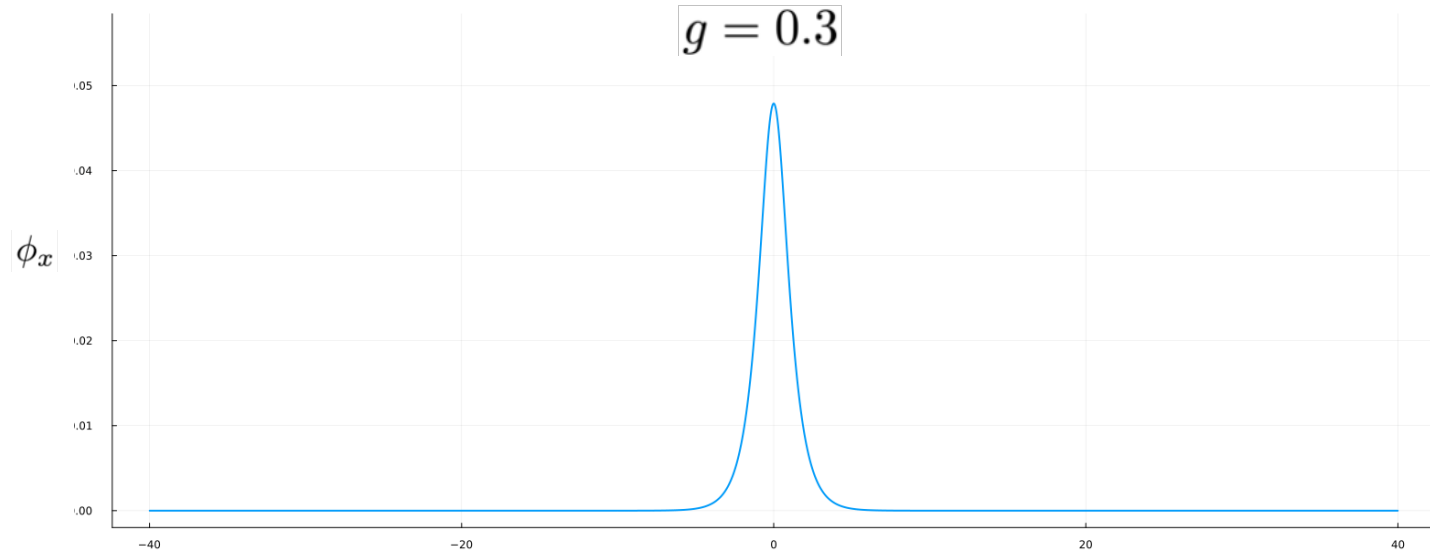
$$g = \frac{e}{\sqrt{\pi}}$$

$$\kappa = \frac{m}{\pi \epsilon}$$

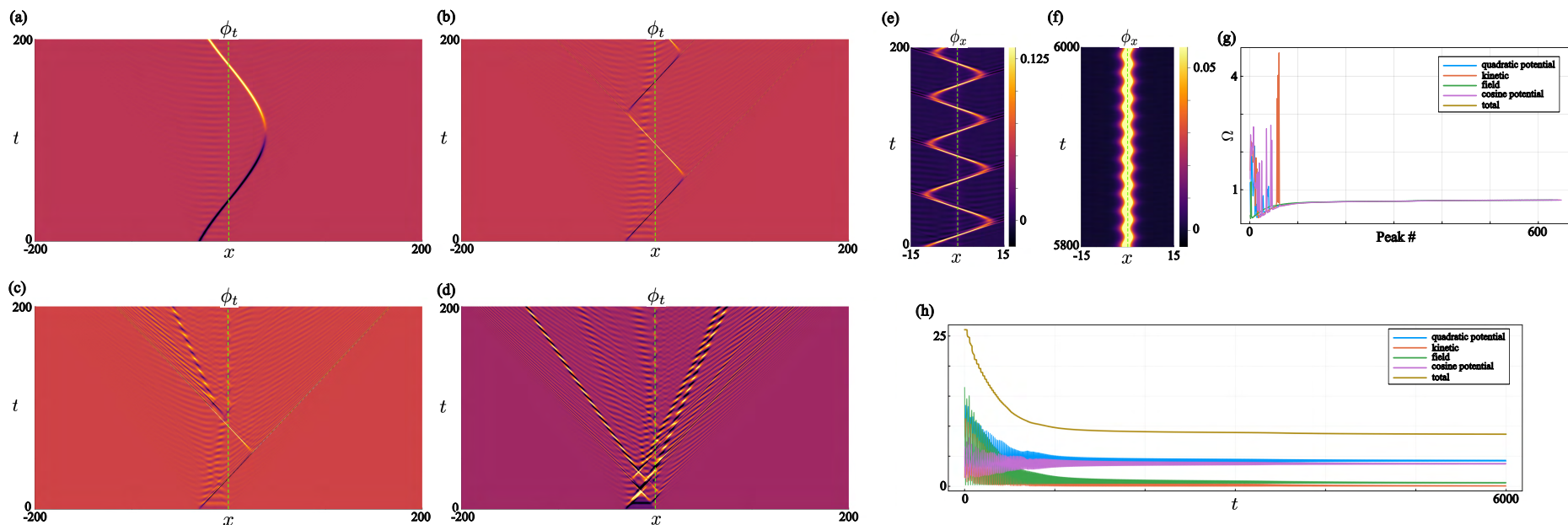
Schwinger Hydrogen Atom



