A Functional Tensor Model for Dynamic Multilayer Networks with Common Invariant Subspaces and the RKHS Estimation

Runbing Zheng, Runshi Tang, Anru R. Zhang, Carey E. Priebe

Dynamic multilayer networks:
Multiple relationships among entities evolving over time

Challenge:

- Capture shared structure across layers
- Handle layer-specific heterogeneity
- Model smooth temporal dynamics

Our idea: Extend COSIE model (Arroyo et al., 2021) to dynamic settings

COSIE: $\mathbf{P}_s = \mathbf{X}\mathbf{R}_s\mathbf{Y}^\top$

$\mathbf{X}, \mathbf{Y}$: common subspaces across layers

$\Downarrow$ $\mathbf{R}_s$: layer-specific core matrices

Our model: $\mathbf{P}_s^{[t]} = \mathbf{X}\mathbf{R}_s^{[t]}\mathbf{Y}^\top$

$\mathbf{R}_s^{[t]}$: temporally smooth cores

Functional tensor interpretation: $\mathbf{P}^{[t]} = \mathbf{R}^{[t]} \times_2 \mathbf{X} \times_3 \mathbf{Y}$, estimated via RKHS framework

- **Observe:** A network with adjacency matrix A such that $A_{ij} \sim \text{Bernoulli}(X_i^\top X_j)$, $1 \leq i < j \leq n$, where $X_1, \ldots, X_n \in \mathbb{R}^d$ and $\sim F$ i.i.d.

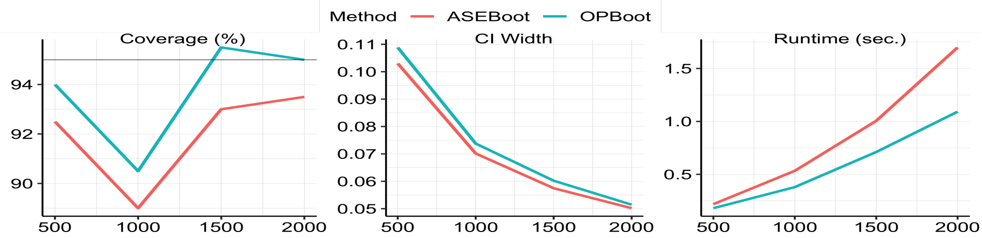
- **Aim 1:** Approximate the distribution of a network statistic $t : [0, 1]^{n \times n} \to \mathbb{R}$.

- **Aim 2:** The algorithm should be computationally efficient.

`OPBoot` (Overlapping Partitions Based Bootstrap)

- Estimate $\hat{X} = \texttt{ASE}(A, d) \in \mathbb{R}^{n \times d}$ and resample from the rows of $\hat{X}$ to obtain $\hat{X}^*$.
- Randomly select an overlap part $S_0 \subset [1:n]$ and partition $[1:n] \setminus S_0 = S_1 \cup \cdots \cup S_s$. Form $S_{0q} = S_0 \cup S_q$, $q \in [1:s]$.
- Estimate $\hat{P}^*_{0q} = \hat{X}^*_{S_{0q}} \hat{X}^{*\top}_{S_{0q}}$, $q \in [1:s]$.
- Compute aggregated statistic $\hat{t}^* = \frac{1}{s} \sum_{q \in [1:s]} t\left(\hat{P}^*_{0q}\right)$.
- Repeat B times.

95% CI for Approximate Trace Using OPBoot



Conclusion:

- CIs achieve good coverage.
- Faster as network statistic is computed on smaller subnetworks.
- Computation gain increases with the network size.

Heterogeneous Treatment Effects in Networks: A Non-Parametric Approach Based on Node Connectivity

Heejong Bong, Colin B. Fogarty, Elizaveta Levina, Ji Zhu

January 12, 2026

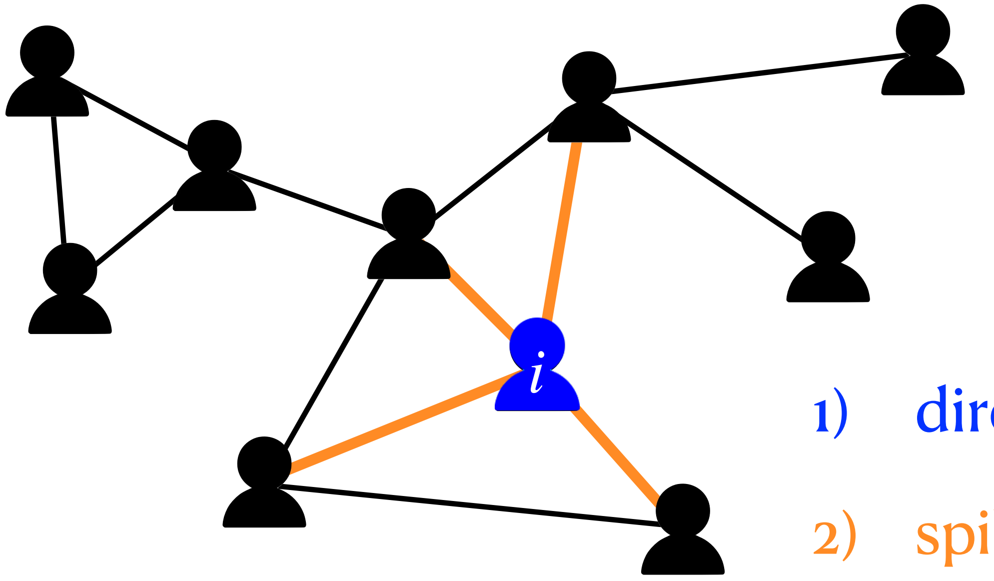
Challenge: interference

Socioeconomic survey on 9,449 villagers (37 villages) in Karnataka, India.

> Banerjee, A., Chandrasekhar, A. G., Duflo, E., & Jackson, M. O. (2013). The diffusion of microfinance. *Science*, *341*(6144), 1236498.

Khatami et al. (2023) studied

savings self-help group participation $\longrightarrow$ **tolerance of financial risk**.

