

Percolation Property of 2D Brownian Motion: CFT Perspective and Exact Solvability

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Lesson 1: Non-probability measure and partition function

- We should view SLE/Brownian path measures as non-probability measures, with partition function being the total mass.
- Markovian property & restriction property become more transparent.

Influence on my work

- Appendix B of *Minkowski content of Brownian cut points* (**Holden-Lawler-Li-S.**): Brownian path measures as non-probability measures; path-decomposition identities.
- **Ang-Holden-S.**: several works connecting conformal welding and Liouville CFT
 - (1). LQG surfaces as non-probability measures with partition functions given by LCFT.
 - (2). conformal welding = concatenating LQG surfaces as for Brownian path measures.

Lesson 2: natural measures and Green function

- conformal invariant random fractals usually carry a natural measure: local and conformal covariant; Minkowski-content; scaling limit of counting measure.
- Green functions (averaged density of these measures) are interesting observables.

Influence on my work

- A key ingredient in the Cardy embedding project by **Holden-S.**: quantum natural measure on pivotal points = GMC over Euclidean natural measure. (Euclidean natural measure on pivotal points with Xinyi Li based on [HLLS])
- **Ang-Cai-S.-Wu**: Green functions of natural measures on CLE loops and clusters realize CFT correlation functions; 3-point functions given by imaginary DOZZ formula.

- The green book on random walk intersections.
(basic reference for **Lawler–S.–Wu** on 4D UST.)

- Lawler-Schramm-Werner's work on non-intersecting exponents for 2D BM.

Starting point of today's talk.