Schwinger Hydrogen Atom



Schwinger Hydrogen Atom

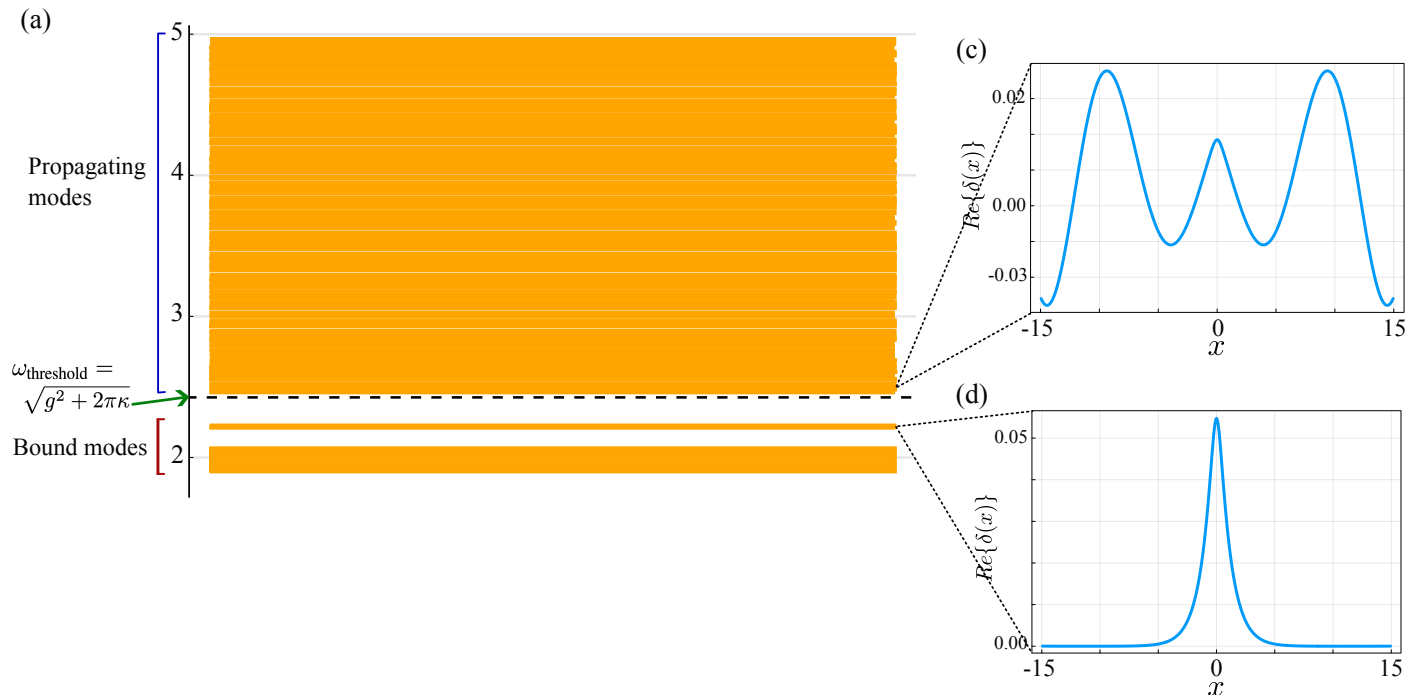


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Spectrum of excitations on solitonic background