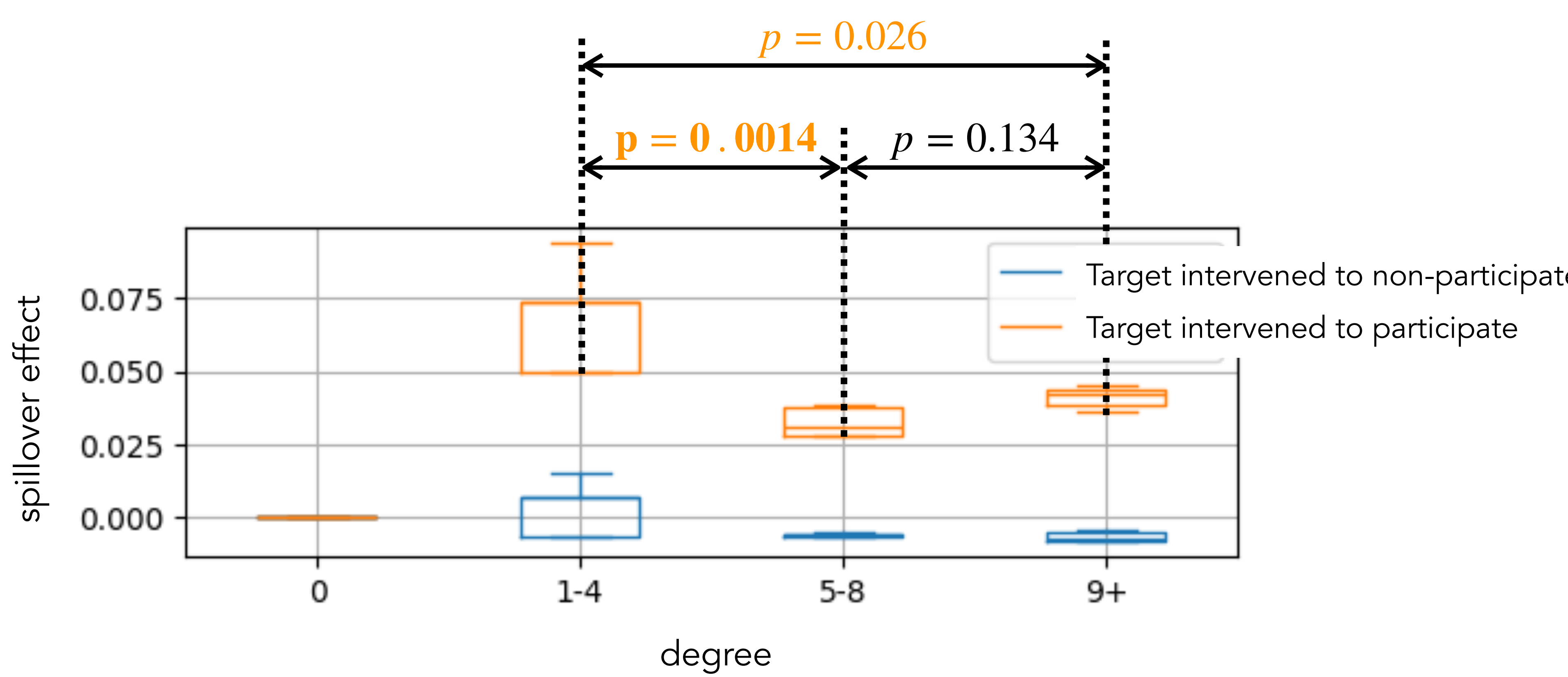


1) direct effect
2) spillover effect

Classical causal inference is not capable to explain the interference.

KECENI estimates node-wise causal effects instead of average across entire population

	KECENI	Khatami *et al.*
Average Direct Effect	0.160 (0.146, 0.170)	0.252 (0.172, 0.332)
Average Spillover Effect, Target intervened as observed	0.011 (-0.001, 0.022)	0.017 (0.007, 0.027)
Average Spillover Effect, Target intervened to non-participate	0.001 (-0.009, 0.012)	-
Average Spillover Effect, Target intervened to participate	0.058 (0.040, 0.075)	-



A Goodness-of-Fit Test for Latent Space Models

Xinyu Li, Yinqiu He

Department of Statistics, University of Wisconsin-Madison

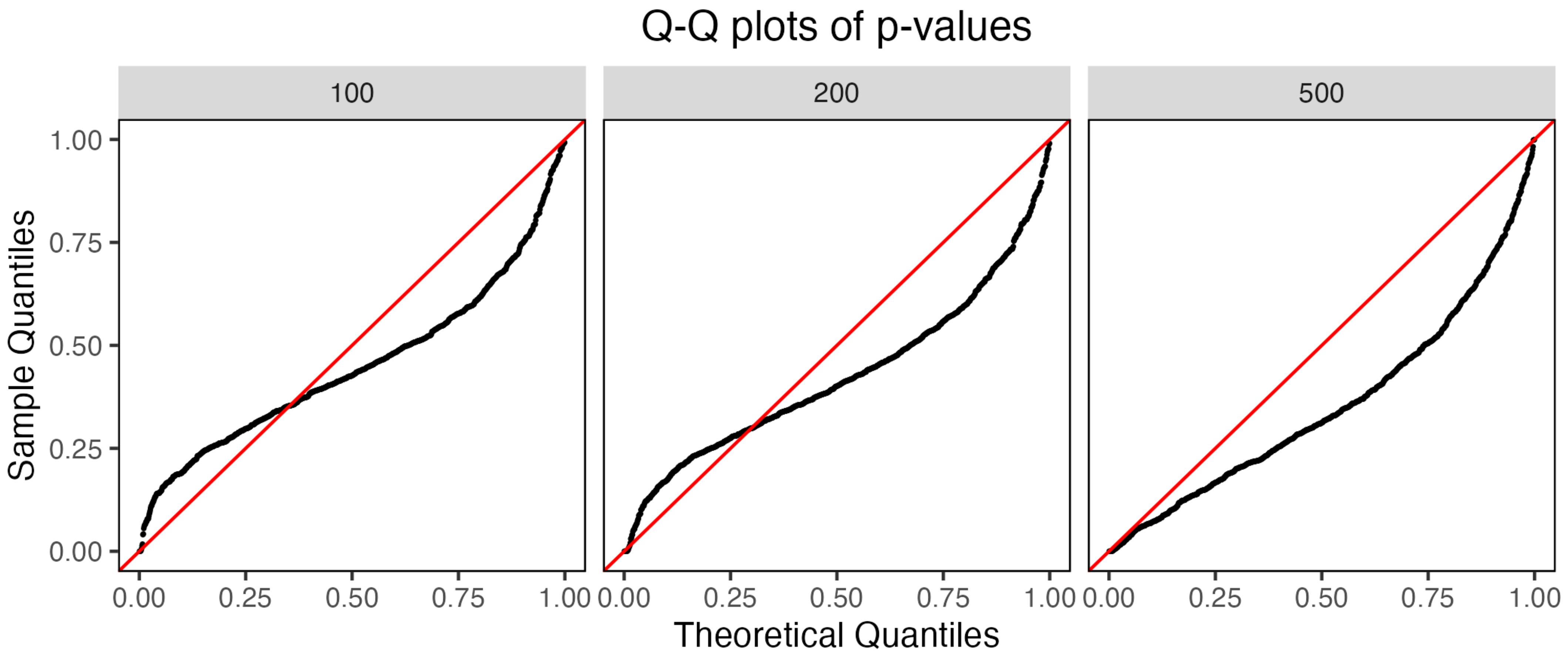
IMSI Recent Advances in Random Networks Workshop Lightning Talk Jan 12th, 2025

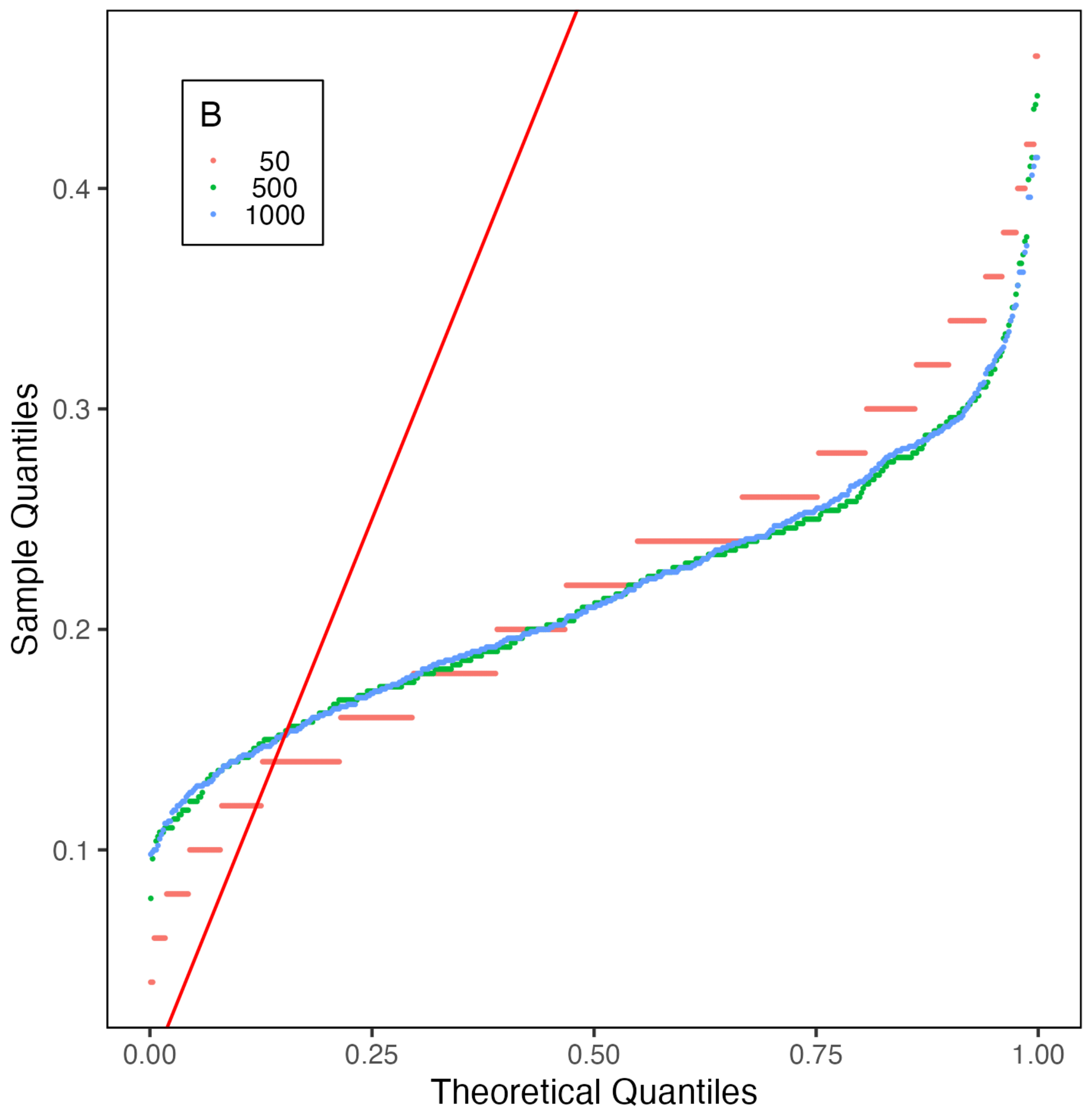
Goal and Limitation of Existing Methods

$H_0 : A_{ij} \sim p(\cdot \mid P_{ij})$ v.s. $H_\alpha : H_0$ is False, where $P_{ij} = \sigma(\alpha_i + \alpha_j + Z_i^T Z_j)$