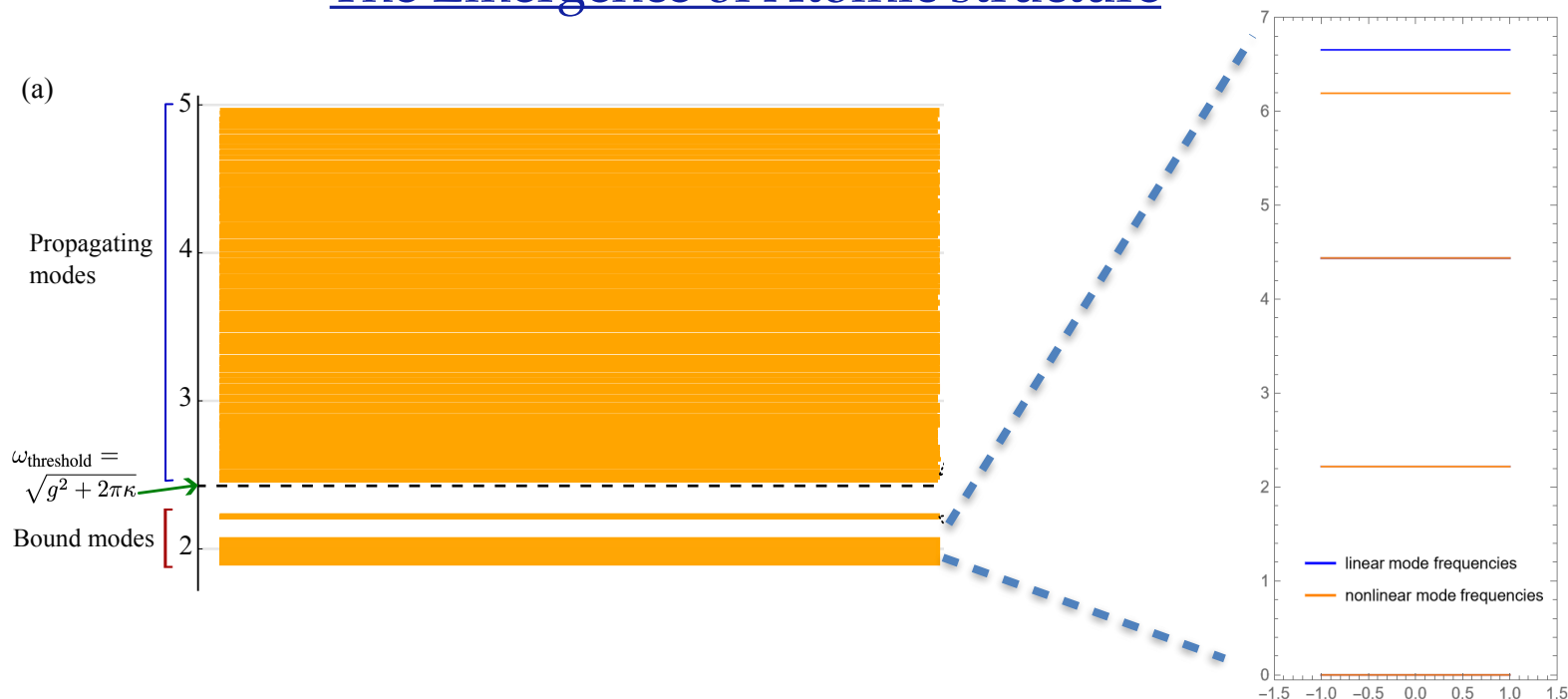


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The Emergence of Atomic structure



Looking ahead

- Measurement-adapted coarse-graining for superconducting quantum circuits
- Discrete spacetime for fundamental QED
- Analog quantum simulator using actual quantum hardware



Software Repositories:

- DEC-QED, <https://github.com/dnpham23/DEC-QED>
- Time-coarse graining: <https://github.com/leonbello/QuantumGraining.jl>

Funding: DOE-BES