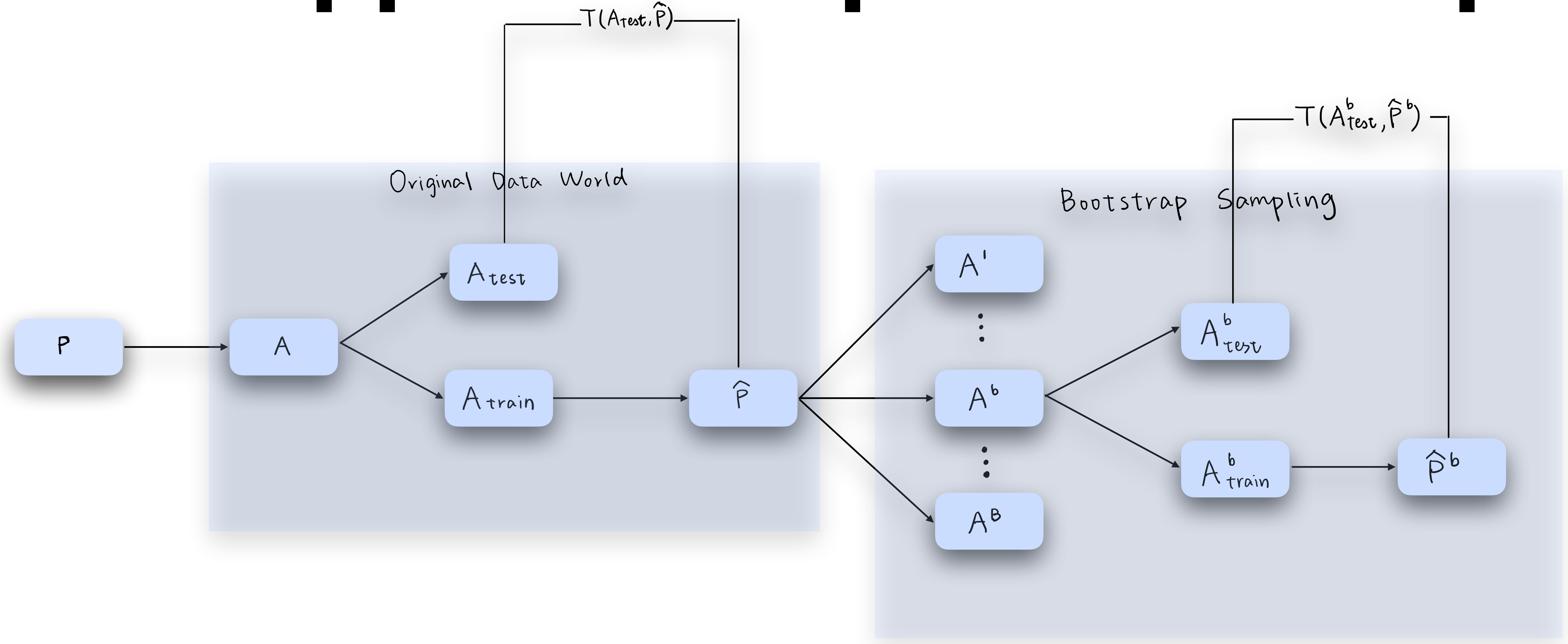
For each node i, $Z_i \in \mathbb{R}^K$ is the latent vector, $\alpha_i \in \mathbb{R}$ is the degree parameter.

- Tracy-Widom Test Statistic
- Naive Parametric Bootstrap





Our Approach: Split-Bootstrap



- Split
- Fit the null model
- Calculate T
- Parametric bootstrap
- Re-split, re-fit, re-calculate
- Centering and scaling

Bootstrap-corrected test statistic:

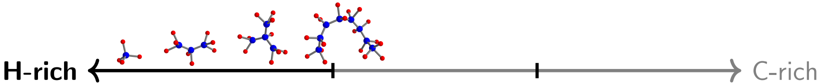
$$T^{\text{boot}} = \frac{T(A_{\text{test}}, \hat{P}) - \frac{1}{B}\sum_{b=1}^{B} T(A^b_{\text{test}}, \hat{P}^b)}{\sqrt{\frac{1}{B-1}\sum_{b=1}^{B}\left(T(A^b_{\text{test}}, \hat{P}^b) - \frac{1}{B}\sum_{b=1}^{B} T(A^b_{\text{test}}, \hat{P}^b)\right)^2}} \to \mathbb{N}(0,1) \text{ under null}$$

Report p-value = $2 \times \left(1 - \Phi(|T^{\text{boot}}|)\right)$, reject null if it's less than α(0.05)

Thanks!

Random cycle networks for hydrocarbon pyrolysis

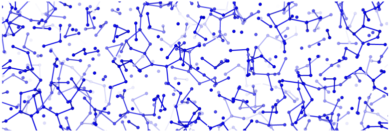
Perrin Ruth, Maria Cameron, University of Maryland, College Park



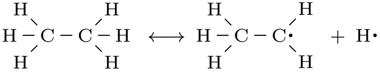
Goal: describe structures and predict the giant molecule in hydrocarbons at extreme temperatures and pressures

Molecular dynamics: simulate with force fields

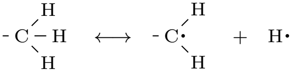
- Computationally expensive
- Data: from ReaxFF [1] force field, permitting reactions



Kinetic Monte Carlo: simulate with learned reactions [2]



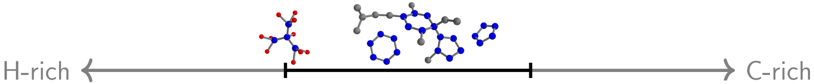
- #reactions grows rapidly with #atoms
- Local reactions [3]: fewer reactions, better prediction of giant mol.



[1] Ashraf and van Duin. *J. Phys. Chem. A.* (2017). [2] Gillespie. *J. Comput. Phys.* (1976).
[3] Dufour-Decieux, Freitas, and Reed. *J. Phys. Chem. A.* (2021).

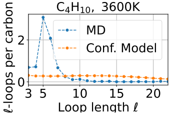
Loops reduce the size of the giant component

Ruth, Dufour-Decieux, Moakler, Cameron. *Phys. Rev. E.* (2025)



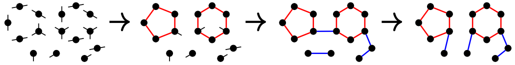
Configuration Model [1]:

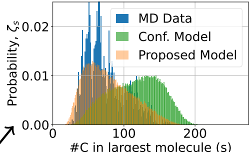
- Predicts mol. size dist. in H-rich systems [2]
- Poorly predicts the size of the giant mol.
- **Issue:** Locally tree-like, lacks small rings



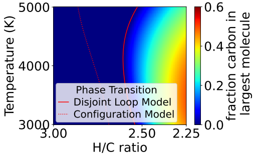
Disjoint Loop Model:

- Add loops & remove added assortativity





- Improved prediction of giant molecule size
- Analytic solution via generating functions
  - Adapted from [3,4]



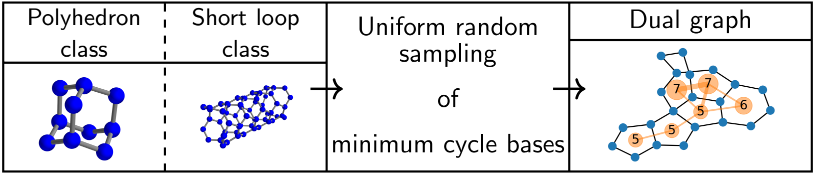
[1] Newman et al. *Phys. Rev. E.* (2001); [2] Dufour Decieux et al. *J. Chem. Phys.* (2023)
[3] Karrer and Newman. *Phys Rev. E* (2010); [4] Newman. *Phys. Rev. Lett.* (2002).

Novel cycle statistics

Perrin Ruth, Maria Cameron. *In review.* arXiv:2511.09732.

Issue: carbon-rich → carbon rings form clusters → loops are not disjoint
Current work: develop statistics sufficient to analyze giant ring clusters

- Conventional approach of using a minimum cycle basis [1] is not unique
- Larger unique sets of cycles grow exponentially with graph size
- **Our approach:** random sampling (inspired by [2-4])



Python codes: `https://github.com/perrineruth/Sampling_Minimum_Cycle_Bases`

[1] Kavitha et al. *Comput. Sci. Rev.* (2009); [2] Vismara. *Electron. J. Comb.* (1997);
[3] Gleiss et al. *Electron. J. Comb.* (2000); [4] Kolodzik et al. *J. Chem. Inf. Model.* (2012).

Procrustes-Wasserstein Matching: Fundamental Limit & Efficient Algorithm

Xiaochun Niu, Tselil Schramm, Jiaming Xu

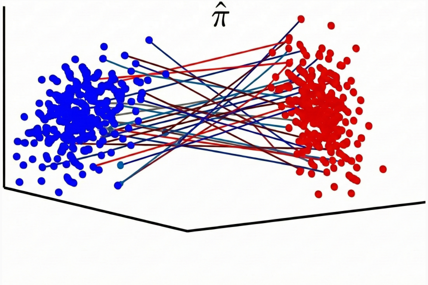
Observation X and Y: $X_i, Z_i \overset{\text{i.i.d.}}{\sim} \mathcal{N}(0, I_d)$;

$$Y_i = (\rho X_{\pi(i)} + \sqrt{1-\rho^2} Z_i) Q \text{ for } i \in [n], \text{ where}$$

- π: **permutation** of $[n]$;
- $Q \sim \mathcal{O}_d$: **orthogonal transformation**;
- $\rho \in [0,1]$: correlation.

Challenge: Both π and Q are unknown.

Equivalent to **geometric graph alignment**: observing XX^T and YY^T.



Goal: Recover π almost exactly.

Procrustes-Wasserstein Matching: Fundamental Limit & Efficient Algorithm

Xiaochun Niu, Tselil Schramm, Jiaming Xu

Focus on **High-Dimensional Regime:** $d \gg \log n$, $n \to \infty$.

	Previous Results	**Our Work**
Statistical	$\rho^2 > 1 - o(1)$	$\rho^2 \gtrsim \frac{\log n}{d}, \ \rho^2 \gg \frac{d}{n}$ (nearly sharp)
Computational	$\rho^2 > 1 - n^{-c}$	$\rho^2 > \sqrt{\alpha} \approx 0.581$

Polynomial-time estimator:

- Assign each data point a **signature** based on **counting rooted bipartite trees**;
- These signatures exhibit much higher correlation for true pairs than for false ones.

Post-selection inference with a single realization of a network

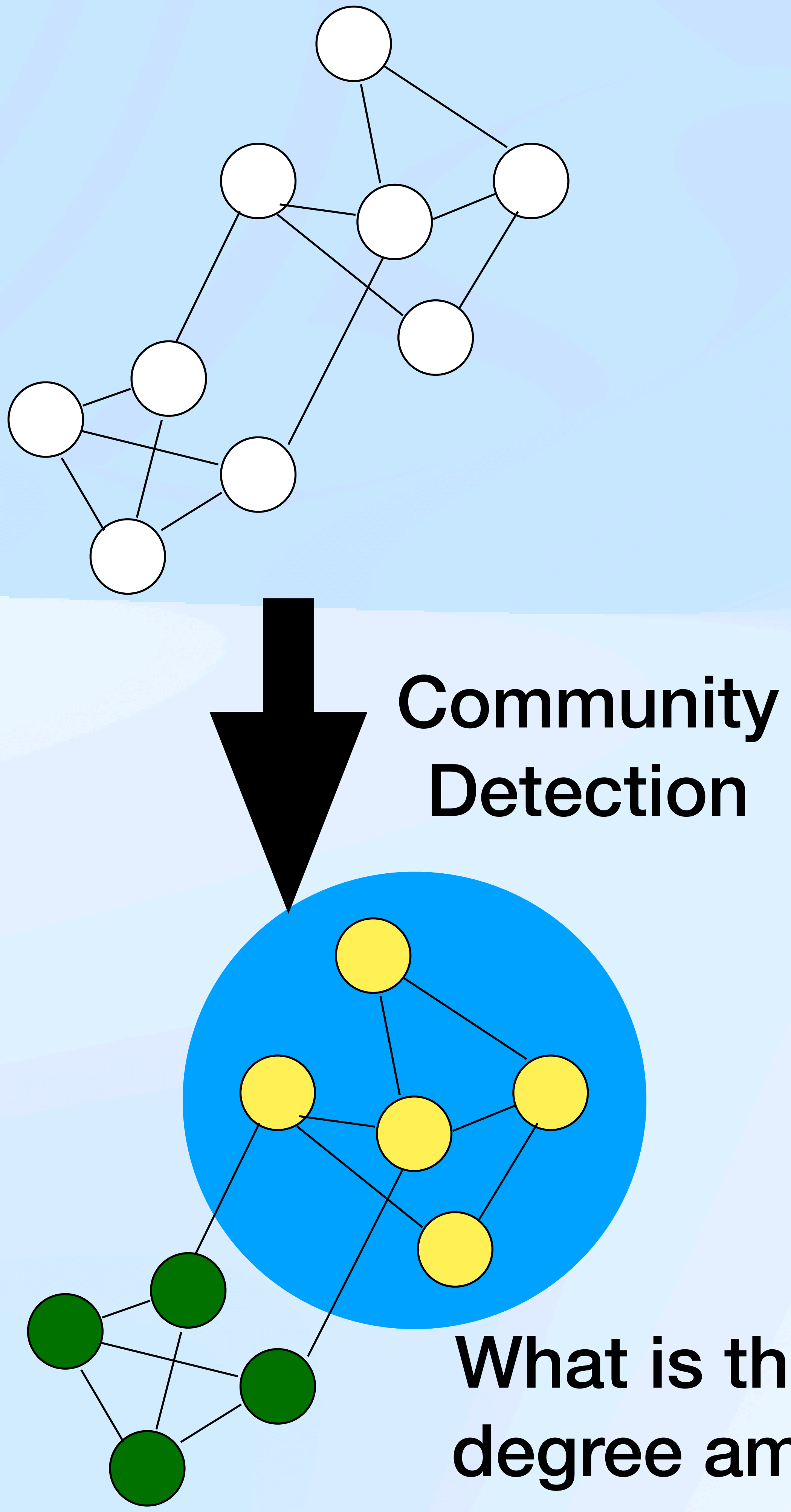
Dan Kessler; www.dankessler.me; kesslerd@unc.edu
Department of Statistics & Operations Research / School of Data Science & Society
University of North Carolina at Chapel Hill

Joint work with Ethan Ancell and Daniela Witten
Department of Statistics / Department of Biostatistics
University of Washington

DK supported by NSF grants DMS-1646108 and DMS-2303371, the Pacific Institute for the Mathematical Sciences, the Simons Foundation, and the eScience Institute at the University of Washington.

Inference after Community Detection

arXiv:2508.11843

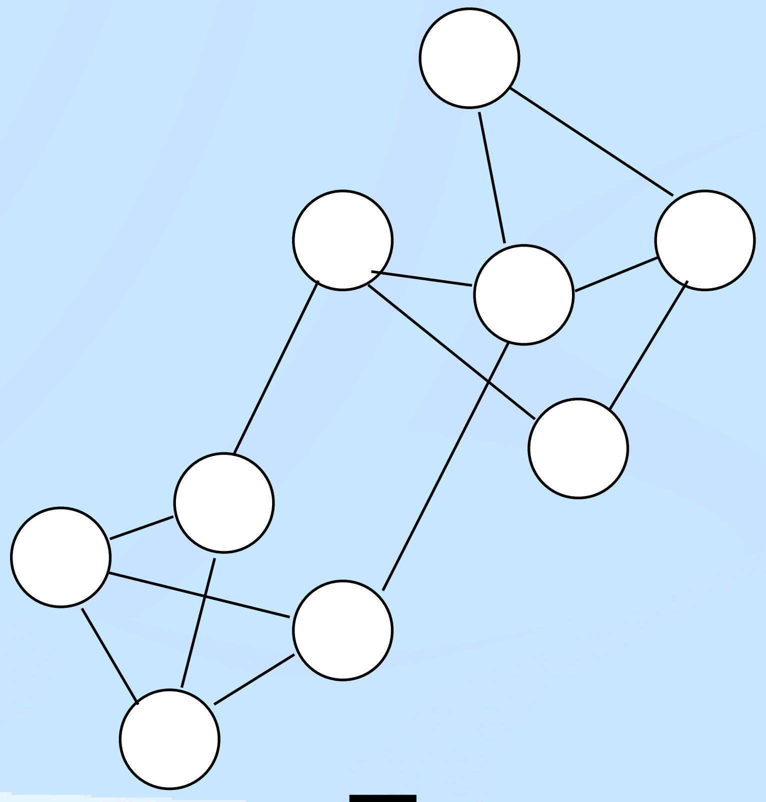


What is the expected degree among yellow nodes?

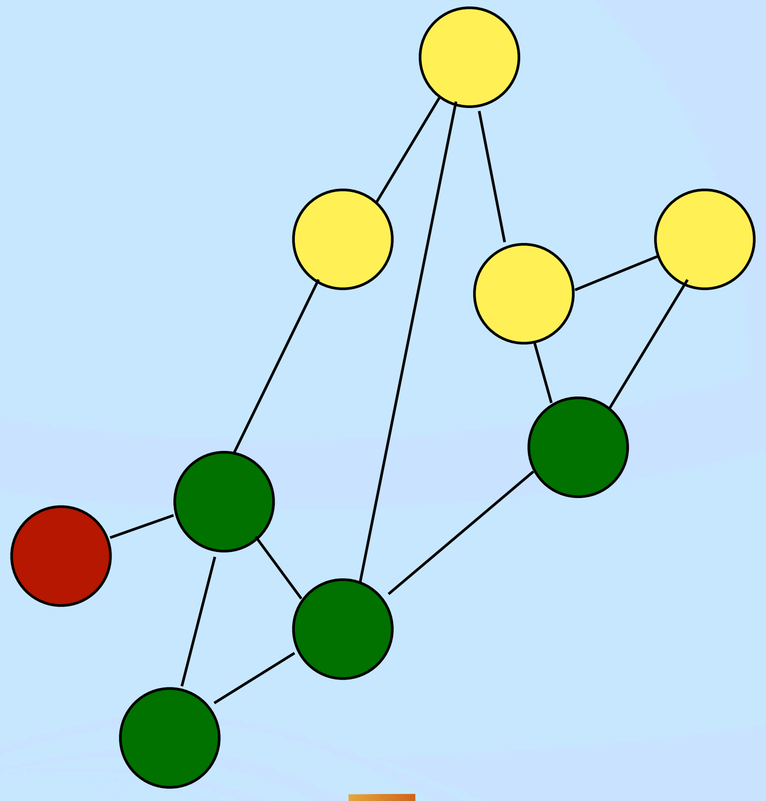
Inference after Community Detection

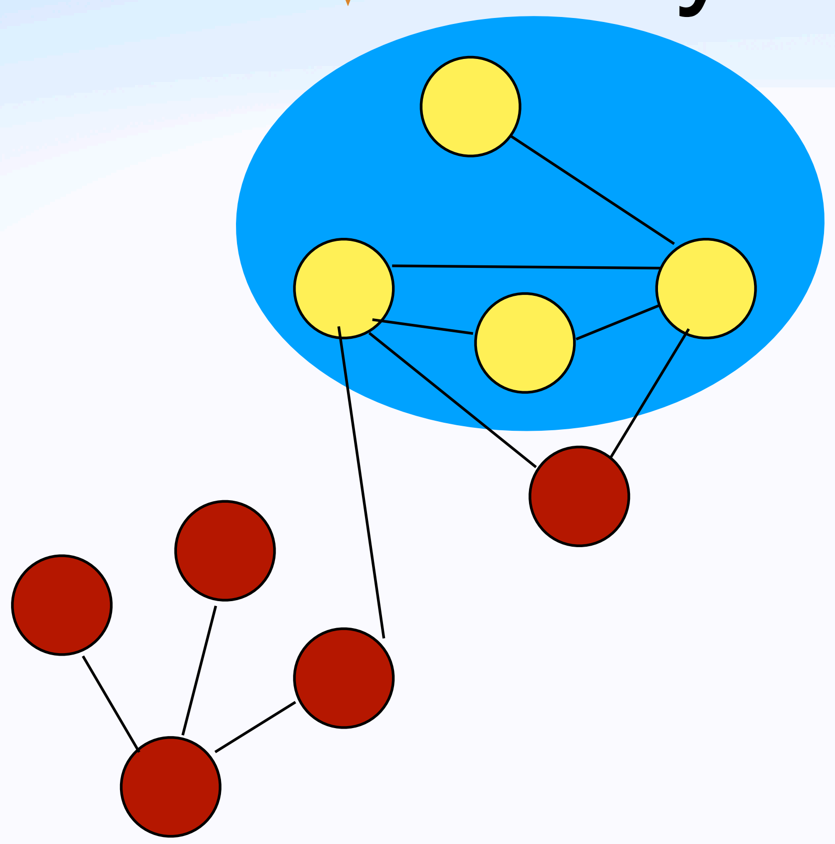
On a Single Network

[arXiv:2508.11843]



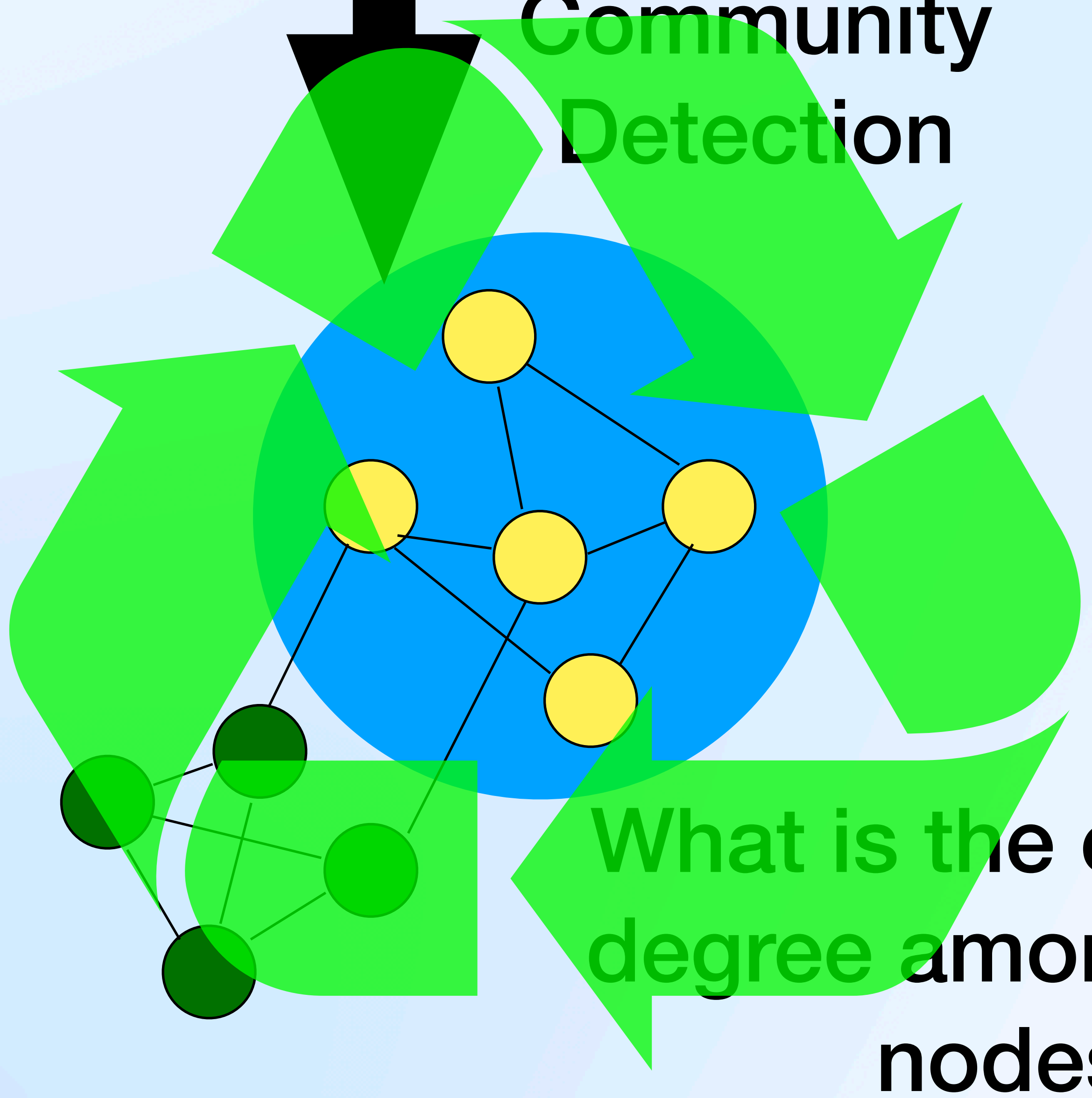
Data Thinning/ Fission



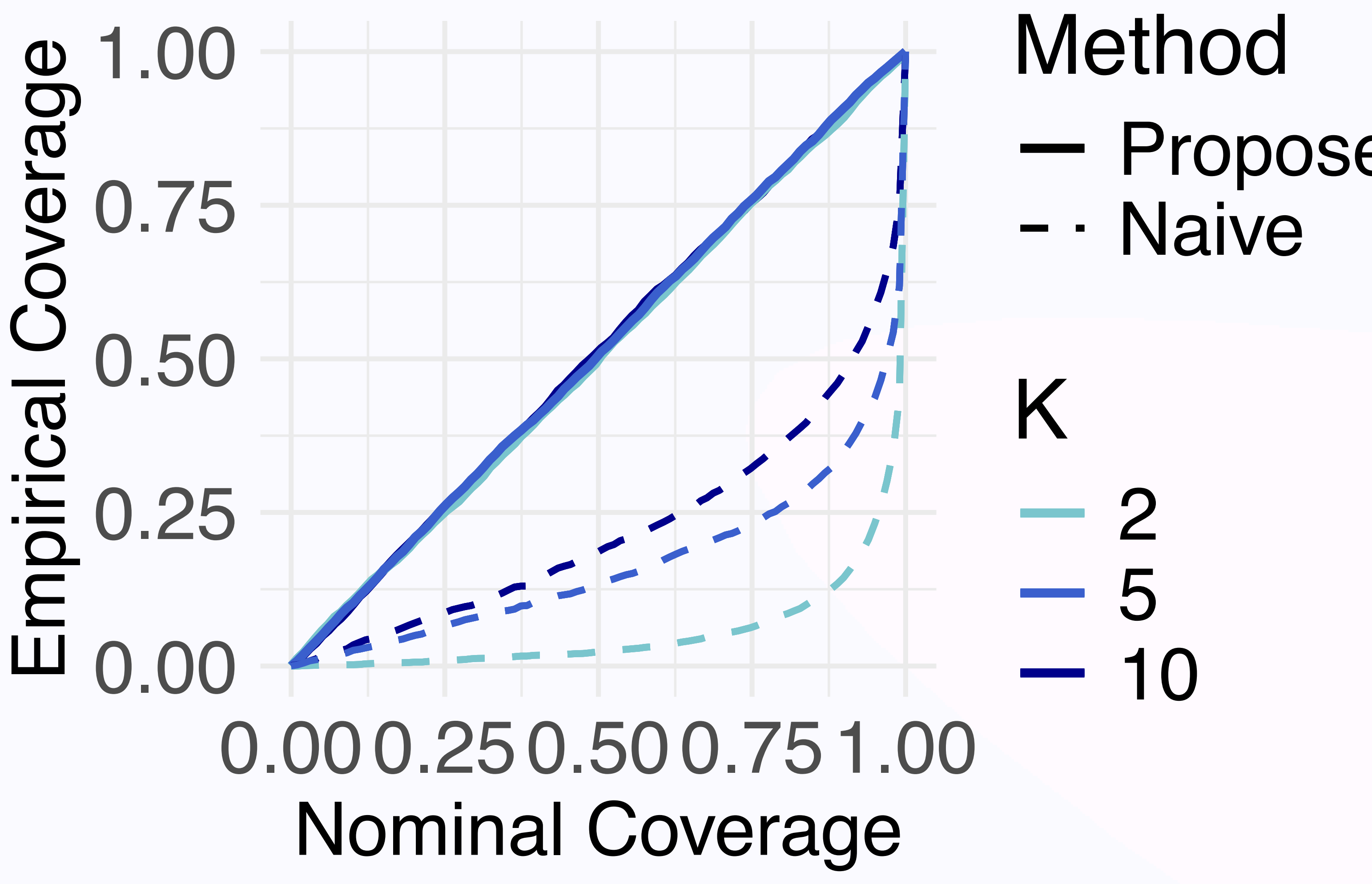


Inference on yellow nodes

Community Detection



What is the expected degree among yellow nodes?



Extreme value theory for singular subspace estimation in the matrix denoising model

Junhyung Chang & Joshua Cape

University of Wisconsin–Madison

IMSI Recent Advances in Random Networks Workshop

Preprint at arXiv:2507.19978

Spectral two-to-infinity distributional theory

- Model: $\widehat{\mathbf{M}} = \mathbf{M} + \mathbf{E} \in \mathbb{R}^{n \times m}$.
- Goal: find normalizing sequences $(a_n)_{n \geq 1}$ and $(b_n)_{n \geq 1}$ such that

$$a_n^{-1}(\|\widehat{\mathbf{U}}\mathbf{R} - \mathbf{U}\|_{2,\infty} - b_n) \rightsquigarrow \mathcal{G}$$

  as $n \to \infty$.
- Non-asymptotic CDF convergence rate to standard Gumbel?
- Convergence under debiased plug-in estimates for signal singular values?
- Hypothesis testing (power analysis)?

$$\mathrm{H}_0 : \mathbf{U} = \mathbf{U}_0 \quad \text{v.s.} \quad \mathrm{H}_\mathrm{A} : \mathbf{U} = \mathbf{U}_1.$$

- Non-Gaussian noise?

Applications

- Powerful against alternatives $\mathbf{U}_1$ that differ from the null $\mathbf{U}_0$ only in a few entries/rows (compared to ℓ_2 or Frobenius norm-based spectral tests).
- Maximum-based spectral test for testing

$$\mathrm{H}_0 : \text{Erdös-Reyni graph} \quad \text{v.s.} \quad \mathrm{H}_\mathrm{A} : \text{Planted dense subgraph}$$

in the setting of Arias-Castro and Verzelen (2014). Powerful for detecting small dense subgraphs (compared to CLT-based tests).

Please see poster for details!
Thank you.

A Unified Framework for Community Detection and Model Selection in Blockmodels

Subhankar Bhadra

Penn State University

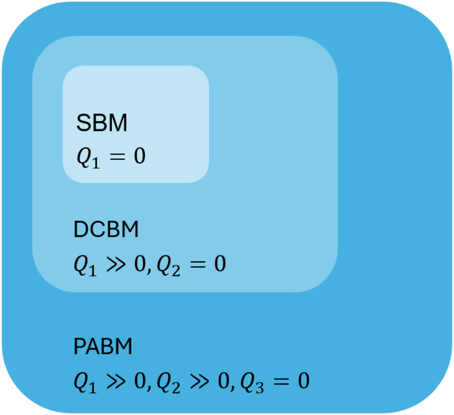
January 12, 2026

Spectral geometry of blockmodels

The spectral embeddings $\{U_i\}$ are a collection of

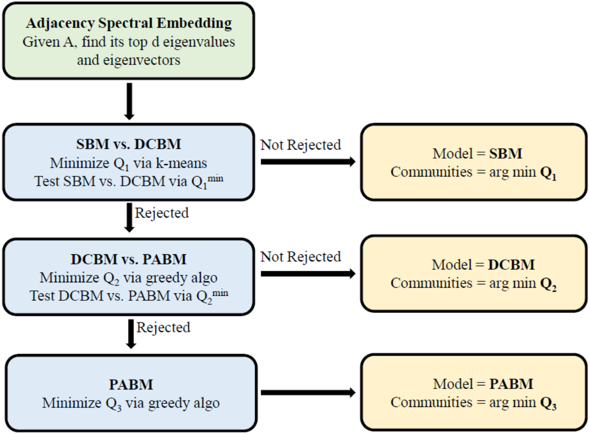
- **point masses** for the SBM
- **rays** for the DCBM
- ***K*-dimensional subspaces** for the PABM

Loss functions: Q_1, Q_2, and Q_3



Nested structure of blockmodels

A two-step testing procedure



Link: doi.org/10.1080/10618600.2025.2590073